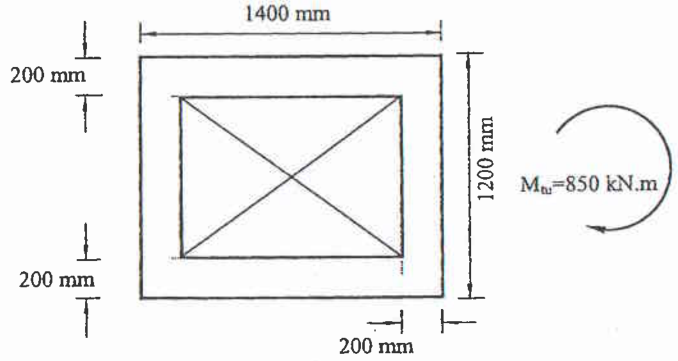
Example 8.3

The figure shown below is for the cross section of a main girder that is subjected to a factored torsional moment of a value of 850 kN.m. It is required to design the girder for torsion.

Data

f_{cu} = 25 N/mm^2

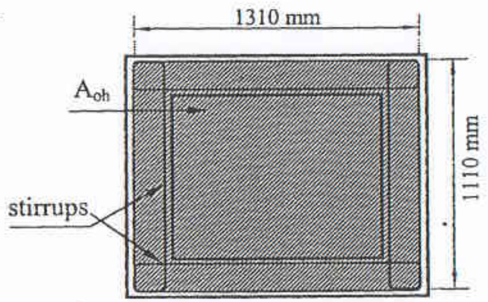
f_{yst} = 360 N/mm^2, f_y=360 N/mm^2



Solution

Step 1: Section properties

Assume concrete cover of 45 mm to the *centerline* of the stirrup all around the cross section



$p_h = 2(1310+1110) = 4840\ mm$

$A_{oh} = 1310 \times 1110 = 1454100\ mm^2$

$A_o = 0.85 A_{oh} = 0.85 \times 1454100 = 1235985\ mm^2$

$t_e = \frac{A_{oh}}{P_h} = \frac{1454100}{4840} = 300.4\ mm > t_{actual}\ (200\ mm)$

Use $t_e = t_{actual}$ = 200 mm

Step 2: Calculations of shear stress due to torsion

$$q_{tu} = \frac{M_{tu}}{2 \times A_o \times t_e} = \frac{850 \times 10^6}{2 \times 1235985 \times 200} = 1.72\ N/mm^2$$

$$q_{tu,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{25}{1.5}} = 0.24\ \ N/mm^2$$

Since $q_{tu}(1.72) > q_{tumin}$ (0.24) then torsion should be considered.

Step 3: Check the adequacy of the cross-section dimensions

$$q_{tu,max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

$$q_{tu,max} = 0.70\sqrt{\frac{25}{1.5}} = 2.86\ N/mm^2 < 4.0\ N/mm^2$$

Since $q_{tu}(1.72) < q_{tu,max}(2.86)$, , the cross section dimensions are adequate.

Step 4: Reinforcement for torsion

A- Stirrups reinforcement

According to clause (4-2-3-5-b) in the code, the spacing of the stirrups should be smaller of: $P_h/8$ (605) mm or 200 mm, <u>try spacing of 200 mm</u>

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst}/\gamma_s} = \frac{850 \times 10^6 \times 200}{2 \times 1235985 \times 360/1.15} = 219.68\ mm^2$$

For box sections having a wall width less than b/6, the code permits dividing the obtained area of stirrups between the two sides of the wall.

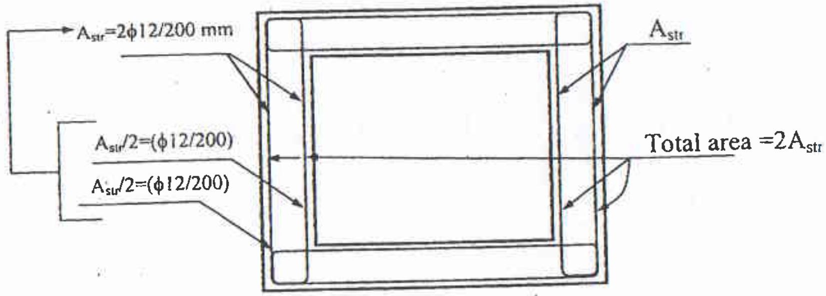
For the two vertical walls (webs) $t_w(200) \le \frac{1200}{6}$

For the two horizontal walls (flanges) $t_w(200) < \frac{1400}{6}$

Hence, area of the cross section of the stirrup on each side of the wall will be equal to 219.68/2=109.8 mm^2.

