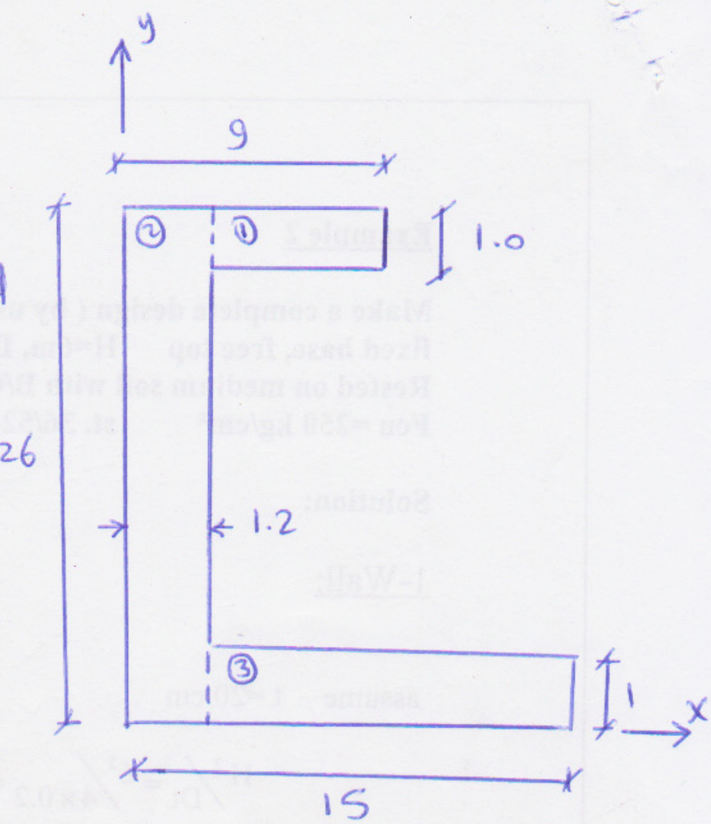
Req.:-

1) Centroid
2) I_x, I_y, I_{xy}, I_p
3) i_x, i_y
4) $I_{max}, I_{min}, \theta_{max}$ → Analytical → graphical
5) at $\theta = 30°$ get $I_{x_{30}}, I_{y_{30}}, I_{xy_{30}}$



Solution

- Put Ref. Axes
- divide the section into Simple Partitions

$x_1 = 5.1$ cm $\qquad y_1 = 25.5$ cm $\qquad A_1 = 8*1 = 7.8\ cm^2$

$x_2 = 0.6$ cm $\qquad y_2 = 13$ cm $\qquad A_2 = 1.2*26 = 31.2\ cm^2$

$x_3 = 8.1$ cm $\qquad y_3 = 0.5$ cm $\qquad A_3 = 13.8*1 = 13.8\ cm^2$

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{7.8*5.1 + 31.2*0.6 + 13.8*8.1}{7.8+31.2+13.8} = 3.2\ cm$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{7.8*25.5 + 31.2*13 + 13.8*0.5}{7.8+31.2+13.8} = 11.6\ cm$$

Moments of Inertia about centeral Axes

$$I_x = \frac{7.8*1^3}{12} + 7.8*(25.5-11.6)^2$$
$$+ \frac{1.2*26^3}{12} + 31.2*(13-11.6)^2$$
$$+ \frac{13.8*1^3}{12} + 13.8*(0.5-11.6)^2 = 5027.88\ cm^4$$

$$I_y = \frac{1*7.8^3}{12} + 7.8(5.1-3.2)^2$$
$$+ \frac{26*1.2^3}{12} + 31.2(0.6-3.2)^2$$
$$+ \frac{1*13.8^3}{12} + 13.8(8.1-3.2)^2 = 832.7 \text{ cm}^4$$

Product moment of Inertia

$$I_{xy} = 0.0 + 7.8*(25.5-11.6)(5.1-3.2)$$
$$+ 0.0 + 31.2*(13-11.6)(0.6-3.2)$$
$$+ 0.0 + 13.8*(0.5-11.6)(8.1-3.2) = -658.15 \text{ cm}^4$$

