HIGHER TECHNOLOGICAL INSTITUTE

H.T.I

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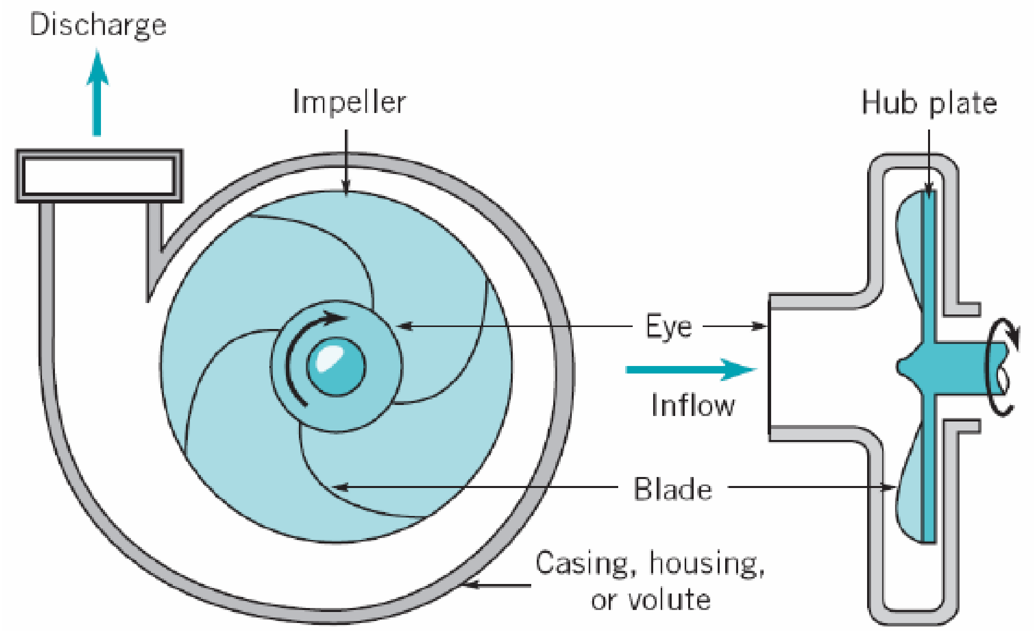
HYDRAULICS (2)

DR. AMIR M. MOBASHER




وقل رب زدني علما

HYDRAULICS (2)

Contents

Best regards,
Dr. Amir Mobasher

CHAPTER 1

FLOW THROUGH PIPES

1.1 Introduction:

Any water conveying system may include the following elements:

- pipes (in series, pipes in parallel)
- elbows
- valves
- other devices

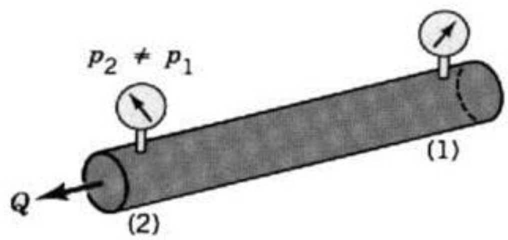
If all elements are connected in series, the arrangement is known as a "***pipeline***". Otherwise, it is known as a "***pipe network***".

1.2 Difference between open-channel flow and the pipe flow:

Pipe flow السريان فى الأنابيب المغلقة :

- The pipe is completely filled with the fluid being transported.
- The main driving force is likely to be a pressure gradient along the pipe.

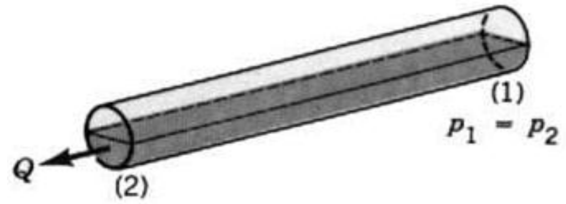
يحدث السريان عادة تحت ضغط ناتج من طلمبة أو خزان.



Open-channel flow السريان فى القنوات المفتوحة:

- Fluid flows without completely filling the pipe.
- Gravity alone is the driving force, the fluid flows down a hill.

يكون الضغط عند اى نقطة على السطح الحر مساويا للضغط الجوى، ويحدث هذا السريان تحت تأثير الجاذبية الأرضية.



1.3 Types of Flow أنواع السريان:

❑ Steady and Unsteady flow السريان المستقر وغير المستقر :

The flow parameters such as velocity (v), pressure (P) and density (r) of a fluid flow are independent of time in a steady flow. In unsteady flow they are independent.

For a steady flow $\left(\partial v/\partial t\right)_{x_o,y_o,z_o} = 0$

For an unsteady flow $\left(\partial v/\partial t\right)_{x_o,y_o,z_o} \neq 0$

❑ Uniform and non-uniform flow السريان المنتظم وغير المنتظم:

A flow is uniform if the flow characteristics at any given instant remain the same at different points in the direction of flow, otherwise it is termed as non-uniform flow.

For a uniform flow $\left(\partial v/\partial s\right)_{t_o} = 0$

For a non-uniform flow $\left(\partial v/\partial s\right)_{t_o} \neq 0$

Steady = time independent
Uniform = constant section

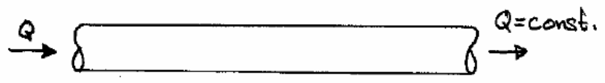
فى السريان المستقر لا تتغير السرعة أو الضغط عند نقطة محددة مع الزمن.

فى السريان المنتظم لا تتغير السرعة و شكل السريان من نقطة إلى أخرى.

Examples of flow types:

Steady uniform flow:

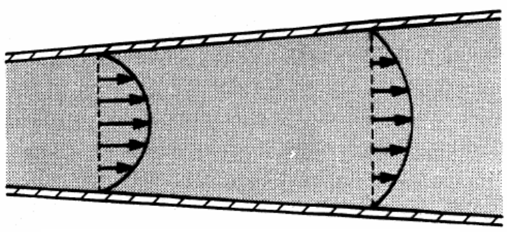
flowrate (Q) and section area (A) are constant



Steady uniform flow

Steady non-uniform flow:

Q = constant, A = A(x).



Steady non-uniform flow

❑ Laminar and turbulent flow السريان الطبقى (اللزج) والسريان المضطرب:

Laminar flow السريان الطبقى (اللزج)

The fluid particles move along smooth well defined path or streamlines that are parallel, thus particles move in laminas or layers, smoothly sliding over each other

Turbulent flow السريان المضطرب

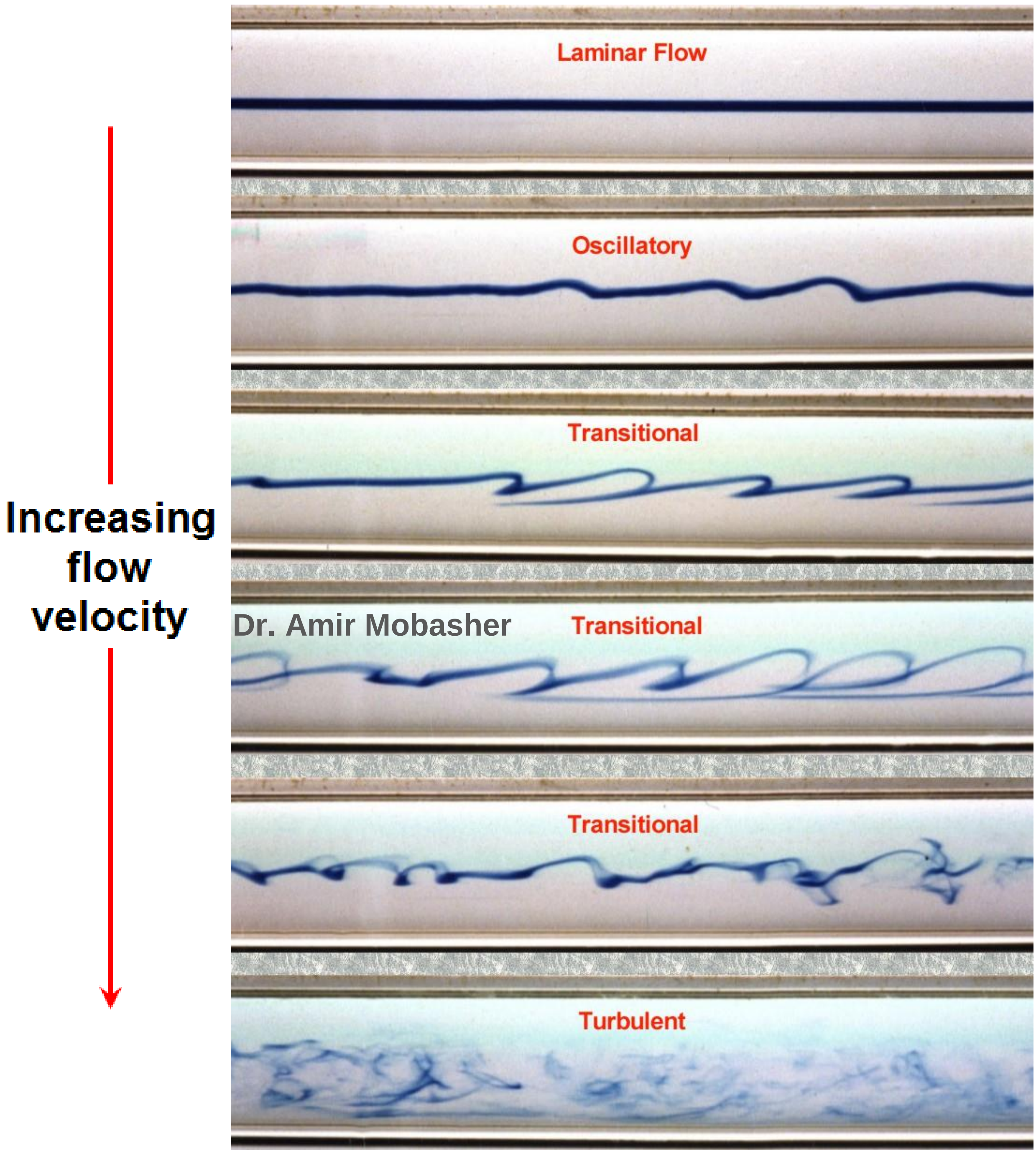
The fluid particles do not move in orderly manner and they occupy different relative positions in successive cross-sections.