Thus choose ϕ12/200 mm

i.e. A_{str}=2x113=266 mm^2 > $A_{str,required}$ (219.68 mm^2)



$$A_{st,min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{360}(2\times 200)\times 200 = 89\ mm^2$$

The minimum area of steel for torsion is: $2A_{str} \geq A_{st,min}$

$4\times 112 > A_{st,min}(89)\quad ok$

Final design ϕ12/200 mm

B-Longitudinal Reinforcement

The area of the longitudinal steel is given by:

$$A_{sl} = \frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right) = \frac{219.7\times 4840}{200}\left(\frac{360}{360}\right) = 5316\ mm^2$$

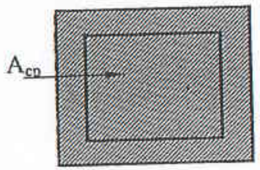
Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$, (use the chosen A_{str})

$$A_{sl,min} = \frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/1.15} - \frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6\times f_{yst}}$ (code 4-2-3-5-c)

$$\frac{219.7}{200} \geq \frac{1400}{6\times 360} \ldots o.k$$

$$A_{cp} = 1400\times 1200 = 1680000\ mm^2$$

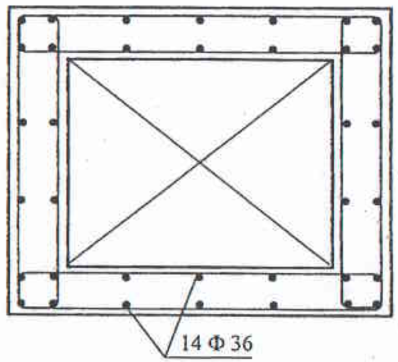


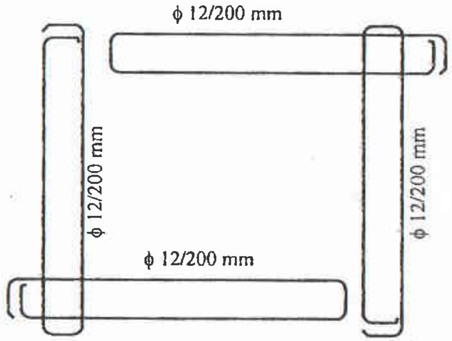
$$A_{sl,min} = \frac{0.4\sqrt{\frac{25}{1.5}}\times 1680000}{360/1.15} - \frac{219.7\times 4840}{200}\left(\frac{360}{360}\right) = 3447\ mm^2$$

Since $A_{sl} > A_{sl,min}$ ...o.k

The bar diameter chosen should be greater than 12mm or s/15(13.3 mm)

Choose 36 Φ 14(5541 mm^2).





Example 8.6

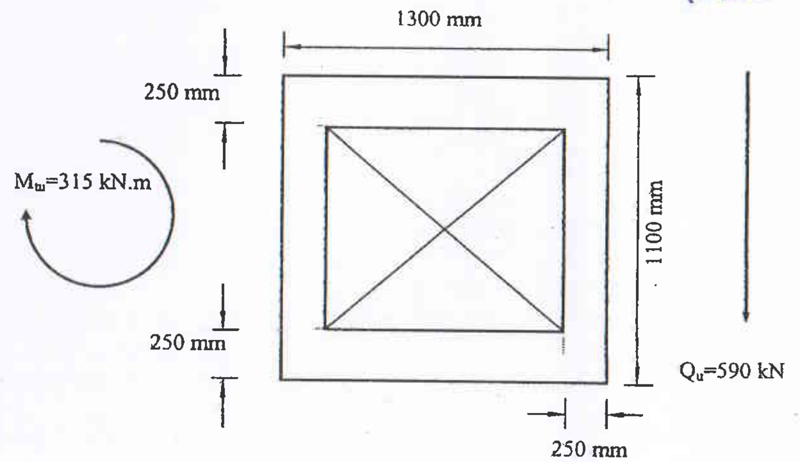
The box section shown in figure is subjected to combined shear and torsion. Check the adequacy of the concrete dimensions and design both web and longitudinal reinforcement.