Polar Moment of Inertia

$$I_p = I_x + I_y$$
$$= 5027.88 + 832.7 = 5860.58 \text{ cm}^4$$

Radius of gyration

$$i_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5027.88}{52.8}} = 9.76 \text{ cm}$$

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{832.7}{52.8}} = 3.97 \text{ cm}$$

Principal Moments of Inertia "Analytically"

$$I_{max} = I_u = \frac{I_x+I_y}{2} + \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2}$$
$$= 2930.29 + \sqrt{(2097.6)^2 + (-658.15)^2}$$
$$= 5128.7 \text{ cm}^4$$

$$I_{min} = I_v = \frac{I_x+I_y}{2} - \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2}$$
$$= 2930.29 - \sqrt{(2097.6)^2 + (-658.15)^2}$$
$$= 731.86 \text{ cm}^4$$

$$\frac{I_x+I_y}{2} = 2930.29$$

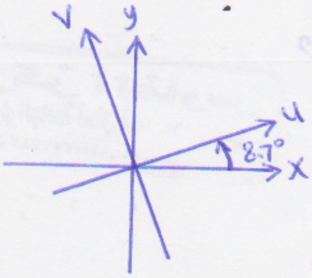
$$\frac{I_x-I_y}{2} = 2097.6$$

$$\tan 2\theta = \frac{-2 I_{xy}}{I_x - I_y} = \frac{-2 * -658.15}{5027.88 - 832.7} = 0.313$$