There is a small fluctuation in magnitude and direction of the velocity of the fluid particles

Transitional flow السريان الانتقالى

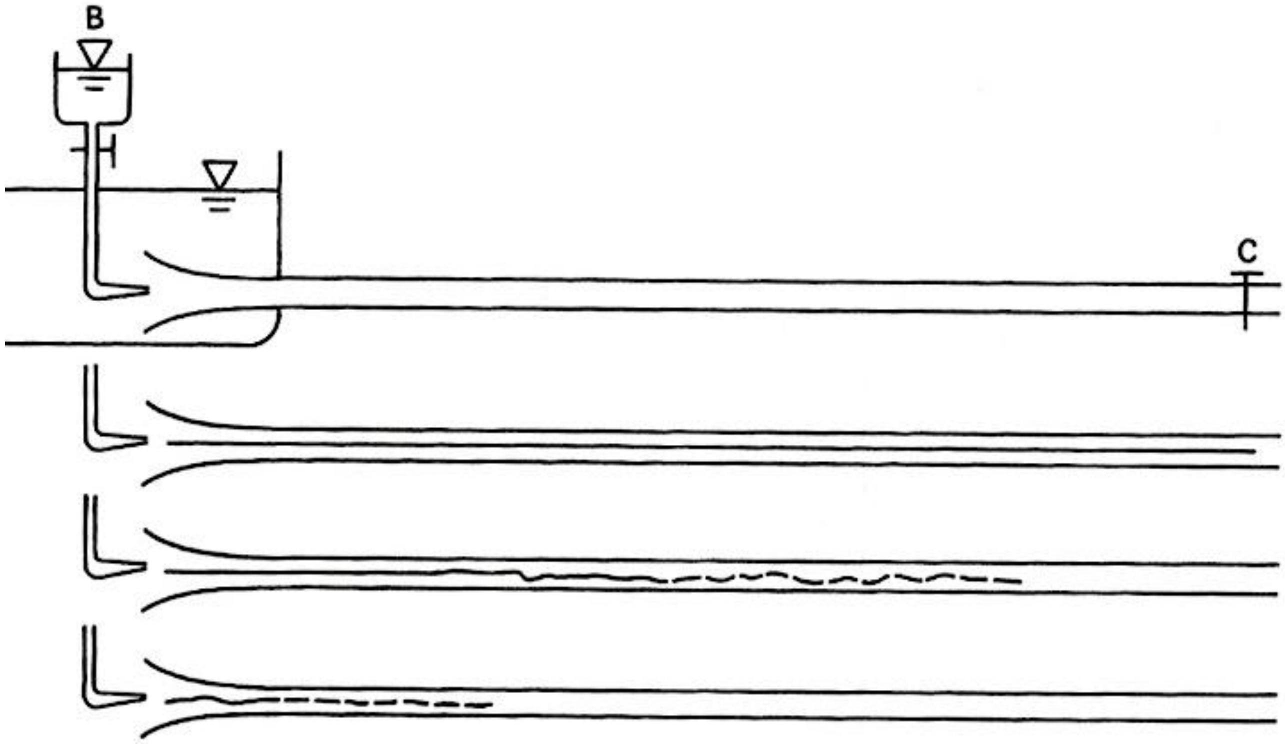
The flow occurs between laminar and turbulent flow

وقد وُجد من التجارب المعملية أن الفاقد فى الضاغط يعتمد على نوع السريان هل هو طبقى أم مضطرب، ويتحدد نوع السريان تبعا لسلوك الجزيئات أثناء حركتها.


Increasing flow velocity
Laminar Flow
Oscillatory
Transitional
Dr. Amir Mobasher
Transitional
Transitional
Turbulent

Reynolds Experiment:

Reynold performed a very carefully prepared pipe flow experiment. Reynold found that transition from laminar to turbulent flow in a pipe depends not only on the velocity, but only on the pipe diameter and the viscosity of the fluid.



Reynolds' apparatus.

Reynolds number is used to check whether the flow is laminar or turbulent. It is denoted by "$\mathbf{R_n}$". This number got by comparing inertial force with viscous force.

$$R_n = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{Inertial\ Forces}{Viscous\ Forces}$$

Where

V: mean velocity in the pipe السرعة المتوسطة [L/T]

D: pipe diameter قطر الماسورة [L]

ρ: density of flowing fluid كثافة السائل [M/L^3]

μ: dynamic viscosity اللزوجة الديناميكية [M/LT]

ν: kinematic viscosity اللزوجة الكينماتيكية [L^2/T]

جمع "رينولد" كل هذه العوامل والتى تؤثر على تحديد نوع السريان فى رقم واحد غير قياسى وسماه "رقم رينولد". وبالتالى يصبح"رقم رينولد" أكثر تعبيرا عن حالة السريان.

The Kind of flow depends on value of "$\mathbf{R_n}$"

If $\mathbf{R_n}$ < 2000 the flow is Laminar

If $\mathbf{R_n}$ > 4000 the flow is turbulent

If 2000 < $\mathbf{R_n}$ < 4000 it is called transition flow.

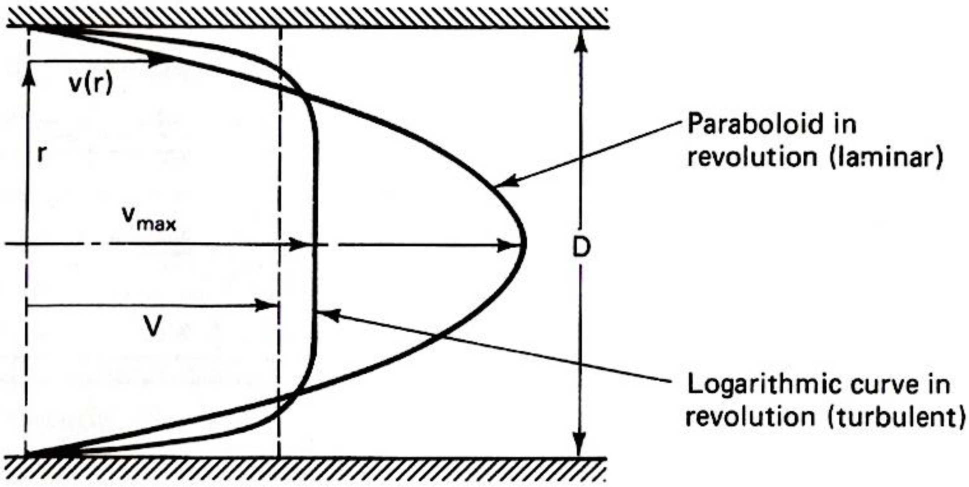
Laminar Vs. Turbulent flows

Laminar flows characterized by:

- low velocities
- small length scales
- high kinematic viscosities
- R_n < Critical R_n
- Viscous forces are dominant

Turbulent flows characterized by

- high velocities
- large length scales
- low kinematic viscosities
- R_n > Critical R_n
- Inertial forces are dominant



Velocity profiles of laminar and turbulent flows in circular pipes

يتضح من منحنى توزيع السرعات أنه فى حالة السريان الطبقى تكون أقصى سرعة عند المنتصف وأقل سرعة عند جدار الماسورة وهى مساوية للصفر. أما فى حالة السريان المضطرب فيمكن تقسيم السريان الى منطقتين: منطقة الطبقة اللزجة ومنطقة الاضطراب. وكلما زاد"رقم رينولد" كلما اقترب توزيع السرعات من الشكل المستطيل (توزيع منتظم).

Example 1-1

40 mm diameter circular pipe carries water ($\upsilon = 1x10^{-6}$ m^2/s). Calculate the largest flow rate (Q) which laminar flow can be expected.

