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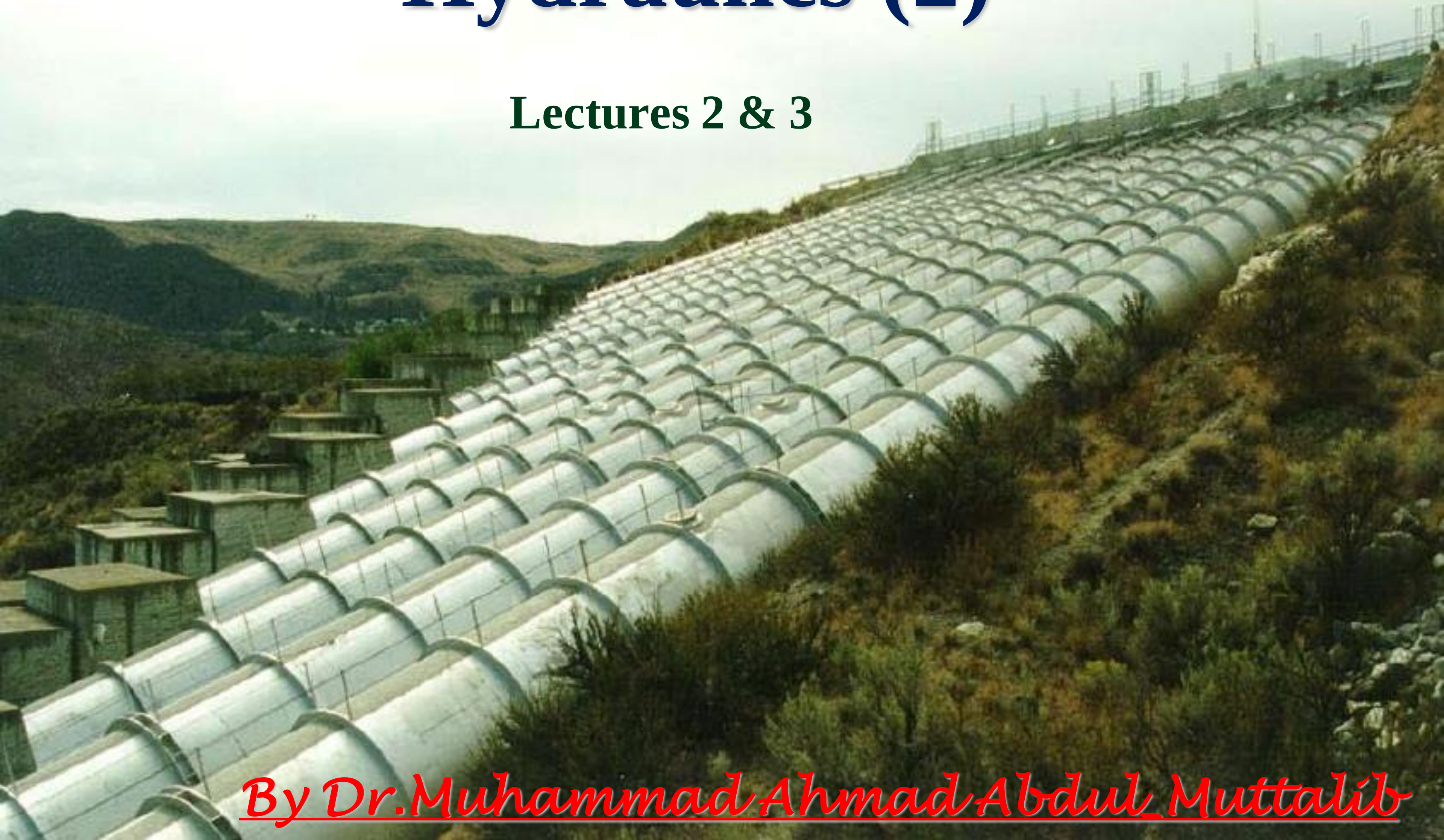
Hydraulics (2)

Lectures 2 & 3



By Dr.Muhammad Ahmad Abdul_Muttalib

Pipe Flow System

- This chapter introduces the fundamental theories of flow in pipelines as well as basic design procedures.
- In this chapter, the pipeline system is defined as a closed conduit with a circular cross-section with water flows (flowing full) inside it.
- It is a closed system, the water is not in contact with air (i.e. no free surface). Flow in a closed pipe results from a pressure difference between inlet and outlet. The pressure is affected by fluid properties and flow rate.
- For circular pipe, D represents the diameter of pipe, R is the pipe radius and L is the pipe length. The cross-sectional area of the pipe can be calculated using $A = \pi R^2$.

Types of Flow

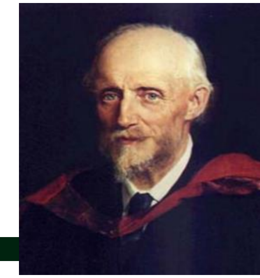
Laminar and Turbulent Flow

- Laminar flow is also referred to as streamline or viscous flow.
  - layers of water flowing over one another at different speeds with virtually no mixing between layers
  - fluid particles move in definite and observable paths or streamlines, and
  - the flow is characteristic of viscous (thick) fluid

Turbulent Flow

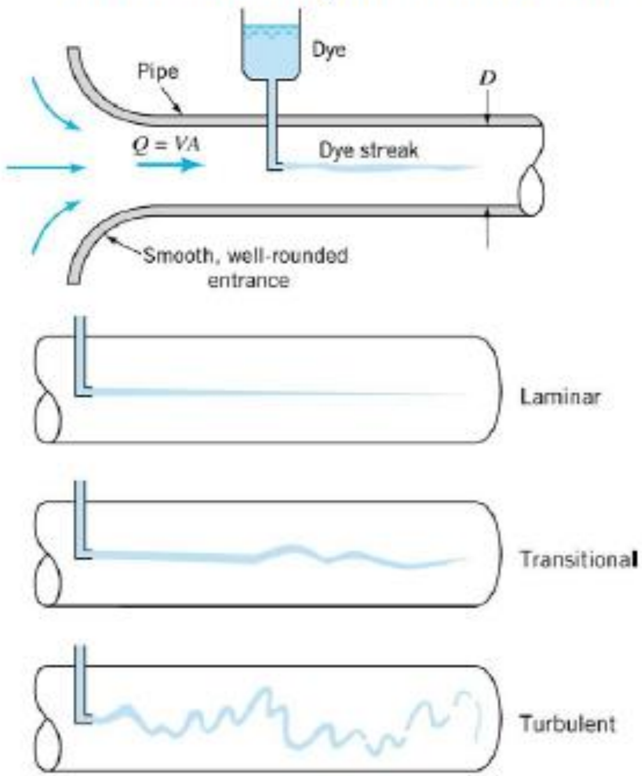
- Turbulent flow is characterized by the irregular movement of particles of the fluid
- There is no definite frequency as there is in wave motion
- The particles travel in irregular paths with no observable pattern and no definite layers

Viscous Pipe Flow: Flow Regime



- Osborne Reynolds (1842-1912)

Osborne Reynolds Experiment to show the three regimes Laminar, Transitional, or Turbulent:



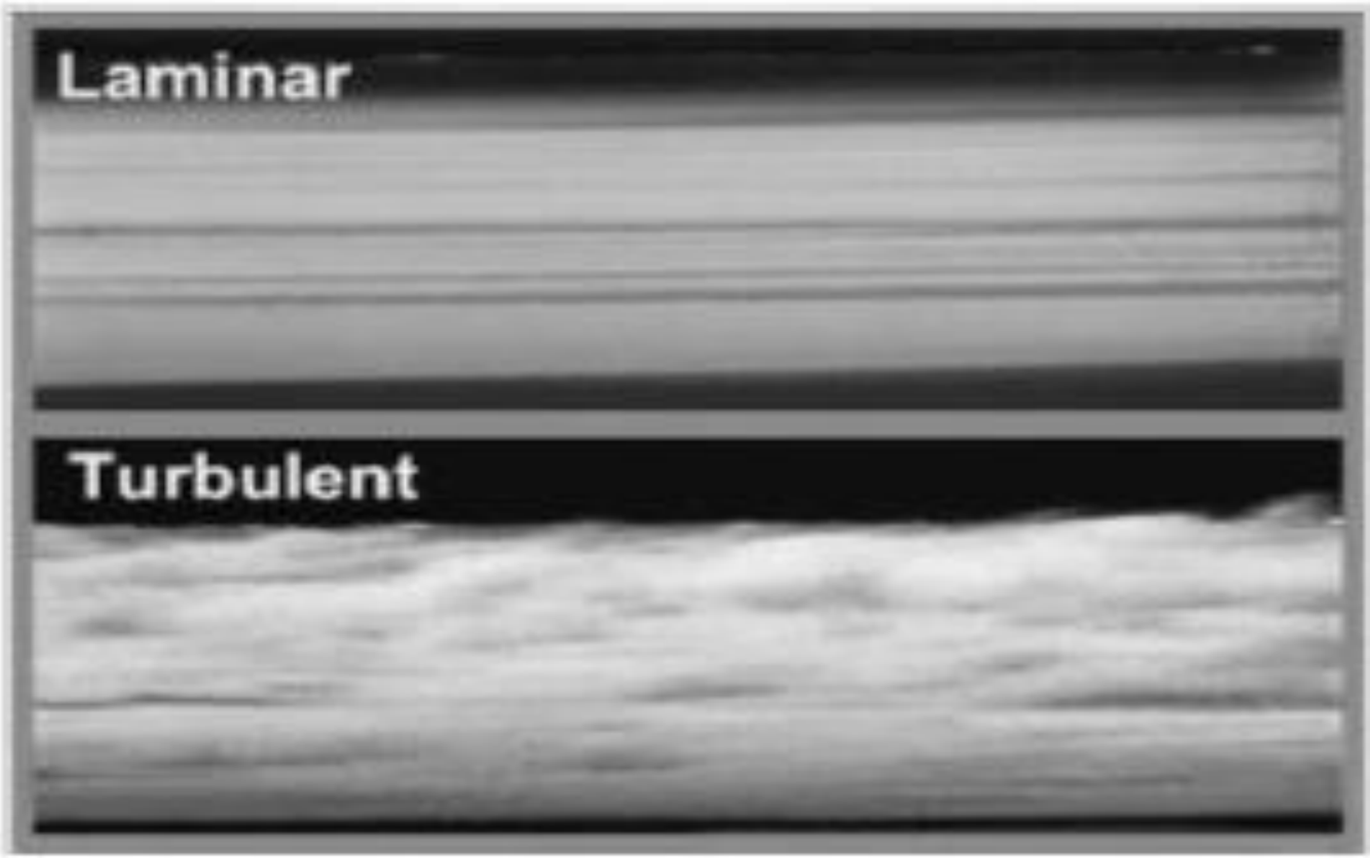
Laminar

Transitional

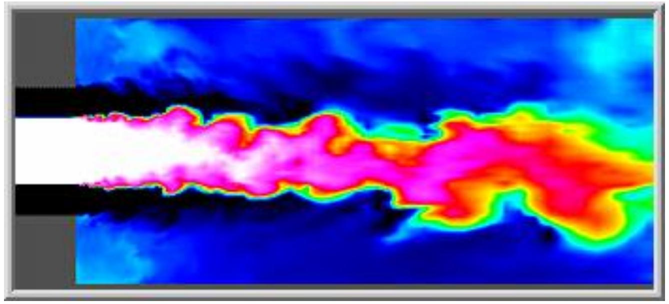
Turbulent

Reynolds Number

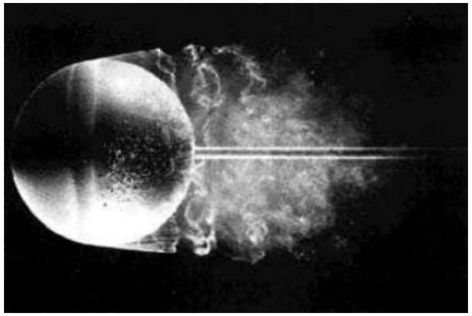
Laminar and turbulent flow visualization



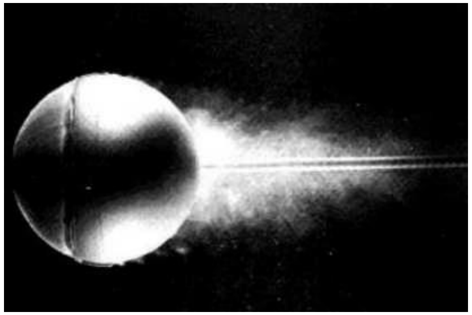
Reynolds Number



Simulation of turbulent flow coming out of a tailpipe



Turbulent flow



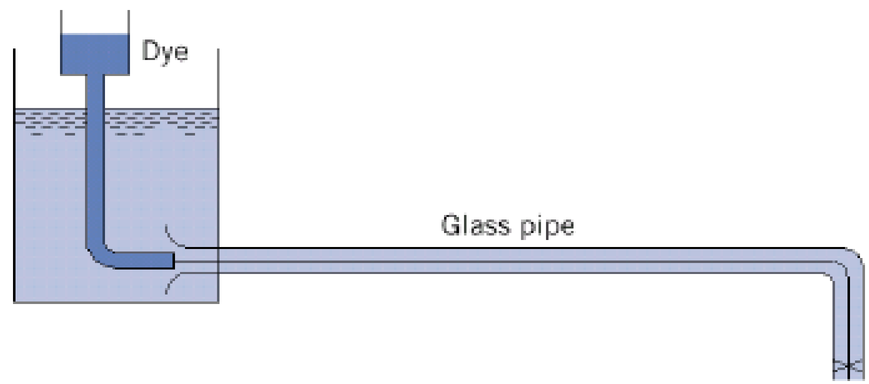
Laminar flow

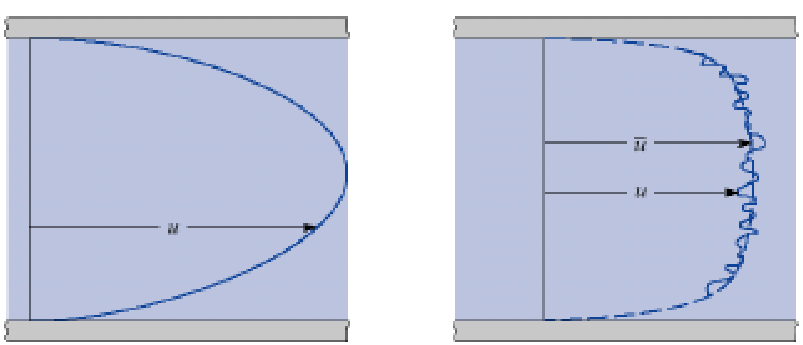
Reynolds Experiment

- Reynolds Number

$$Re = \frac{\rho V D}{\mu}\begin{cases} < 2000 & \text{Laminar flow} \quad h_f \propto V \\ 2000-4000 & \text{Transition flow} \\ > 4000 & \text{Turbulent flow} \quad h_f \propto V^2 \end{cases}$$

- Laminar flow: Fluid moves in smooth streamlines
- Turbulent flow: Violent mixing, fluid velocity at a point varies randomly with time
- Transition to turbulence in a 2 in. pipe is at V=2 ft/s, so most pipe flows are turbulent





(a) (b)