Data

f_{cu} = 30 N/mm^2

f_{yst} = 240 N/mm^2, f_y=400 N/mm^2

Combined shear and Torsion



Solution

Step 1: Shear and Torsional Stresses

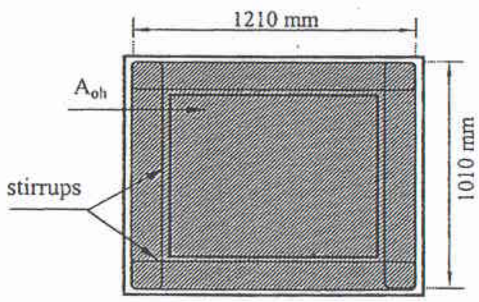
Step 1.1: Shear Stresses

For calculating shear stresses, only the web width will be considered thus: b=250+250=500mm

$$q_u = \frac{Q_u}{b \times d} = \frac{590 \times 1000}{500 \times 1050} = 1.124\ N/mm^2$$

Step 1.2: Torsional Stresses

Assume concrete cover of 45 mm to the *centerline* of the stirrup all around the cross section



$p_h = 2(1210+1010) = 4440\ mm$

$A_{oh} = 1210 \times 1010 = 1222100\ mm^2$

$A_o = 0.85 A_{oh} = 0.85 \times 1222100 = 1038785\ mm^2$

$t_e = \frac{A_{oh}}{p_h} = \frac{1222100}{4440} = 275.2\ mm > t_{actual}(250\ mm)$

Use $t_e = t_{actual} = 250$ mm

$M_{tu} = 315$ kN.m

$$q_{tu} = \frac{M_{tu}}{2 \times A_o \times t_e} = \frac{315 \times 10^6}{2 \times 1038785 \times 250} = 0.606\ N/mm^2$$

$$q_{tu,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{30}{1.5}} = 0.27$$

Since $q_{tu}(0.606) > q_{tumin}$ (0.27) then torsion can not be neglected

Step 2: Check the adequacy of the cross-section dimensions

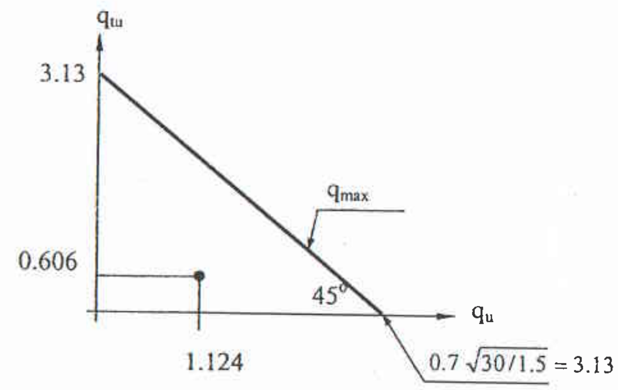
For box section, use the following equations is applied

$q_{max} = 0.7\sqrt{30/1.5} = 3.13 \quad \leq 4\ N/mm^2$

$q_{tu} + q_u \leq q_{max}$ ⇒ check max

$0.606 + 1.124 = 1.73 \leq 3.13$ ....o.k

Since the previous equation is satisfied, the cross section dimensions are adequate for resisting combined shear and torsion.



Graphical representation of the interaction of the maximum stresses due to shear and torsion for box sections

Step 3: Design of closed stirrups for shear and torsion

Step 3.1: Area of stirrups for shear

The concrete shear strength q_{cu} equals

$$q_{cu} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since the applied shear (1.124)is greater than q_{cu} (1.07)shear reinforcement is needed

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.124 - \frac{1.07}{2} = 0.59\ N/mm^2$$

Assume spacing s=100 mm

$$A_{st} = \frac{q_{su} \times b \times s}{f_y/1.15} = \frac{0.59 \times 500 \times 100}{240/1.15} = 140.67\ mm^2$$

Since two stirrups is used and each one have two branches as shown in figure
Area required for shear for one branch of the stirrup equals $A_{st}/4 = 35.17\ mm^2$

Step 3.2: Area of stirrups for torsion

The area of one branch A_{str}

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst} / \gamma_s} = \frac{315 \times 10^6 \times 100}{2 \times 1038785 \times 240/1.15} = 72.65\ mm^2$$

For box section the code permits (4-2-3-5-b) the use of reinforcement along the interior and exterior sides of each web if the wall thickness t_w is less or equal than the section width/6.

$\because t_w(250) > \frac{1100}{6}$ *and* $t_w(250) > \frac{1300}{6}$, only the external leg is considered in calculations of torsional reinforcement as shown in the figure below.