Solution

$$D = 0.04m$$

$$R_n = \frac{VD}{\nu} = 2000 \Rightarrow \frac{V(0.04)}{1\times10^{-6}} = 2000 \Rightarrow V = 0.05m/\sec$$

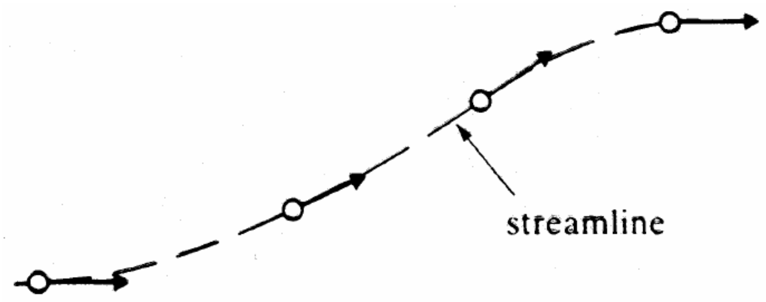
$$Q = V.A = 0.05\times\frac{\pi}{4}(0.04)^2 = 6.28\times10^{-5}m^3/\sec$$

❑ Streamlines and Streamtubes:

Streamline "التدفق" خط السريان:

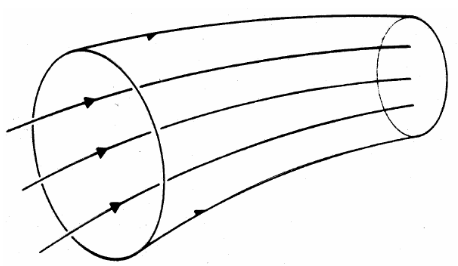
A curve that is drawn in such a way that it is tangential to the velocity vector at any point along the curve. A curve that is representing the direction of flow at a given time. No flow across a stream line.

هو ذلك الخط الذى يعبر عن اتجاه حركة الجزيئات الموجودة عليه فى لحظة زمنية معينة.



Streamtube"التدفق" أنبوب السريان :

A set of streamlines arranged to form an imaginary tube. Ex: The internal surface of a pipeline.



مجموعة من خطوط السريان تمثل فيما بينها أنبوب.

❑ Compressible And Incompressible Flows:

Incompressible Flow is a type of flow in which the density (ρ) is constant in the flow field.

Compressible Flow is the type of flow in which the density of the fluid changes in the flow field.

❑ Ideal and Real Fluids:

a- Ideal Fluids

- It is a fluid that has no viscosity, and incompressible
- Shear resistance is considered zero
- Ideal fluid does not exist in nature e.g. Water and air are assumed ideal

b- Real Fluids

- It is a fluid that has viscosity, and compressible
- It offers resistance to its flow e.g. All fluids in nature

عند دراستنا لسائل نعتبر ان السائل غير قابل للانضغاط ومثالى وبذلك نهمل قوى الاحتكاك الداخلى للحصول على معادلات بسيطة، ثم يتم تصحيحها بعد ذلك لأخذ قوى الاحتكاك فى الاعتبار.

1.4 Volume flow rate – Discharge :

The discharge is the volume of fluid passing a given cross-section per unit time.

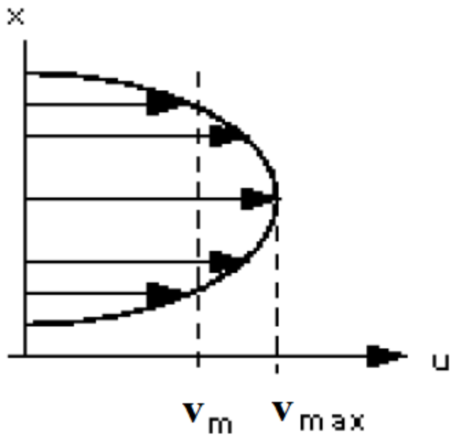
$$\text{discharge, } Q = \frac{\text{volume of fluid}}{\text{time}}$$

1.5 Mean Velocity:

It is the average velocity passing a given cection.

The velocity in the pipe is not constant across the cross section. Crossing the centreline of the pipe, the velocity is zero at the walls increasing to a maximum at the centre then decreasing symmetrically to the other wall. This variation across the section is known as the velocity profile or distribution.

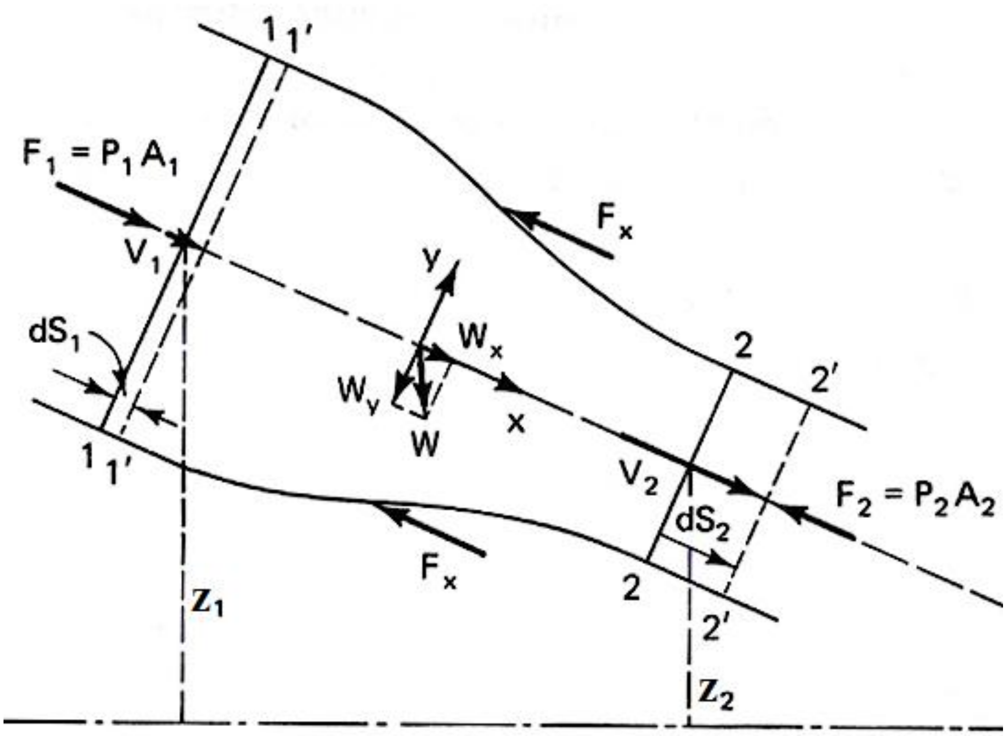
$$v_m = \frac{Q}{A}$$



A typical velocity profile across a pipe

1.6 Continuity equation for Incompressible Steady flow معادلة الاستمرار للسريان المستقر:

Cross section and elevation of the pipe are varied along the axial direction of the flow.



General description of flow in pipes

Conservation law of mass

$$\rho.dVol_{1-1'} = \rho.dVol_{2-2'} = mass\ \ flux(\,fluid\ \ mass)$$

Mass enters the control volume

Mass leaves the control volume

$$\rho.\frac{dVol_{1-1'}}{dt} = \rho.\frac{dVol_{2-2'}}{dt}$$

$$\rho.A_1\frac{dS_1}{dt} = \rho.A_2\frac{dS_2}{dt} \Rightarrow \rho.A_1.V_1 = \rho.A_2.V_2 = \rho.Q$$

Continuity equation for Incompressible Steady flow

$$A_1.V_1 = A_2.V_2 = Q$$

اذا استمر السائل فى تدفقه خلال مجرى مائى ما مثل قناة أو ماسورة منتظمة أو متغيرة المقطع فإن كمية السائل المارة خلال وحدة الزمن تكون متساوية عند جميع القطاعات خلال المجرى، و هذا هو ما يعرف بمعادلة الاستمرار للسريان المستقر.

Apply Newton's Second Law:

$$\sum \vec{F} = M\,\vec{a} = M\,\frac{d\vec{V}}{dt} = \frac{M\vec{V}_2 - M\vec{V}_1}{\Delta t}$$

$$\sum F_x = P_1A_1 - P_2A_2 - F_x + W_x$$

F_x is the axial direction force exerted on the control volume by the wall of the pipe.

$$but\ M/\Delta t = \rho.Q = mass\ \ flow\ rate$$

$$\sum F_x = \rho.Q(V_{x_2} - V_{x_1})$$

$$\sum F_y = \rho.Q(V_{y_2} - V_{y_1})$$

$$\sum F_z = \rho.Q(V_{z_2} - V_{z_1})$$

$$\sum \vec{F} = \rho.Q(\vec{V}_2 - \vec{V}_1)$$

Conservation of moment equation

5.1 Energy Head in Pipe Flow

Water flow in pipes may contain energy in three basic forms:

1- Kinetic energy طاقة الحركة,

2- Potential energy طاقة الوضع,

3- Pressure energy طاقة الضغط .

- **Consider the control volume:**
- **In time interval dt:**

➢ Water particles at sec.1-1 move to sec. 1\`-1\` with velocity V_1.

➢ Water particles at sec.2-2 move to sec. 2\`-2\` with velocity V_2.