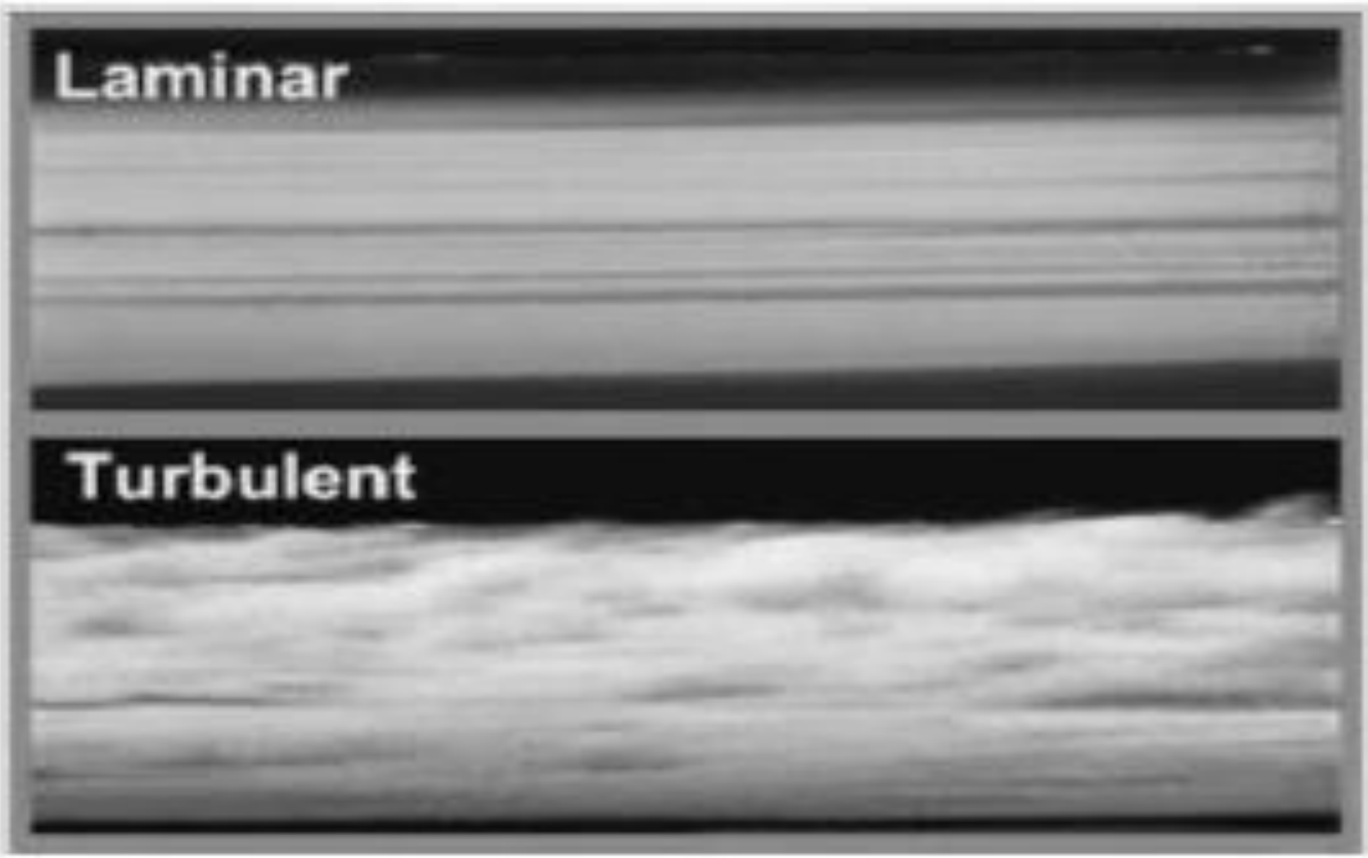
Laminar Turbulent

Reynolds Number

- The Reynolds Number is important in analyzing any type of flow when there is substantial velocity gradient - shear force.
- The Reynolds Number indicates the relative significance of the viscous effect compared to the inertia effect.
- The Reynolds number is defined as the ratio of the inertial force and the viscous force.

Reynolds Number

Laminar and turbulent flow visualization



Reynolds Number

- Reynolds Number can be expressed as:

$$R_e = \frac{LV\rho}{\mu} = \frac{LV}{\nu}$$

L : characteri ctic length

V : velocity

ρ : density

μ : dynamic viscosity or obsolute viscosity

ν : kinematic viscosity

Critical Reynolds Number

$$R_e = \frac{LV\rho}{\mu} = \frac{LV}{\nu} \qquad (1)$$

True critical Reynolds Number

$$R_{crit} \cong 2000 \qquad (2)$$

- For water at 59°F (15°C)
- When D = 1 in $\quad V_{crit}$ = 0.3 fps
- When V = 3 fps $\quad D_{crit}$ = 0.1 in

Critical Reynolds Number

$$R_e = \frac{LV\rho}{\mu} = \frac{LV}{\nu} \qquad (1)$$

True critical Reynolds Number

$$R_{crit} \cong 2000 \qquad (2)$$

- For water at 59°F (15°C)
- When D = 1 in $\quad V_{crit}$ = 0.3 fps
- When V = 3 fps $\quad D_{crit}$ = 0.1 in

Example

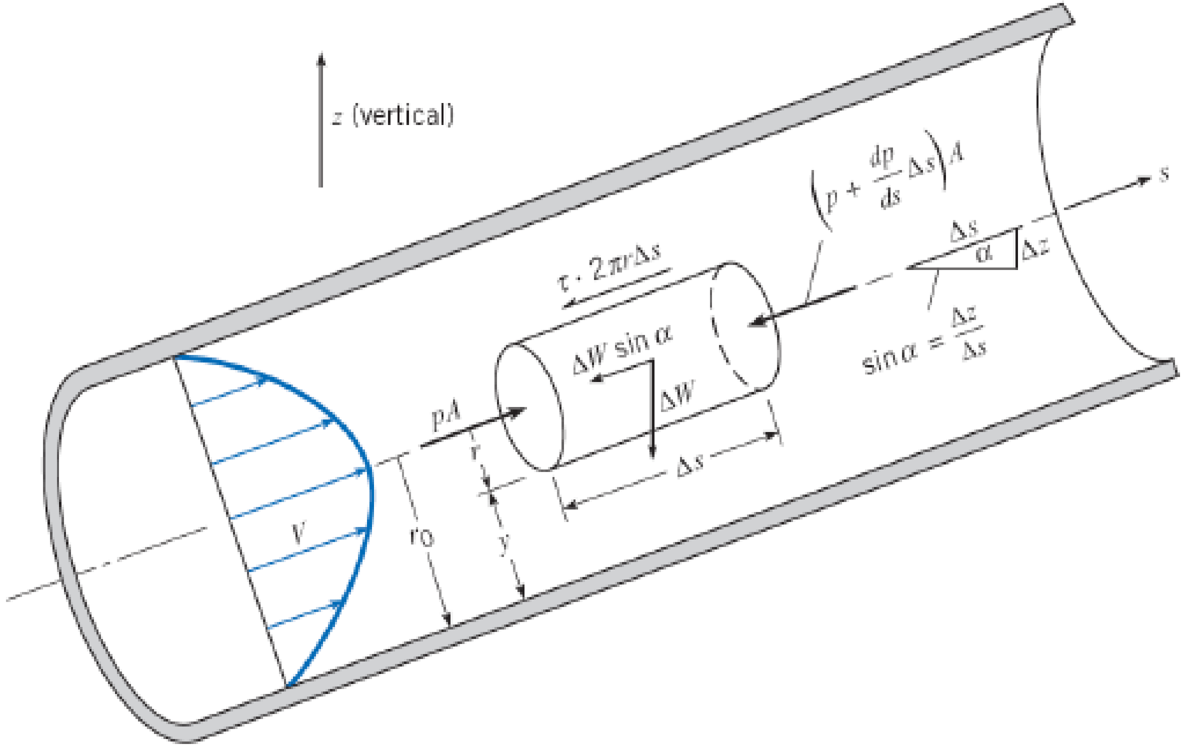
In a refinery oil (S=0.85, $\upsilon=1.8x10^{-5}$ m^2/s) flows through a 100-mm-diameter pipe at 0.5 l/s. Is the flow laminar or turbulent?