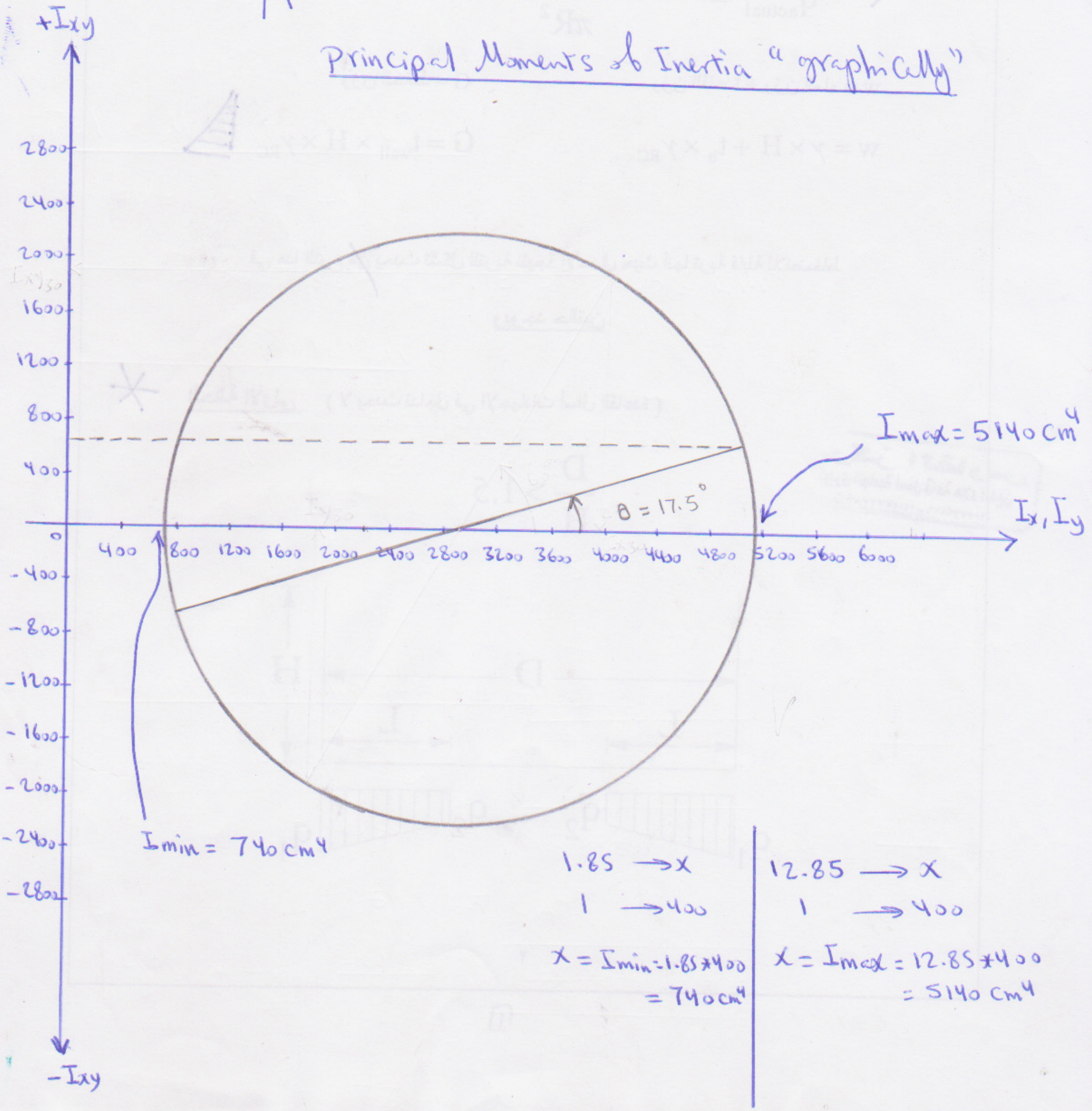
$$2\theta = \tan^{-1}(0.313)$$

$$2\theta = 17.4^\circ \qquad \theta = 8.7^\circ$$

$$= 8^\circ\ 42'\ 36.04''$$



زاوية موجبة ← دوران عكس عقارب الساعة



Principal Moments of Inertia "graphically"

$1.85 \rightarrow x$	$12.85 \rightarrow x$
$1 \rightarrow 400$	$1 \rightarrow 400$
$x = I_{min} = 1.85 * 400 = 740\ cm^4$	$x = I_{max} = 12.85 * 400 = 5140\ cm^4$

لرسم دائرة مور MoHR's Circle

[1] نختار scale مناسب بحيث يغطي أعلى قيمة متوقعة لل Imax

ولكن $1\,cm \rightarrow 400\,cm^4$
على الرسم

[2] نحدد مكان مركز الدائرة ولكنه على مسافة C من نقطة (0,0)

$$C = \frac{I_x + I_y}{2} = 2930.29\ cm^4$$

باستخدام ال scale نحول C إلى بعد على الرسم

$1 \longrightarrow 400$

$x \longrightarrow 2930.29$

$$x = \frac{2930.29}{400} = 7.3\ cm$$

[3] نحسب نصف قطر الدائرة الرسم من ال Center ولكنه R

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{(2097.6)^2 + (-658.15)^2} = 2198.4\ cm^4$$

$1 \longrightarrow 400$

$x \longrightarrow 2198.4$

$$x = \frac{2198.4}{400} = 5.5\ cm$$

[4] باستخدام المنقلة نقيس الزاوية 2θ

بعد مد خط افقي ليقطع الدائرة من قيمة I_{xy}

سنجد $2\theta = 17.5$

$\theta = 8.75^\circ$

There is alittle deffbenence between Analytical values and graphical values Due to small plotting accuracy دقة الرسم

At $\theta = 30°$

$$I_{x_{30}} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= 2930.29 + 2097.6 * 0.5 + 658.15 * 0.866$$

$$= 4549.05 \text{ cm}^4$$

$\cos 2\theta = 0.5$

$\sin 2\theta = 0.866$

$$I_{y_{30}} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= 2930.29 - 2097.6 * 0.5 + (-658.15) * 0.866$$

$$= 1311.5 \text{ cm}^4$$

$$I_{xy_{30}} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= 2097.6 * 0.866 + (-658.15) * 0.5$$

$$= 1487.45 \text{ cm}^4$$