To satisfy continuity equation:

$$A_1.V_1.dt = A_2.V_2.dt$$

<u>The work done by the pressure force:</u>

$$P_1.A_1.ds_1 = P_1.A_1.V_1.dt$$ ……. on section 1-1

$$-P_2.A_2.ds_2 = -P_2.A_2.V_2.dt$$ ……. on section 2-2

-ve sign because P_2 is in the opposite direction to distance traveled ds_2

The work done by the gravity force:

$$\rho g.A_1.V_1 dt.(z_1 - z_2)$$

The kinetic energy:

$$\frac{1}{2}M.V_2^2 - \frac{1}{2}M.V_1^2 = \frac{1}{2}\rho.A_1.V_1.dt(V_2^2 - V_1^2)$$

The total work done by all forces is equal to the change in kinetic energy:

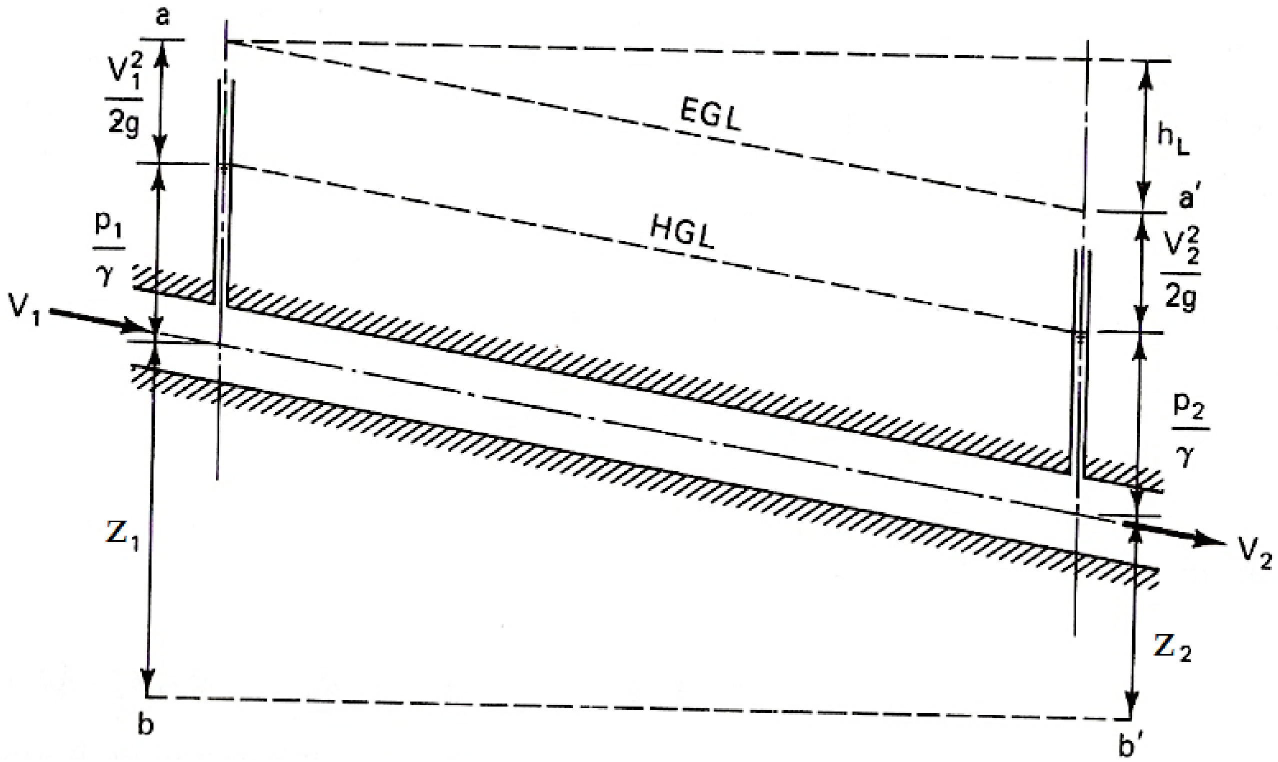
$$P_1.Q.dt - P_2.Q.dt + \rho g.Q.dt.(z_1 - z_2) = \frac{1}{2}\rho.Q.dt(V_2^2 - V_1^2)$$

Dividing both sides by ρgQdt

Bernoulli Equation

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2$$

Energy per unit weight of water
OR: Energy Head



Energy head and Head loss in pipe flow

$$H_2 = \frac{V_2^{\,2}}{2g} + \frac{P_2}{\gamma} + z_2$$

Energy head = Kinetic head + Pressure head + Elevation head

$$H_1 = \frac{V_1^{\,2}}{2g} + \frac{P_1}{\gamma} + z_1$$

Notice that:

- In reality, certain amount of energy loss (h_L) occurs when the water mass flow from one section to another.
- The energy relationship between two sections can be written as:

$$\frac{V_1^{\,2}}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^{\,2}}{2g} + \frac{P_2}{\gamma} + z_2 + h_L$$

1.7 Flow Through A Single Pipe:

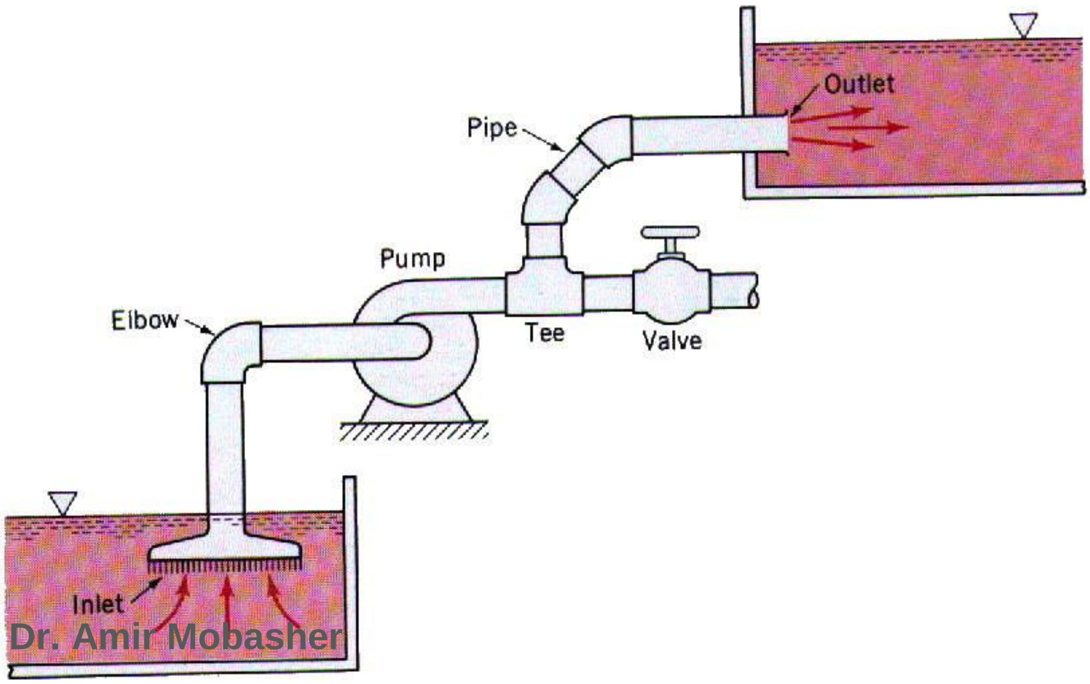
1.7.1 Calculation of Head (Energy) Losses الطاقة المفقودة :

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy (head) of fluid is lost.

Energy Losses (Head losses)

- **Major Losses**: loss of head due to pipe friction and to viscous dissipation in flowing water
- **Minor losses**: Loss due to the change of the velocity of the flowing fluid in the magnitude or in direction as it moves through fitting like Valves, Tees, Bends and Reducers.

يمكن تقسيم الفواقد فى الضاغط الى نوعين: الأولى وهى فواقد رئيسية ناتجة عن الاحتكاك على طول المجرى، والثانية فواقد ثانوية ناتجة عن وجود عائق فى المجرى أو نتيجة تغير شكل المجرى.