$$V=\frac{Q}{\pi\frac{D^2}{4}}=\frac{4(0.0005m^3/s)}{\pi(0.1m)^2}=0.0637m/s$$

$$R_e=\frac{DV}{\upsilon}=\frac{0.1m(0.0637m/s)}{1.8x10^{-5}m^2/s}=354$$

$$R_e<R_{crit}=2000 \qquad \rightarrow \text{the flow is laminar}$$

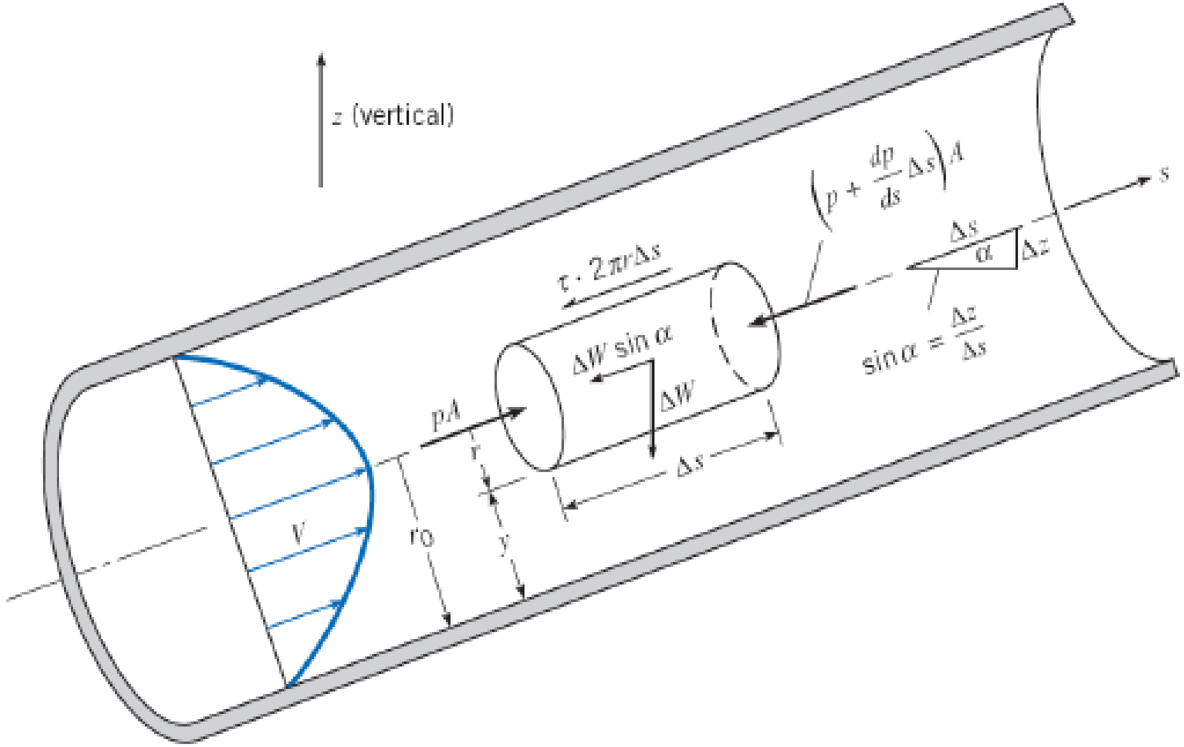
Shear stress distribution across a pipe section



For steady, uniform flow, the momentum balance in s for the fluid cylinder yields

$$\sum F_s = F_{\text{pressure}} + F_{\text{gravity}} + F_{\text{viscous}} = 0$$

Shear stress distribution across a pipe section



For steady, uniform flow, the momentum balance in s for the fluid cylinder yields

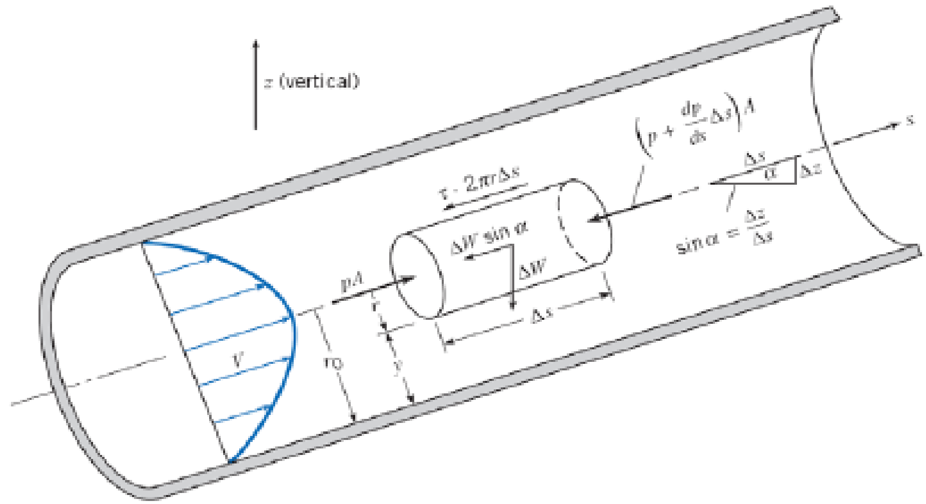
$$\sum F_s = F_{\text{pressure}} + F_{\text{gravity}} + F_{\text{viscous}} = 0$$

$$pA - \left(p + \frac{dp}{ds}\Delta s\right) A - \Delta W \sin\alpha - \tau\,(2\pi r)\,\Delta s = 0$$

with $\Delta W = \gamma A \Delta s$

and $\sin\alpha = \dfrac{dz}{ds}$

we solve for τ to get:



$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

regardless of whether flow is laminar or turbulent.

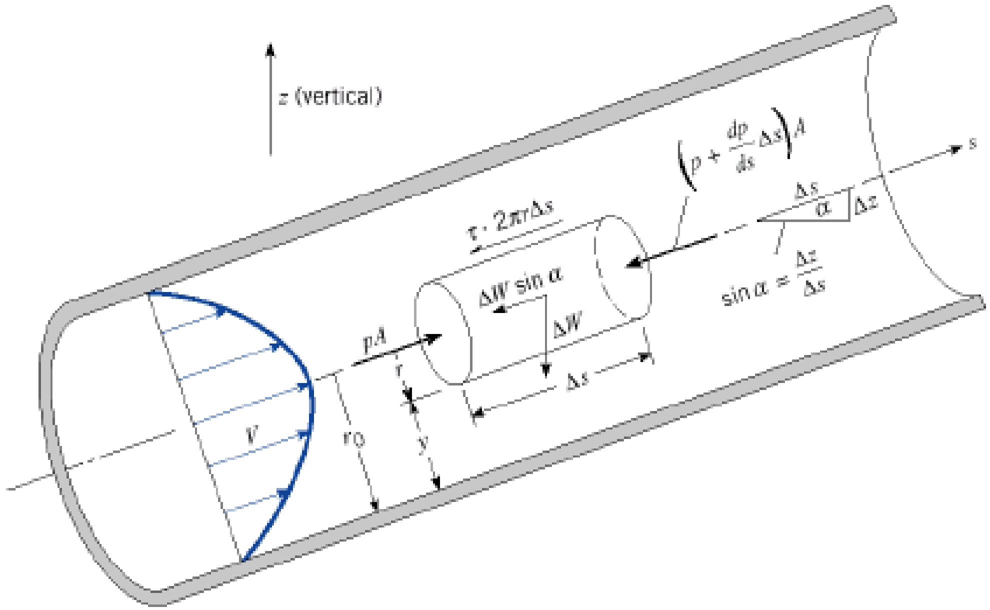
(Technically, turbulent flow is neither uniform nor steady, and there are accelerations; we neglect this).

Shear Stress in Pipes

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}\gamma\left(\frac{p}{\gamma}+z\right)\right]$$

$$\tau = -\frac{r\gamma}{2}\frac{dh}{ds}$$

$$h_1 - h_2 = h_f = \frac{2L\tau}{\gamma r} = \frac{4L\tau}{\gamma D}$$



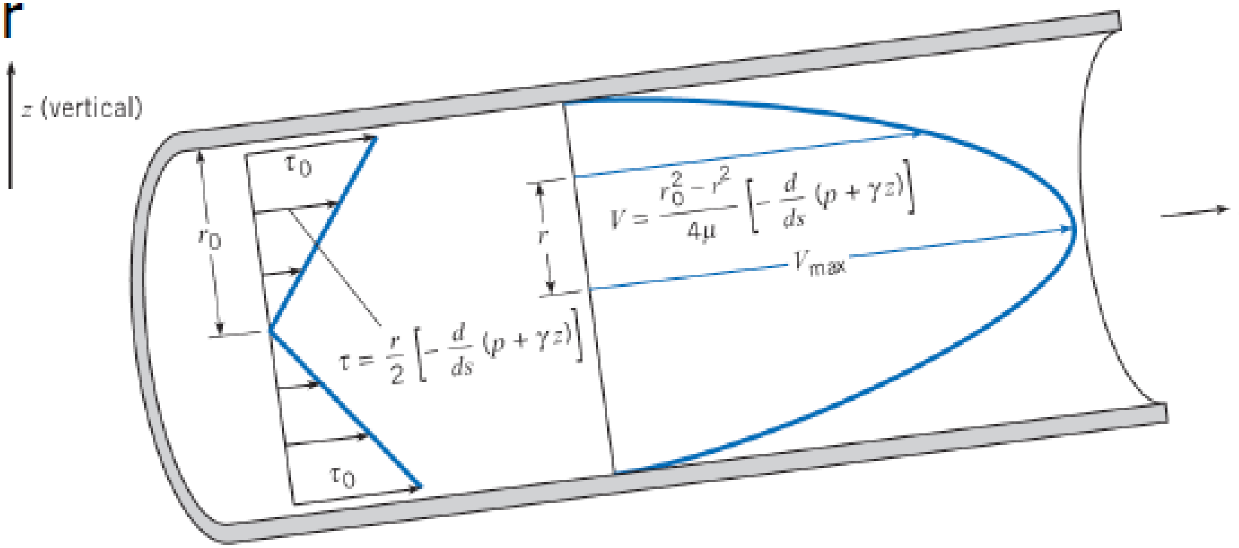
- Since $-dh/ds > 0$, then the shear stress will be zero at the center ($r = 0$) and increase linearly to a maximum at the wall.
- Head loss is due to the shear stress.
- Applicable to either laminar or turbulent flow
- Now we need a relationship for the shear stress in terms of the R_e and pipe roughness