A_{str} for one branch =72.65 mm^2

الفرع الخارجي من الكانة يعطي الـ Torsion

Step 3.3: Stirrups for combined shear and torsion

A-Flanges

The area of the stirrups is required for torsion only (one branch) =72.65 mm^2

Thus choose ϕ 10/100 mm (78.5 mm^2).

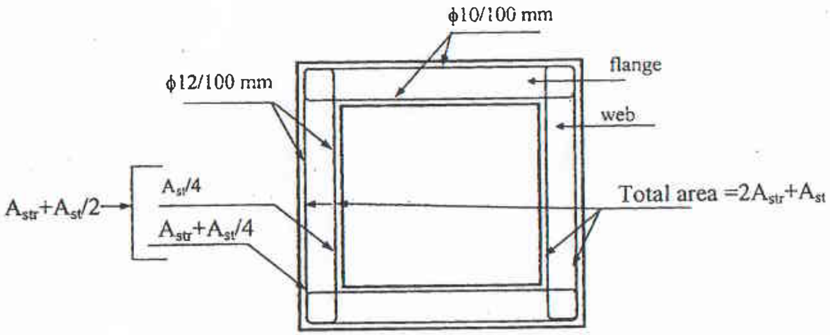
B-Webs

The area required for one branch of the exterior leg for shear and torsion $= A_{str} + A_{st}/4$

$= 72.65 + 35.17 = 107.8$ mm^2

Thus choose ϕ12/100 mm (*113 mm*^{2})

The area required for one branch of the interior leg for shear = 35.17 mm^2



Check A_{smin}

$$2A_{str} + A_{st})_{min} = \frac{0.40}{f_y} b \times s = \frac{0.40}{240} 500 \times 100 = 83.33\ mm^2$$

$A_{st,chosen} = 4 \times 113 = 452\ mm^2 > 83.33$ ......ok

Step 4: Design of Longitudinal reinforcement

$$A_{sl} = \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right) = \frac{72.65 \times 4440}{100}\left(\frac{240}{400}\right) = 1935\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

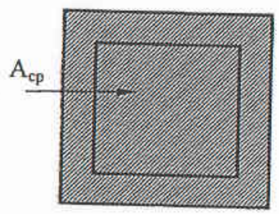
Since the chosen stirrup is for combined shear and torsion, use the calculated A_{str}

$$A_{sl,min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y / \gamma_s} - \left(\frac{A_{str}}{s}\right) \times p_h \left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$ (code 4-2-3-5-c)

$\frac{72.65}{100} < \frac{1300}{6 \times 240}$ thus use $\frac{b}{6 \times f_{yst}}$

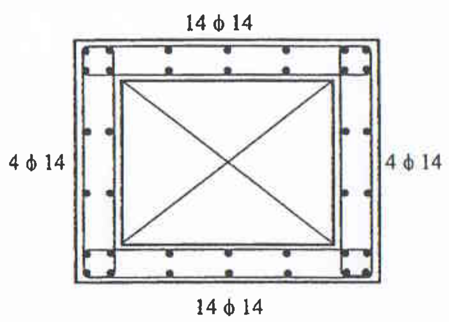
$A_{cp} = 1300 \times 1100 = 1430000\ mm^2$

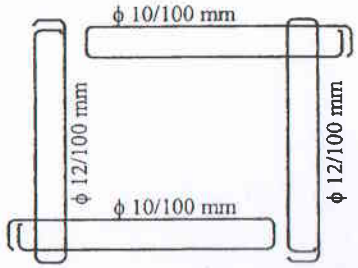


$$A_{sl,min} = \frac{0.40\sqrt{\frac{30}{1.5}} \times 1430000}{400/1.15} - \frac{1300}{6 \times 240} \times 4440 \times \left(\frac{240}{400}\right) = 4949\ mm^2$$

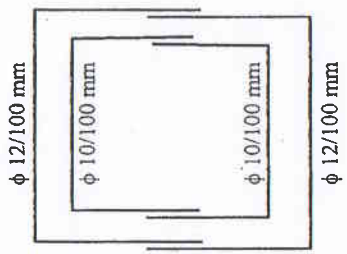
Since $A_{sl} < A_{sl,min}$ use $A_{sl,min}$

The bar diameter chosen should be greater than 12mm or s/15(6.67 mm)
Choose 36 ϕ 14(5541 mm^2) such that the maximum spacing between longitudinal steel is less than 300 mm.





Note : Another alternative for stirrups arrangement is given below. Note also that the internal stirrup is taken as ϕ 10/100 mm since it is only resist shear stresses.



Alternative stirrups detail