1.7.2 Losses of Head due to Friction الفاقد فى الضاغط بالاحتكاك

- Energy loss through friction in the length of pipeline is commonly termed the major loss h_f
- This is the loss of head due to pipe friction and to the viscous dissipation in flowing water.
- The resistance to flow in a pipe is a function of:
  - The pipe length, L
  - The pipe diameter, D
  - The mean velocity, V
  - The properties of the fluid ( )
  - The roughness of the pipe, (the flow is turbulent).
- Several formulas have been developed in the past. Some of these formulas have faithfully been used in various hydraulic engineering practices.
  - Darcy-Weisbach formula
  - The Hazen -Williams Formula
  - The Manning Formula
  - The Chezy Formula

Darcy-Weisbach Equation

$$h_L = F\frac{L}{D} \times \frac{V^2}{2g} = \frac{8F\,L\,Q^2}{g\,D^5\,\pi^2}$$

Where:
F is the friction factor
L is pipe length
D is pipe diameter
Q is the flow rate
h_L is the loss due to friction

Friction Factor: (F) معامل الاحتكاك

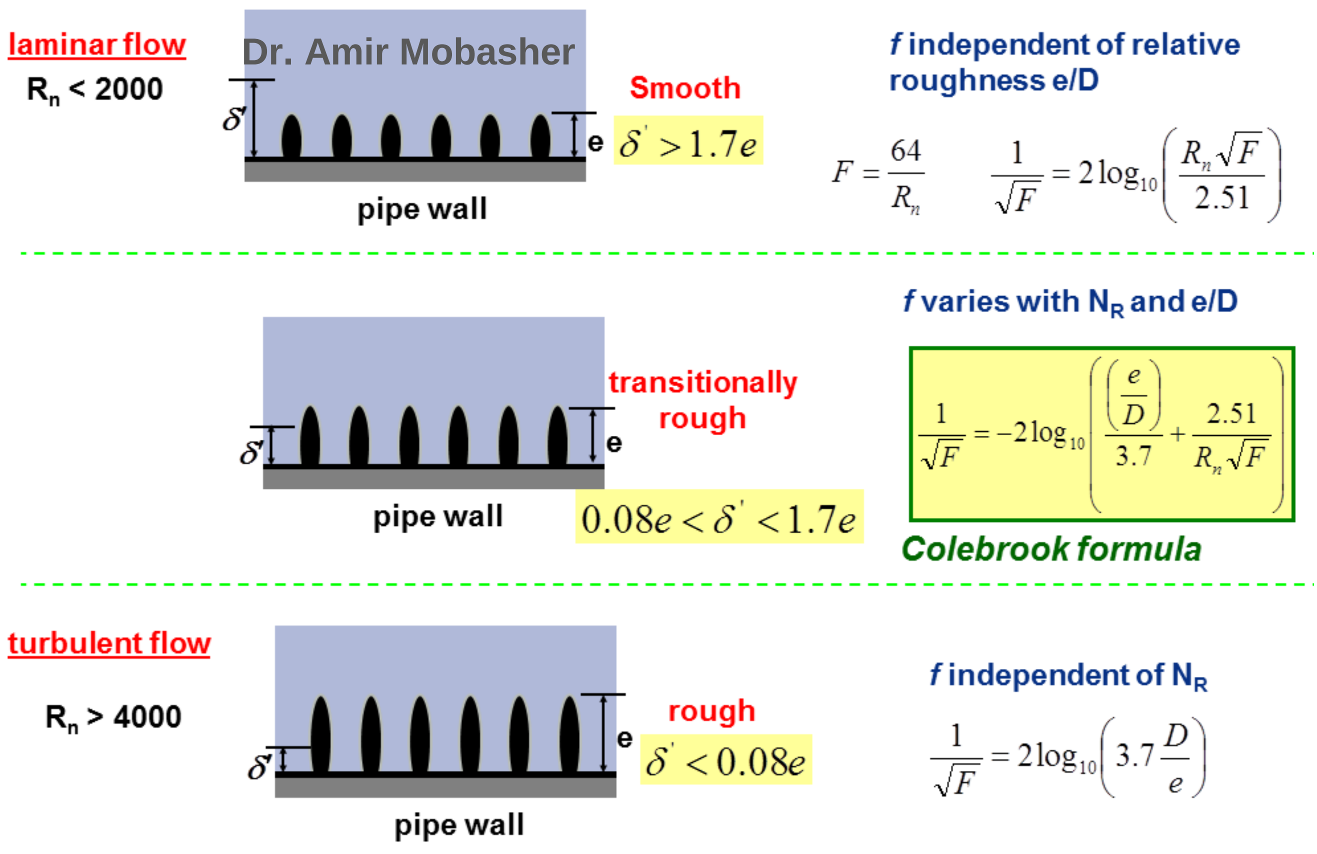
- For Laminar flow: (R_n < 2000) [depends only on Reynolds' number and not on the surface roughness]