Velocity for laminar flow in pipes

Using the result for τ, we substitute

$$\tau = \mu \frac{dV}{dy}$$

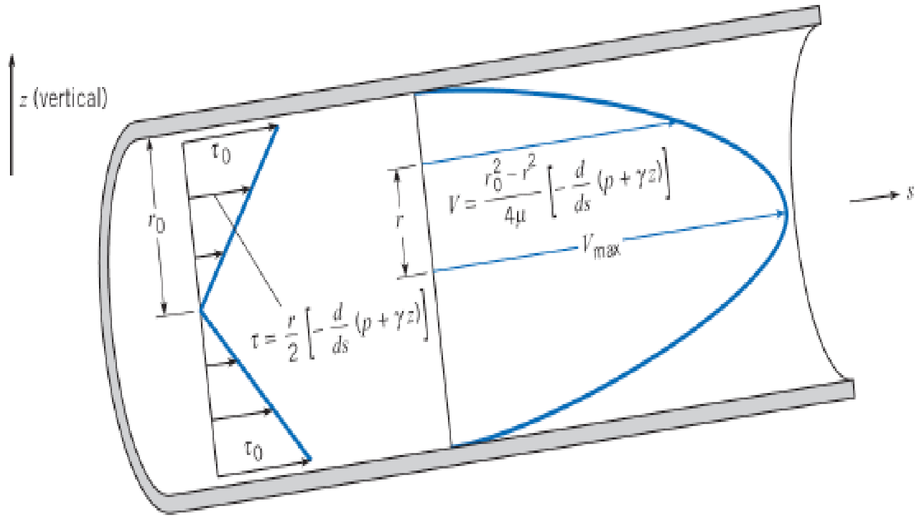
$$= -\mu \frac{dV}{dr}$$



$$\Rightarrow \quad -\mu \frac{dV}{dr} = \frac{r}{2}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

Integration yields $$V = -\frac{r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right] + C$$

Velocity for laminar flow in pipes



The velocity is 0 at the boundary,

One boundary condition:

$$V = 0 \quad \text{at} \quad r = r_0$$

$$\Rightarrow \quad V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

(parabolic profile)

Laminar Flow in Pipes

- Laminar flow -- Newton's law of viscosity is valid:

$$\tau = \mu \frac{dV}{dy} = -\frac{r\gamma}{2}\frac{dh}{ds}$$

$$\frac{dV}{dy} = -\frac{dV}{dr}$$

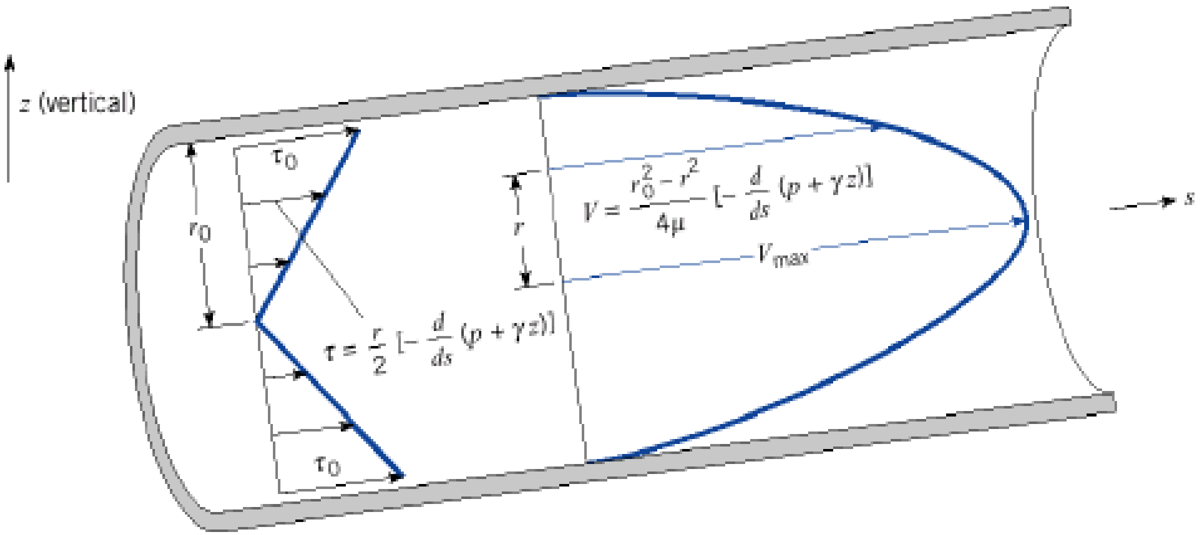
$$\frac{dV}{dr} = \frac{r\gamma}{2\mu}\frac{dh}{ds}$$

$$dV = \frac{r\gamma}{2\mu}\frac{dh}{ds}dr$$

$$V = \frac{r^2\gamma}{4\mu}\frac{dh}{ds} + C \qquad C = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}$$

$$V = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

$$V = V_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$



- Velocity distribution in a pipe (laminar flow) is parabolic with maximum at center

Example

Oil flows steadily in a vertical pipe. Pressure at z=100m is 200 kPa, and at z=85m it is 250 kPa.

Given: Diameter D = 3 cm

Viscosity μ = 0.5 Ns/m^2

Density ρ = 900 kg/m^3

Assume laminar flow.

Is the flow upward or downward? What is the velocity at the center and at r=6mm?

Example: Solution

First determine rate of change of p + γz

$$\frac{d}{ds}(p+\gamma z) = \frac{(p_{100}+\gamma z_{100})-(p_{85}+\gamma z_{85})}{z_{100}-z_{85}}$$

$$= \frac{[200\times 10^3 + 8830\ (100)] - [250\times 10^3 + 8830\ (85)]}{15} = 5.53\ \mathrm{kN/m^3}$$

Since the velocity is given by

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

the flow velocity is negative, i.e., downward.

Example: Solution

The velocity at any point r is found from

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

where we have already determined the value of

$$\frac{d}{ds}(p+\gamma z) = 5.53\ \mathrm{kN/m^3}$$

For r = 0, V = -0.622 m/s
For r = 6 mm, V = -0.522 m/s

Note that the velocity is in the direction of pressure increase. The flow direction is determined by the combination of pressure gradient and gravity. In this problem, the effect of gravity is stronger.

Head loss for laminar flow in a pipe

The mean velocity in the pipe is given by

$$\bar{V} = \frac{Q}{A} = \frac{1}{A}\int V\, dA$$

$$= \frac{1}{\pi r_0^2}\int_0^{r_0} \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right](2\pi r\, dr)$$

$$= \frac{r_0^2}{8\mu}\left[-\frac{d}{ds}(p+\gamma z)\right] \Rightarrow Q = -\frac{\pi \gamma r_0^4}{8\mu}\frac{dh}{ds} = -\frac{\pi r_0^4}{8\mu}\frac{d(p+\gamma z)}{ds}$$

$$= \frac{D^2}{32\mu}\left[-\frac{d}{ds}(p+\gamma z)\right] \Rightarrow Q = -\frac{\pi \gamma D^4}{128\mu}\frac{dh}{ds}$$

Rearranging gives

$$\frac{d}{ds}(p+\gamma z) = -\frac{32\mu\bar{V}}{D^2}$$

which we integrate along s between sections 1 and 2:

$$p_2 - p_1 + \gamma(z_2 - z_1) = -\frac{32\mu\bar{V}}{D^2}(s_2 - s_1)$$

Identify the length of pipe section L = s_2 - s_1

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \frac{32\mu L\bar{V}}{\gamma D^2}$$