$$F = \frac{64}{R_n}$$

- For turbulent flow in smooth pipes (e/D = 0) with $4000 < R_n < 10^5$ is

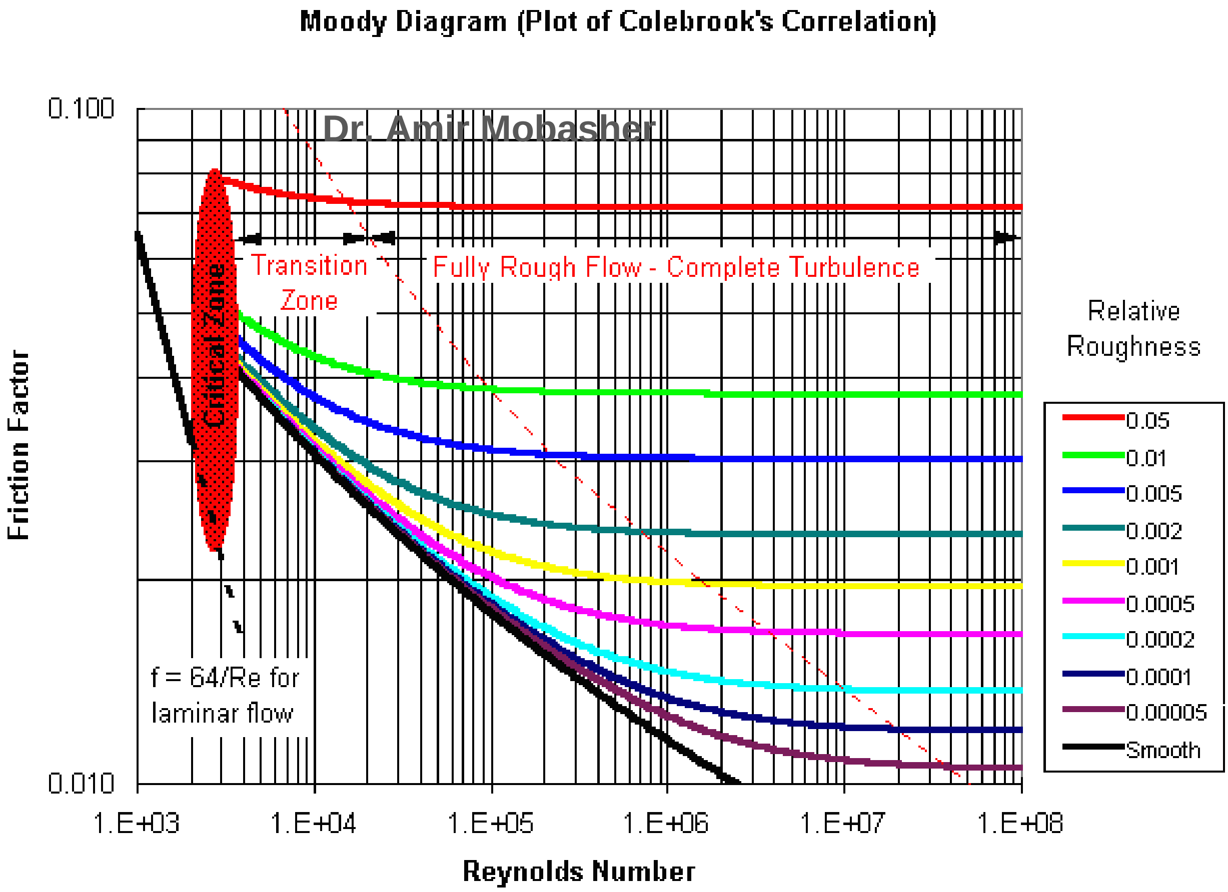
$$F = \frac{0.316}{R_n^{1/4}}$$

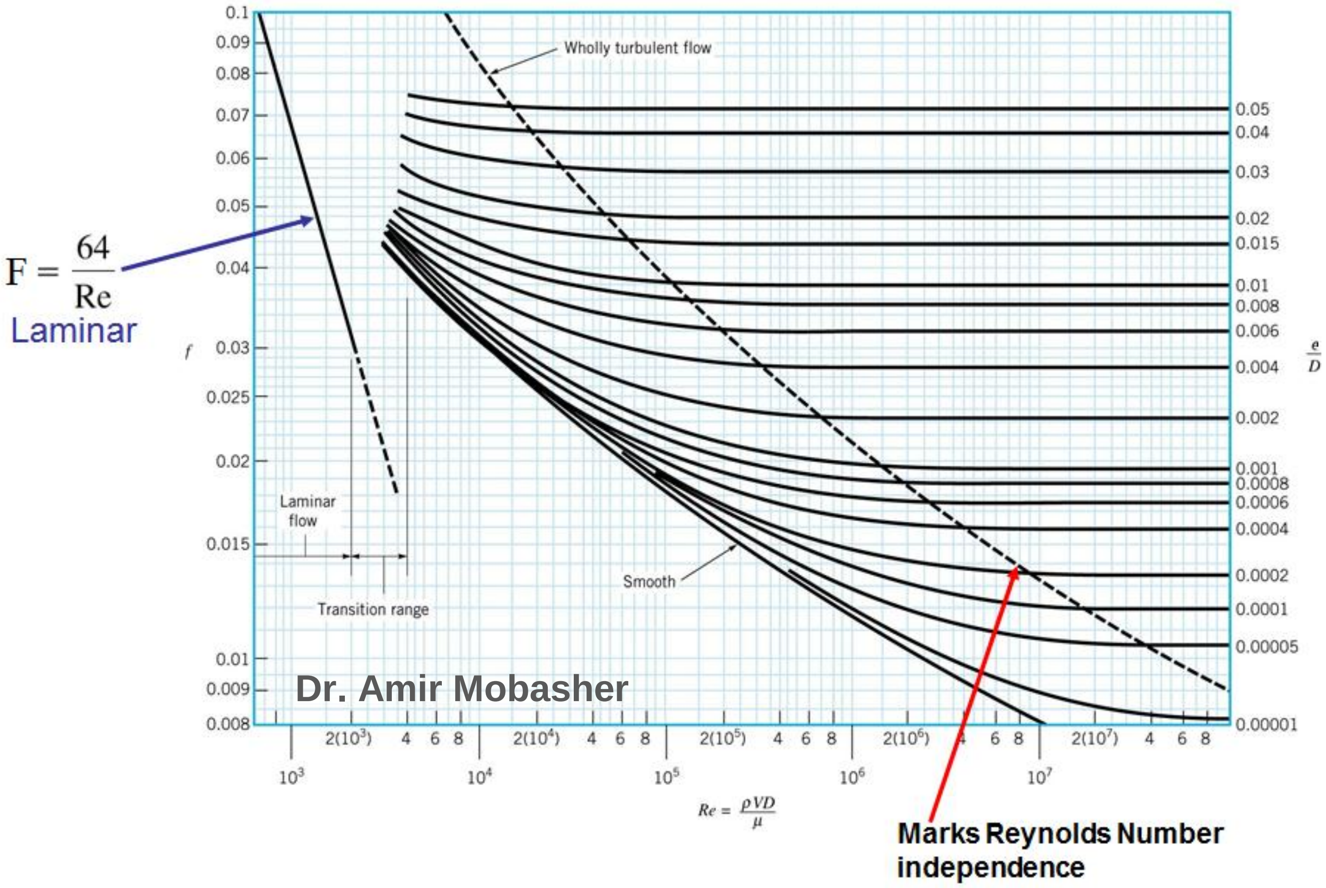
- The thickness of the laminar sublayer δ decrease with an increase in R_n.



Moody diagram

- A convenient chart was prepared by Lewis F. Moody and commonly called the Moody diagram of friction factors for pipe flow, There are 4 zones of pipe flow in the chart:
  - A laminar flow zone where F is simple linear function of R_n
  - A critical zone (shaded) where values are uncertain because the flow might be neither laminar nor truly turbulent
  - A transition zone where F is a function of both R_n and relative roughness
  - A zone of fully developed turbulence where the value of F depends solely on the relative roughness and independent of the Reynolds Number





Typical values of the absolute roughness (e) are given in the next table

Roughness Height, *e*, for Certain Common Pipe Materials

Pipe Material	*e(mm)*	*e(ft)*
Glass, drawn brass, copper (new)	0.0015	0.000005
Seamless commercial steel (new)	0.004	0.000013
Commercial steel (enamel coated)	0.0048	0.000016
Commercial steel (new)	0.045	0.00015
Wrought iron (new)	0.045	0.00015
Asphalted cast iron (new)	0.12	0.0004
Galvanized iron	0.15	0.0005
Cast iron (new)	0.26	0.00085
Wood Stave (new)	0.18 ~ 0.9	0.0006 ~ 0.003
Concrete (steel forms, smooth)	0.18	0.0006
Concrete (good joints, average)	0.36	0.0012
Concrete (rough, visible, form marks)	0.60	0.002
Riveted steel (new)	0.9 ~ 9.0	0.003-0.03
Corrugated metal	45	0.15

Example 1-2

The water flow in Asphalted cast Iron pipe (e = 0.12mm) has a diameter 20cm at $20^{o}C$. Is $0.05\ m^3/s$. determine the losses due to friction per 1 km

Solution

$$V = \frac{0.05 m^3/s}{(\pi/4)(0.2^2 m^2)} = 1.59 m/s$$

$$T = 20^{o} C \Rightarrow \upsilon = 1.01 \times 10^{-6} m^2/s$$

$$e = 0.12 mm$$

$$\frac{e}{D} = \frac{0.12 mm}{200 mm} = 0.0006 \rightarrow \text{Moody}$$

$$R_n = \frac{VD}{\upsilon} = \frac{1.59 \times 0.2}{1.01 \times 10^{-6}} = 314852 = 3.15 \times 10^5$$

Moody → *F= 0.018*

$$h_f = F \frac{L}{D} \frac{V^2}{2g} = 0.018 \left(\frac{1{,}000\ m}{0.20\ m}\right)\left(\frac{1.59^2}{2(9.81\ m/s^2)}\right)$$

$$= 11.55\ m$$

$$D \geq 5cm - - - - V \leq 3.0 m/\sec$$

$$V = 1.318\, C_{HW} R_h^{0.63} S^{0.54} \qquad \textit{British Units}$$

$$V = 0.85\, C_{HW} R_h^{0.63} S^{0.54} \qquad \textit{SI Units}$$

Simplified ↓

$$R_h \rightarrow \text{hydraulic Radius} = \frac{\text{wetted A}}{\text{wetted P}} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}$$

$$S = \frac{h_f}{L}$$

$$C_{HW} \rightarrow \text{Hazen Williams Coefficient}$$

$$h_f = \frac{10.7\, L}{C_{HW}^{1.852}\, D^{4.87}} Q^{1.852} \qquad \text{SI Units}$$

Hazen-Williams Coefficient, C_{HW}, for Different Types of Pipe

Pipe Materials	C_{HW}
Asbestos Cement	140
Brass	130–140
Brick sewer	100
Cast-iron	
New, unlined	130
10 yr. old	107–113
20 yr. old	89–100
30 yr. old	75–90
40 yr. old	64–83
Concrete or concrete lined	
Steel forms	140
Wooden forms	120
Centrifugally spun	135
Copper	130–140
Galvanized iron	120
Glass	140
Lead	130–140
Plastic	140–150
Steel	
Coal-tar enamel lined	145–150
New unlined	140–150
Riveted	110
Tin	130
Vitrified clay (good condition)	110–140
Wood stave (average condition)	120

❑ Manning Formula

This formula has extensively been used for open channel design, it is also quite commonly used for pipe flows

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$R_h \rightarrow$ hydraulic Radius $= \frac{\text{wetted A}}{\text{wetted P}} = \frac{D}{4}$

$S = \frac{h_f}{L}$

$n \rightarrow$ *Manning* Coefficient

Simplified

$$h_f = \frac{10.3\, L\, (nQ)^2}{D^{5.33}} \qquad \text{SI Units}$$

Manning's Roughness Coefficient, *n*, for Pipe Flows

Type of Pipe	*Manning's n* Min.	*Manning's n* Max.
Glass, brass, or copper	0.009	0.013
Smooth cement surface	0.010	0.013
Wood-stave	0.010	0.013
Vitrified sewer pipe	0.010	0.017
Cast-iron	0.011	0.015
Concrete, precast	0.011	0.015
Cement mortar surfaces	0.011	0.015
Common-clay drainage tile	0.011	0.017
Wrought iron	0.012	0.017
Brick with cement mortar	0.012	0.017
Riveted-steel	0.017	0.020
Cement rubble surfaces	0.017	0.030
Corrugated metal storm drain	0.020	0.024

❑ The Chezy Formula

$$V = C\, R_h^{1/2}\; S^{1/2}$$

$$h_f = 4\frac{L}{D}\left(\frac{V}{C}\right)^2$$

Where C = Chezy coefficient

- It can be shown that this formula, for circular pipes, is equivalent to Darcy's formula with the value for

$$C=\sqrt{\frac{8g}{F}}$$

F is Darcy Weisbeich coefficient

- The following formula has been proposed for the value of C:

$$C=\frac{23+\frac{0.00155}{S}+\frac{1}{n}}{1+(23+\frac{0.00155}{S})\frac{n}{\sqrt{R_h}}}$$

n is the Manning coefficient

Example 1-3

New Cast Iron (C_{HW} = 130, n = 0.011) has length = 6 km and diameter = 30cm. Q= 0.32 m^3/s, T=30^o. Calculate the head loss due to friction.