This is simply the energy equation for a pipe with head loss

$$h_f = \frac{32\mu L\bar{V}}{\gamma D^2}$$

Head Loss in Laminar Flow

$$\bar{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$

$$\frac{dh}{ds} = -\bar{V}\frac{32\mu}{\gamma D^2}$$

$$dh = -\bar{V}\frac{32\mu}{\gamma D^2}ds$$

$$h_2 - h_1 = -\bar{V}\frac{32\mu}{\gamma D^2}(s_2 - s_1)$$

$$h_1 = h_2 + h_f$$

$$h_f = \frac{32\mu L\bar{V}}{\gamma D^2}$$

$$f = \frac{64}{\mathrm{Re}} \Rightarrow f = \frac{8\tau_w}{\rho\bar{V}^2}$$

$$h_f = \frac{32\mu L\bar{V}}{\gamma D^2}$$

$$= \frac{32\mu L\bar{V}}{\gamma D^2}\frac{\rho\bar{V}^2/2}{\rho\bar{V}^2/2}$$

$$= 64\left(\frac{\mu}{\rho\bar{V}D}\right)\left(\frac{L}{D}\right)\bar{V}^2/2g$$

$$= \frac{64}{R_e}\left(\frac{L}{D}\right)\bar{V}^2/2g$$

$$h_f = f\frac{L}{D}\frac{\bar{V}^2}{2g} \qquad f = \frac{64}{R_e}$$

Example

- Given: Oil ($S = 0.97$, $\mu = 10^{-2}$ lbf-s/ft^2) in 2 inch pipe, $Q = 0.25$ cfs.
- Find: Pressure drop per 100 ft of horizontal pipe.

Solution:

$$V = \frac{Q}{A} = \frac{0.25}{\pi(2/12)^2/4} = 11.46\ ft/s$$

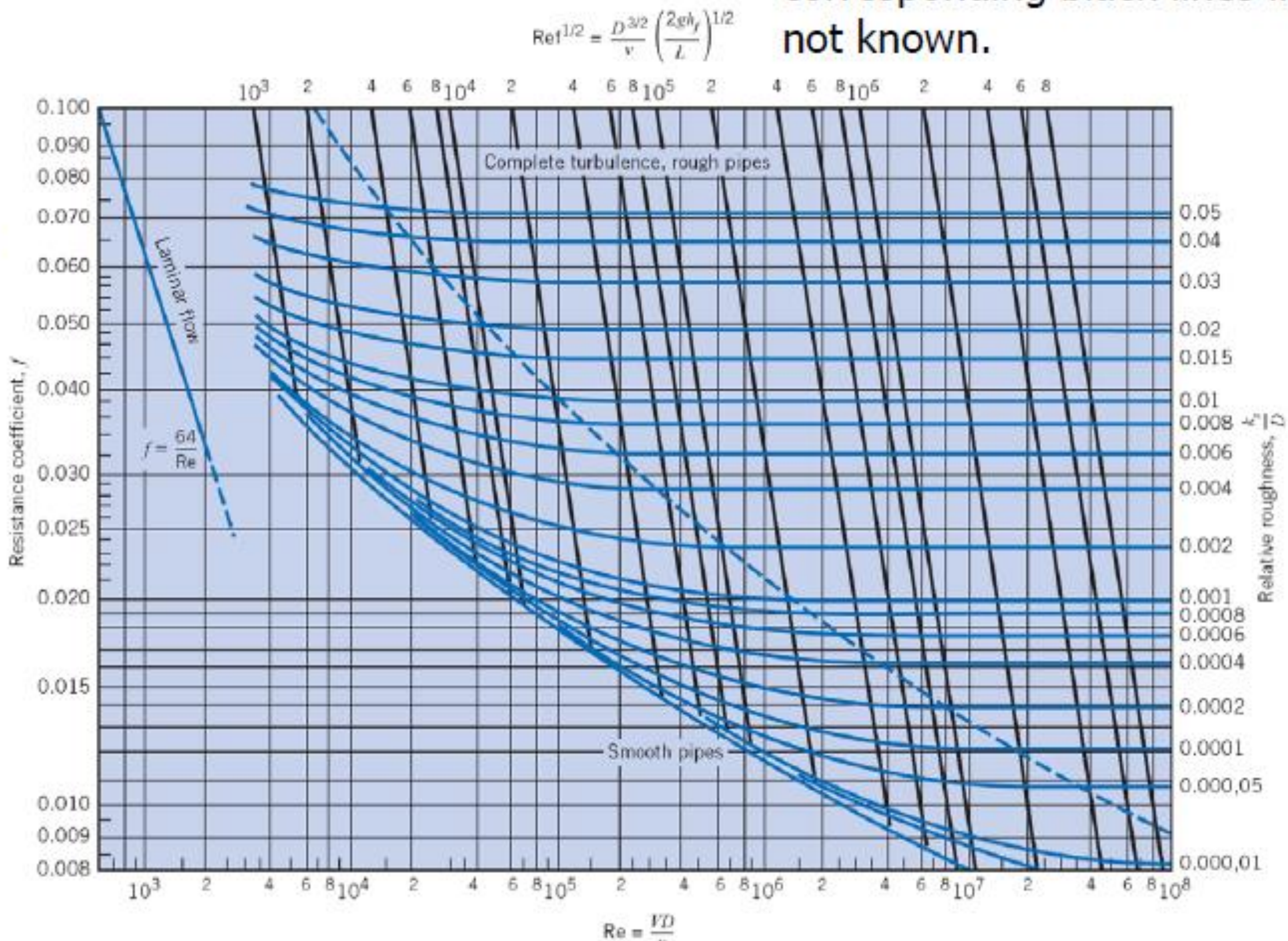
$$R_e = \frac{\rho V D}{\mu} = \frac{(0.97 \times 62.4) \times 11.46 \times (2/12)}{10^{-2} \times 32.2\,(pdl.s/ft^2)} = 360\,(\text{laminar})$$

$$\Delta p = \gamma\, h_f = \frac{32 \mu L V}{D^2} = \frac{32 \times 10^{-2} \times 100 \times 11.46}{(2/12)^2} \times \frac{1}{144} = 91.7\ \text{psi/100 ft}$$

How to find f for rough pipes? Moody diagram:

use this parameter and the corresponding black lines if velocity is not known.

f; the value we're looking for



Reynolds number (if velocity is known)

Get value for ks from table; each value of ks/D corresponds to one of the blue curves

Example:

Find head loss per kilometer of pipe.
Pipe is a 20-cm asphalted cast-iron pipe.
Fluid is water.
Flow rate is $Q = 0.05 \, m^3/s$.

Solution:

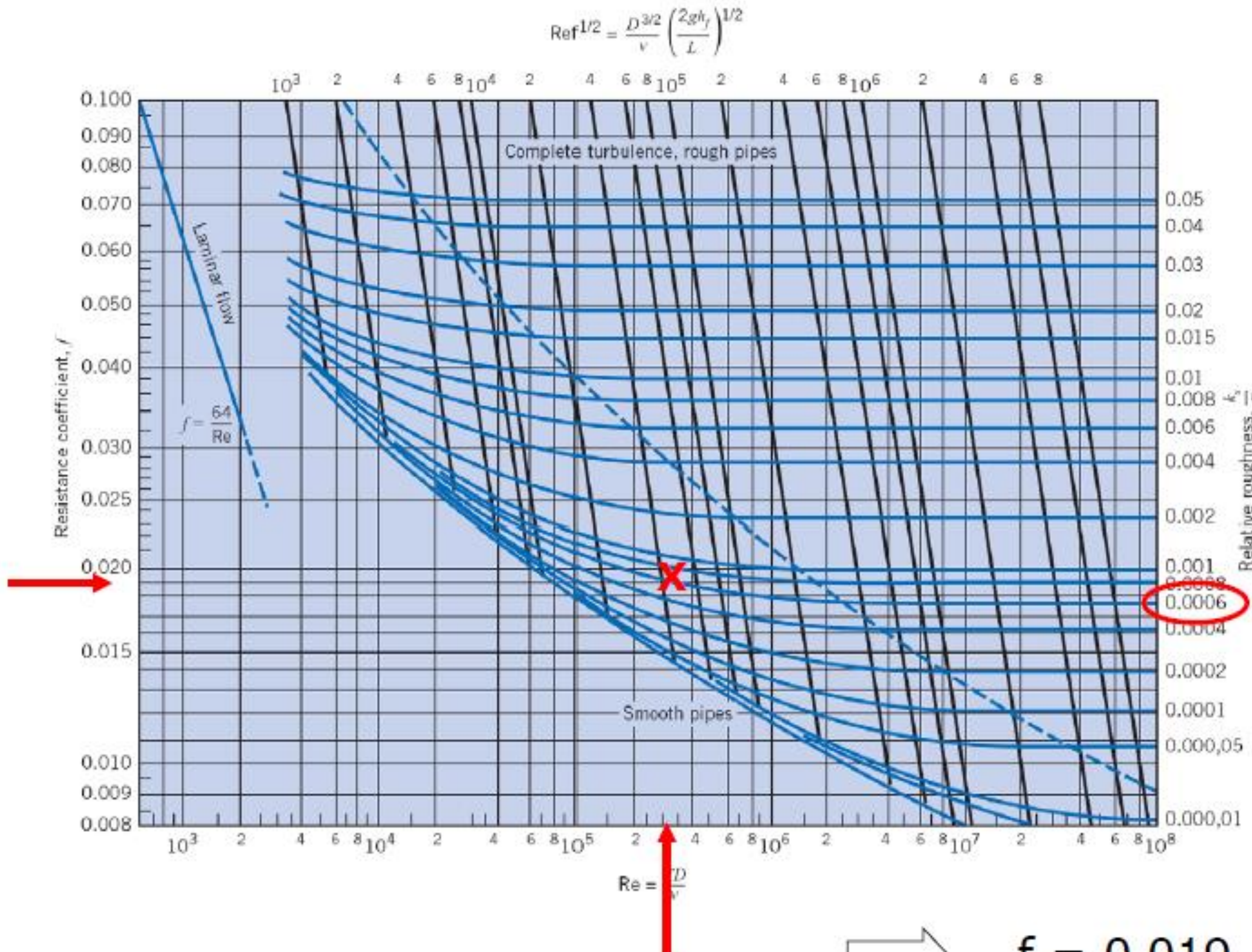
First compute Reynolds number

$$Re = \frac{VL\rho}{\mu} = \frac{QL\rho}{A\mu} = 3.18 \times 10^5$$

From Table, $k_s = 0.12$ mm for asphalted cast-iron pipe.

So, $k_s/D = 0.0006$

Material	Condition	ϵ ft	ϵ mm	Uncertainty, %
Steel	Sheet metal, new	0.00016	0.05	± 60
	Stainless, new	0.000007	0.002	± 50
	Commercial, new	0.00015	0.046	± 30
	Riveted	0.01	3.0	± 70
	Rusted	0.007	2.0	± 50
Iron	Cast, new	0.00085	0.26	± 50
	Wrought, new	0.00015	0.046	± 20
	Galvanized, new	0.0005	0.15	± 40
	Asphalted cast	0.0004	0.12	± 50
Brass	Drawn, new	0.000007	0.002	± 50
Plastic	Drawn tubing	0.000005	0.0015	± 60
Glass	—	Smooth	Smooth	
Concrete	Smoothed	0.00013	0.04	± 60
	Rough	0.007	2.0	± 50
Rubber	Smoothed	0.000033	0.01	± 60
Wood	Stave	0.0016	0.5	± 40



$Ref^{1/2} = \frac{D^{3/2}}{\nu}\left(\frac{2gh_f}{L}\right)^{1/2}$
Complete turbulence, rough pipes
Laminar flow
$f = \frac{64}{Re}$
Smooth pipes
Resistance coefficient, f
Relative roughness, $\frac{k_s}{D}$
Re = $\frac{VD}{\nu}$
0.100
0.090
0.080
0.070
0.060
0.050
0.040
0.030
0.025
0.020
0.015
0.010
0.009
0.008
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.000,05
0.000,01

f = 0.019

Example: Solution

With f = 0.019, we get the head loss h_f from the Darcy-Weisbach equation:

$$h_f = f\frac{LV^2}{2Dg} = 0.0019\left(\frac{1000\text{m}}{0.20\text{m}}\right)\left(\frac{(1.59\text{m/s})^2}{2(9.81\text{m/s}^2)}\right) = 12.2\text{m}$$

Example: Find volume flow rate Q

Similar to last problem:

Pipe is 20-cm asphalted cast-iron.
Fluid is water.
Head loss per kilometer is 12.2 m.

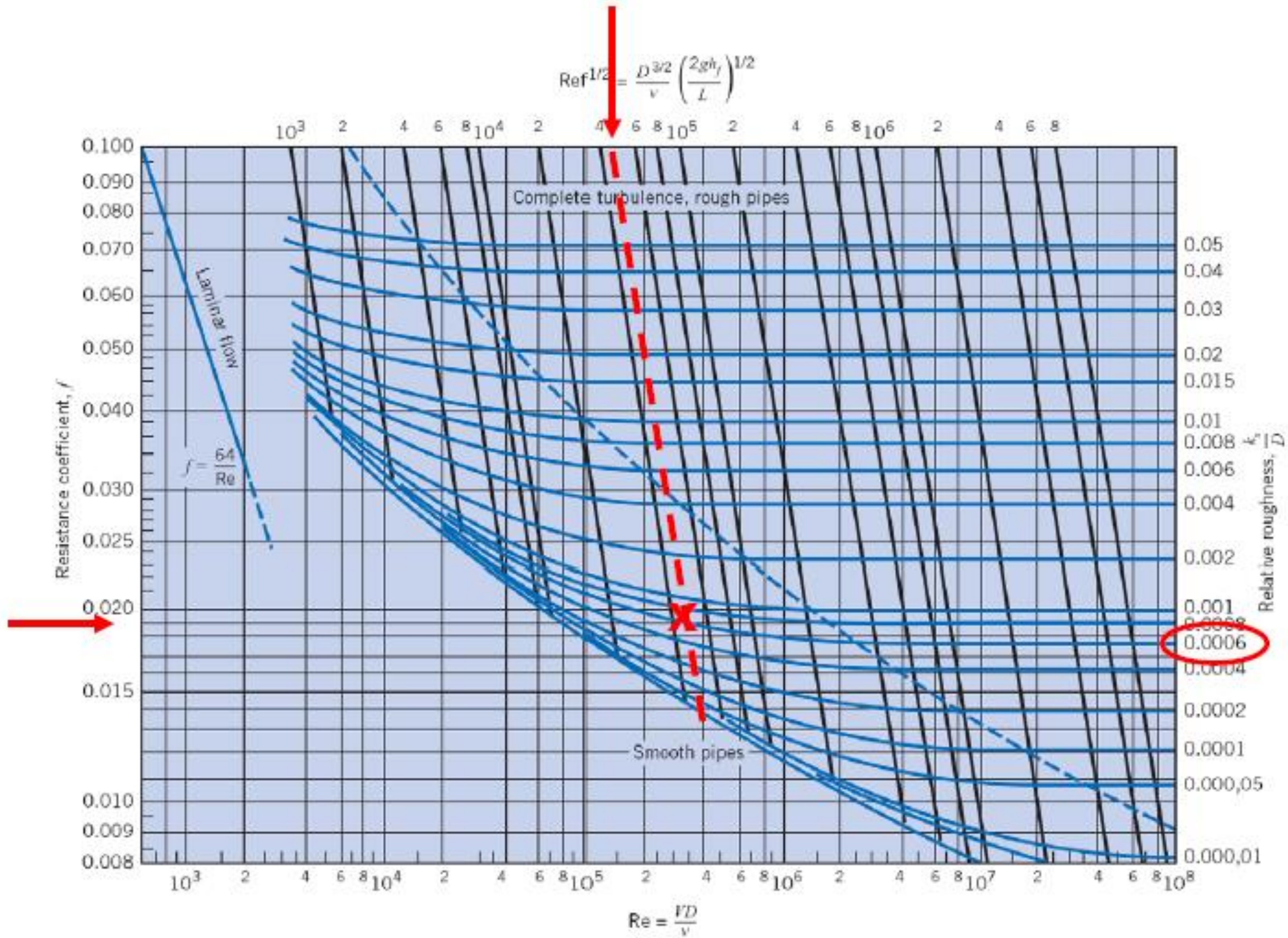
The difference to the previous problem is that we don't know the velocity, so we can't compute Re.

Compute instead

$$D^{3/2}\frac{\sqrt{gh_f/L}}{\nu} = 4.38 \times 10^4$$

where $\nu = \dfrac{\mu}{\rho}$

is the kinematic viscosity.



Relative roughness same as previous problem

again ⇨ $f = 0.019$

Now we use the Darcy-Weisbach equation again to get V

$$h_f = f \frac{LV^2}{2Dg}$$

$$\Rightarrow V = \sqrt{\frac{2Dgh_f}{fL}}$$

$$V = 1.59\text{m/s}$$

$$Q = A\,V = \frac{\pi D^2}{4} V = 0.050\text{m}^3\text{/s}$$

Empirical (experimental) relations for determining friction factor (f):

- In general, friction factor is function of (Re and relative roughness height)
- Laminar region is Independent of roughness

$$f = F(\mathrm{Re}, \frac{e}{D}) \qquad f = \frac{64}{\mathrm{Re}}$$

- Turbulent region
  - Smooth zone

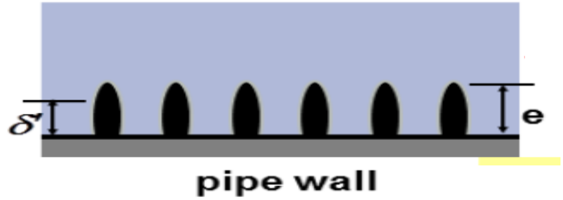
If $\mathrm{Re} > 10^5$ $$\frac{1}{\sqrt{f}} = 2Log(\mathrm{Re}\sqrt{f}) - 0.8$$ **Von Karman & Prandtl**

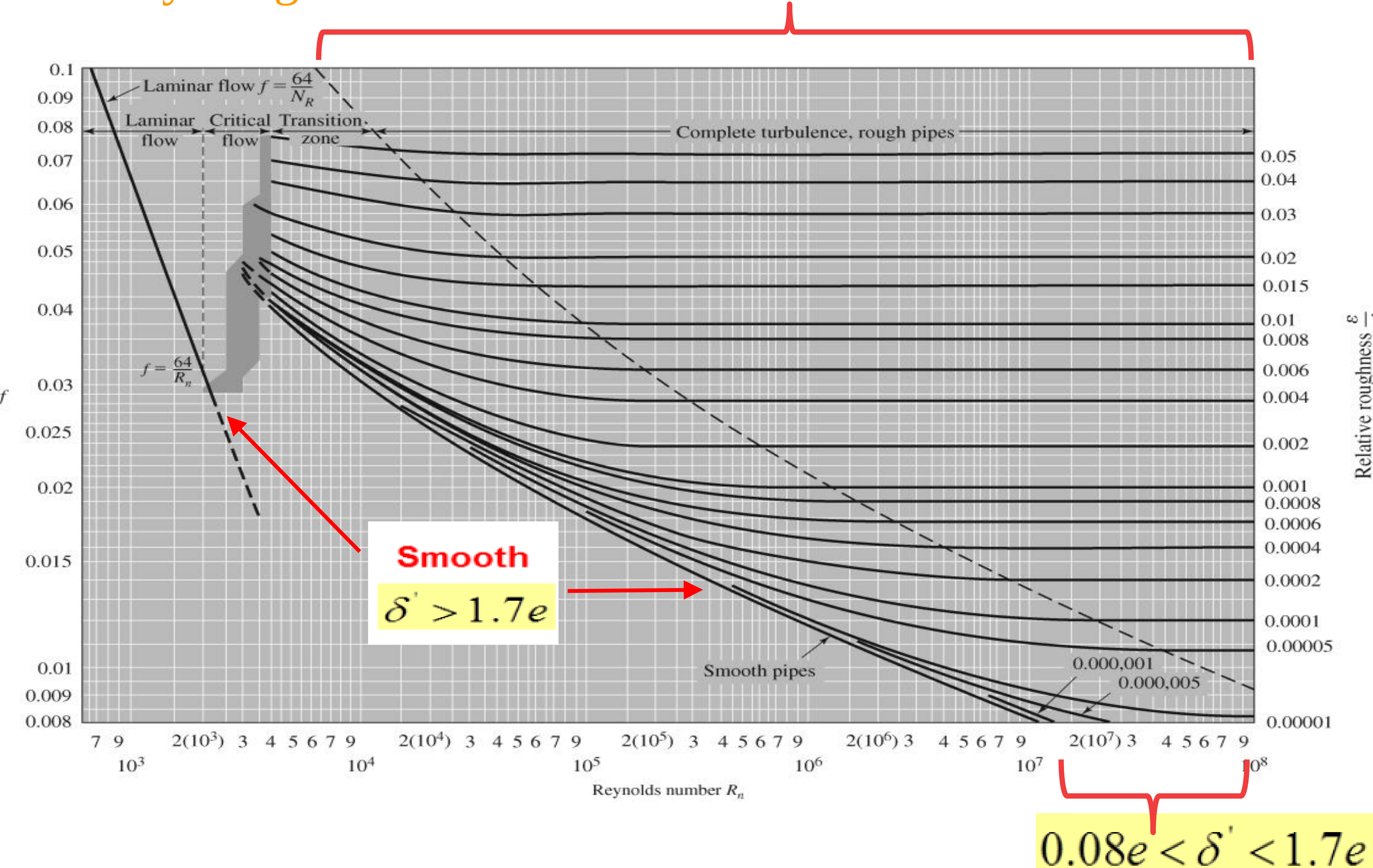
If $\mathrm{Re} \leq 10^5$ $$f = 0.079/\mathrm{Re}^{0.25}$$ **Blasius**

  - Transition zone $$\frac{1}{\sqrt{f}} - 2\log(r/\varepsilon) = 1.74 - 2\log(1 + 18.7\frac{r/\varepsilon}{R_n\sqrt{f}})$$ **Colebrook & White**

  - Rough zone $$\frac{1}{\sqrt{f}} = 2Log(r_o/\varepsilon) + 1.74$$ **Nikuradse**

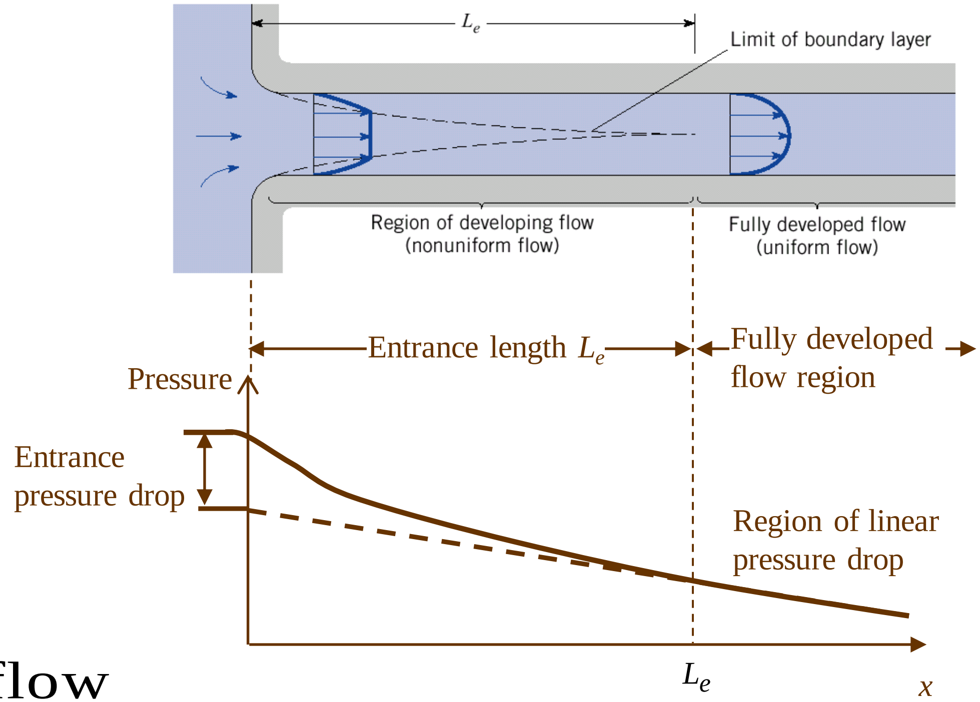
Moody Diagram





Pipe Entrance

- Developing flow
  - Includes boundary layer and core,
  - viscous effects grow inward from the wall
- Fully developed flow
  - Shape of velocity profile is same at all points along pipe



$$\frac{L_e}{D} \approx \begin{cases} 0.06\,\mathrm{Re} & \text{Laminar flow} \\ 4.4\,\mathrm{Re}^{1/6} & \text{Turbulent flow} \end{cases}$$