Solution

- Hazen-William Method

$$h_f = \frac{10.7\ L}{C_{HW}^{\ 1.852}\ D^{4.87}}\ Q^{1.852}$$

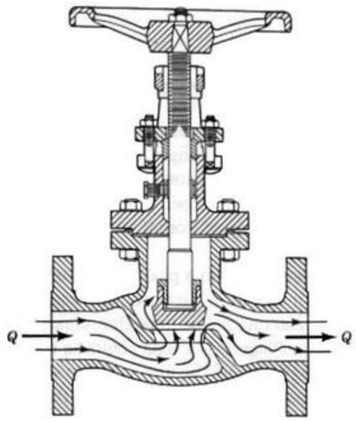
$$h_f = \frac{10.7\ \times 6000}{130^{1.852}\ 0.3^{4.87}}\ 0.32^{1.852} = 333m$$

- Manning Method

$$h_f = \frac{10.3\ L\ (nQ)^2}{D^{5.33}}$$

$$h_f = \frac{10.3\ \times 6000\ (0.011 \times 0.32)^2}{0.3^{5.33}} = 470\ m$$

1.7.3 Minor losses الفواقد الثانوية:



Flow pattern through a valve

- It is due to the change of the velocity of the flowing fluid in the magnitude or in direction [turbulence within bulk flow as it moves through and fitting]
- The minor losses occurs du to :
  - Valves
  - Tees, Bends
  - Reducers
  - And other appurtenances
- It has the common form

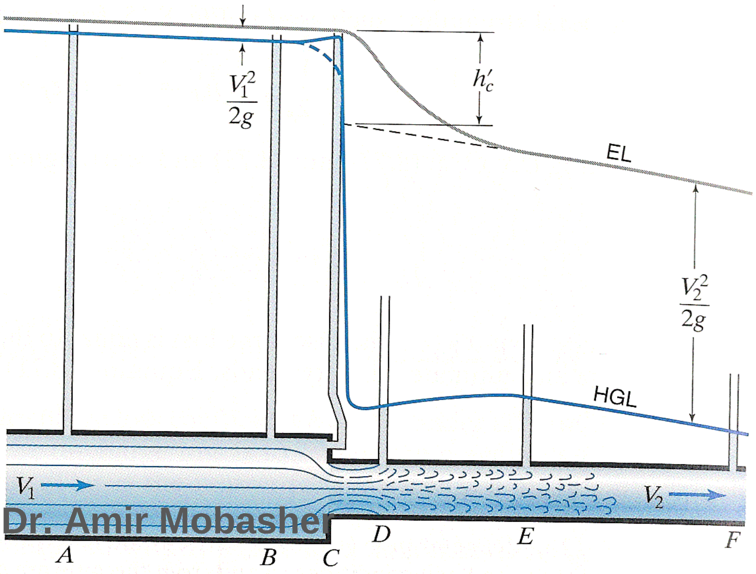
$$h_m = k_L \frac{V^2}{2g} = k_L \frac{Q^2}{2gA^2}$$

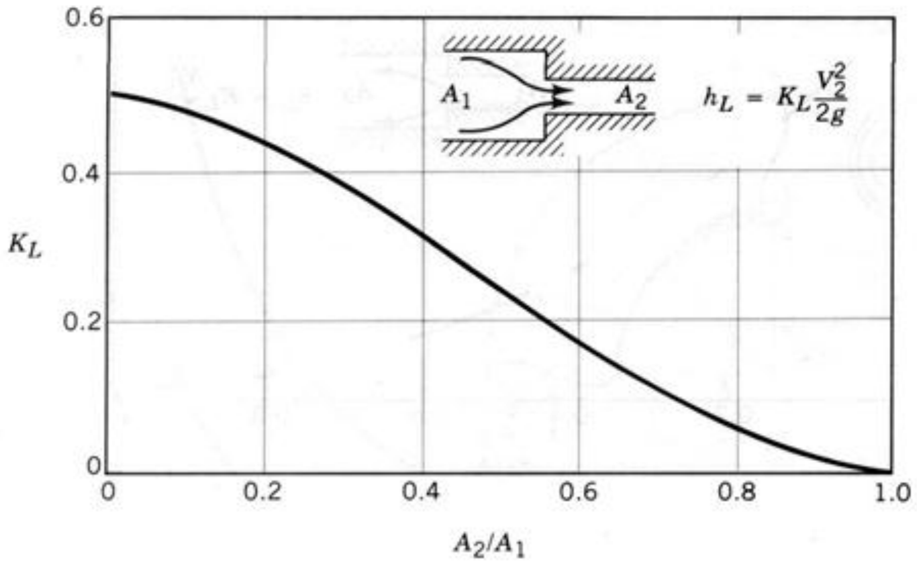
- "***minor***" compared to friction losses in long pipelines but, can be the dominant cause of head loss in shorter pipelines.

❑ Losses due to Sudden contraction الفاقد فى الضاغط نتيجة ضيق مفاجئ:

A sudden contraction in a pipe usually causes a marked drop in pressure in the pipe due to both the increase in velocity and the loss of energy to turbulence.

نتيجة وجود ضيق مفاجئ تحدث مناطق انفصال تُحدث فاقد فى الطاقة.

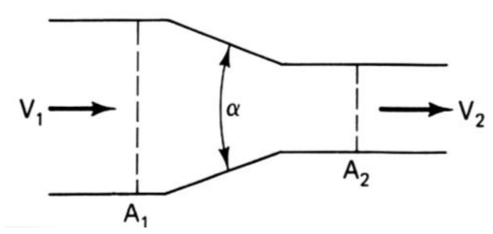




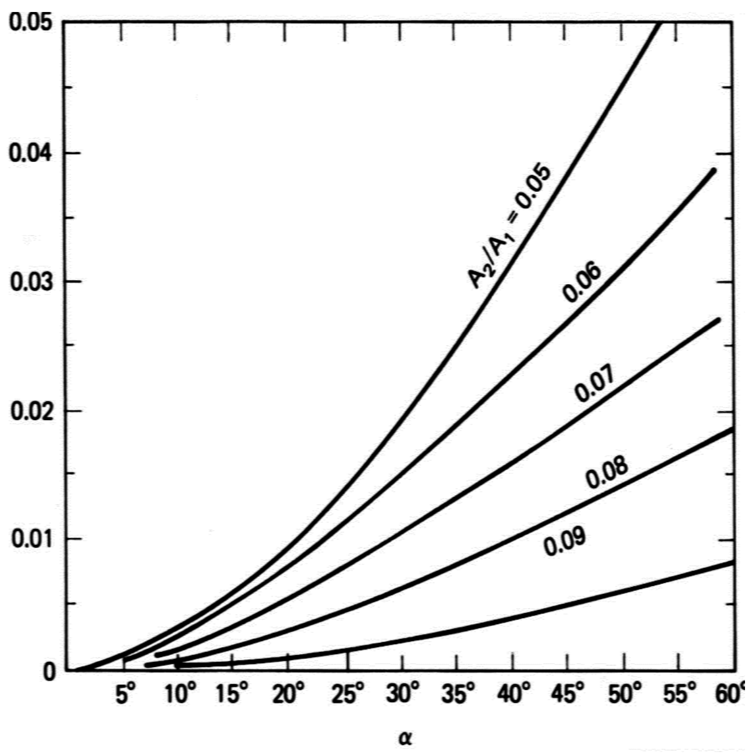
$$h_L = 0.5\,\frac{V_2^{\,2}}{2g}$$

❑ Head Loss Due to Gradual Contraction ضيق تدريجى:

Head losses due to pipe contraction may be greatly reduced by introducing a ***gradual pipe transition*** known as a ***confusor***



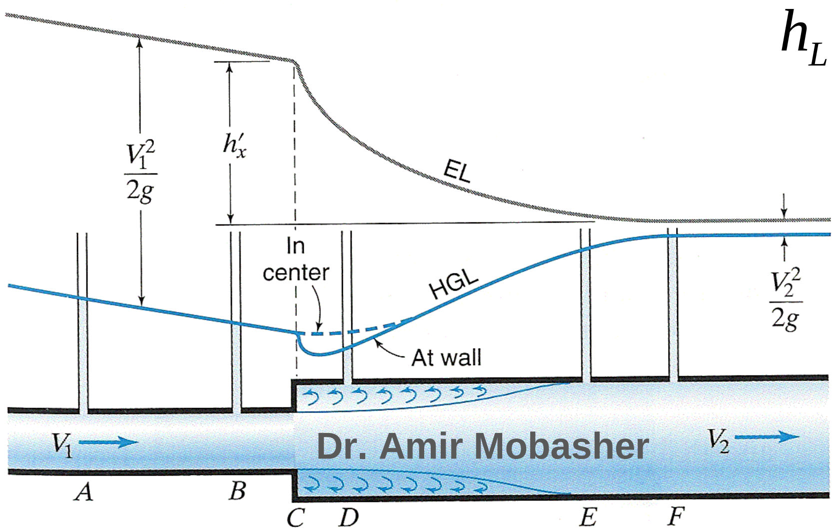
$$h_c' = k_c' \frac{V_2^{\,2}}{2g}$$



❑ Losses due to Sudden Enlargement الفاقد فى الضاغط نتيجة الاتساع المفاجئ:

Note that the drop in the energy line is much larger than in the case of a contraction

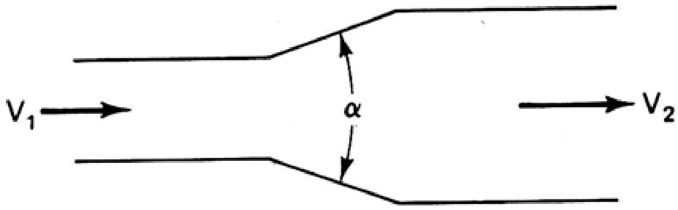
$$h_L = \frac{(V_1 - V_2)^2}{2g}$$



❑ Head Loss Due to Gradual Enlargement اتساع تدريجى:

Head losses due to pipe enlargement may be greatly reduced by introducing a ***gradual pipe transition*** known as a ***diffusor***

$$h_E' = k_E' \frac{V_1^2 - V_2^2}{2g}$$

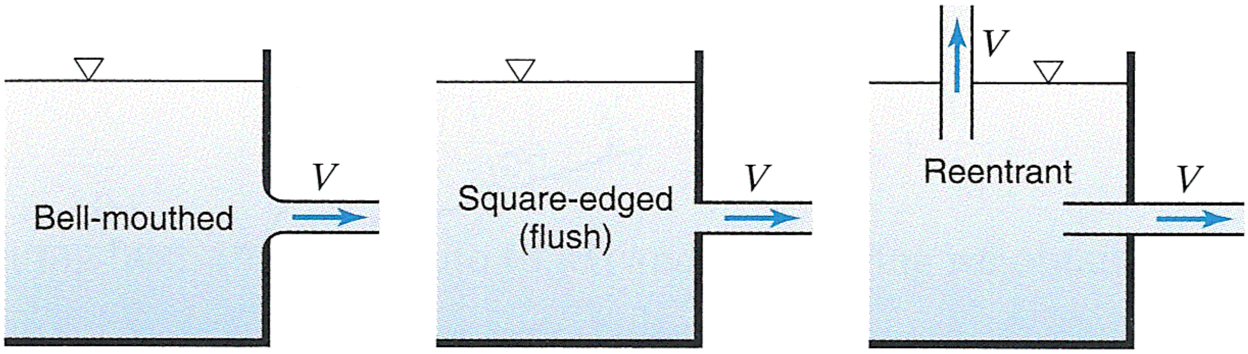


α	10°	20°	30°	40°	50°	60°	75°
K_E'	.078	.31	.49	.60	.67	.72	.72

❑ Loss due to pipe entrance الفاقد فى الضاغط عند مدخل ماسورة:

General formula for head loss at the entrance of a pipe is also expressed in term of velocity head of the pipe

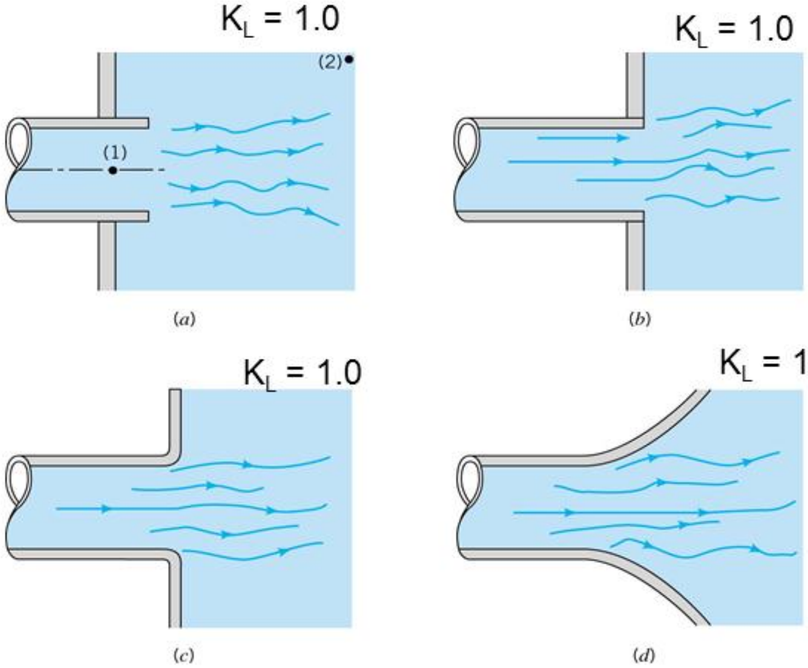
$$h_{ent} = K_{ent} \frac{V^2}{2g}$$



(a) $k_e = 0.04$ (b) $k_e = 0.5$ (c) $k_e \approx 0.8$

❑ **Head Loss at the Exit of a Pipe الفاقد فى الضاغط عند مخرج ماسورة:**

$$h_L = \frac{V^2}{2g}$$



❑ **Head Loss Due to Bends in Pipes الفاقد فى الضاغط نتيجة الانحناء:**

$$h_b = k_b \frac{V^2}{2g}$$

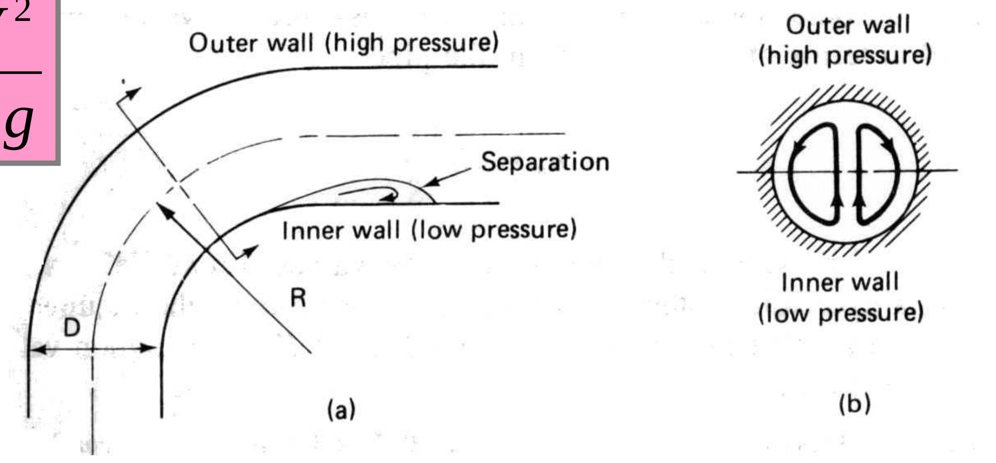


Figure Head loss at a bend: (a) flow separation in a bend; (b) secondary flow at a bend.

R/D	1	2	4	6	10	16	20
K_b	0.35	0.19	0.17	0.22	0.32	0.38	0.42

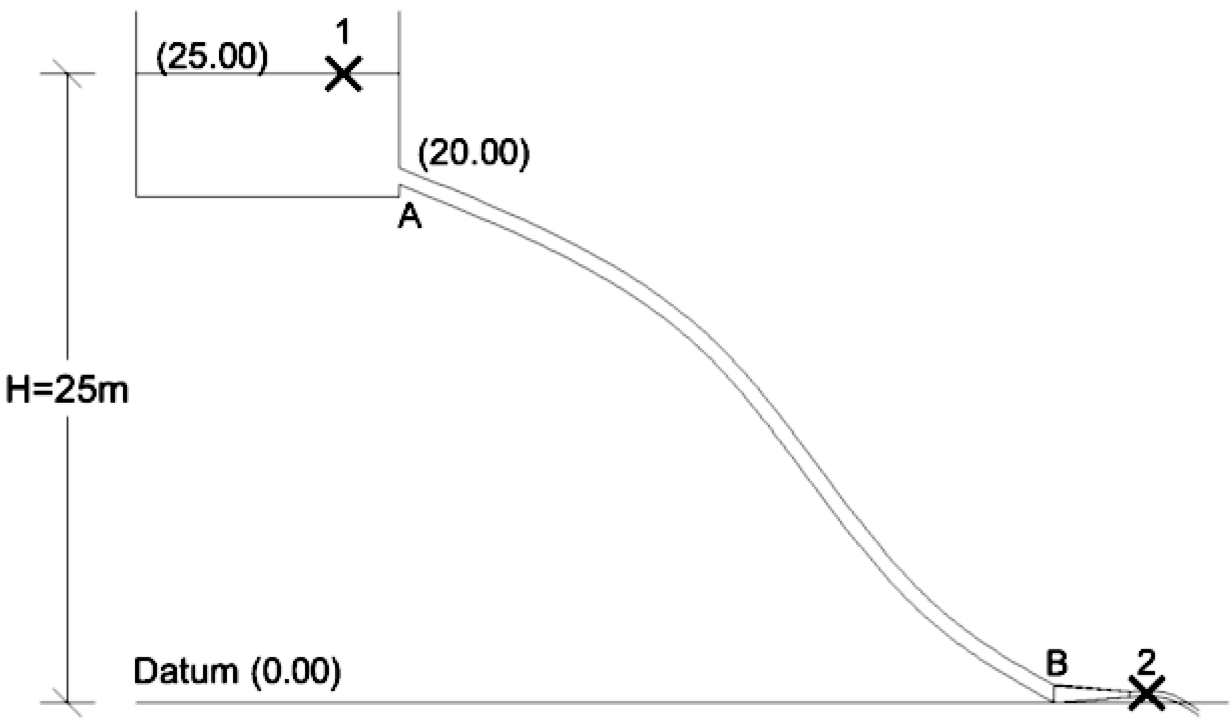
Example 1-4

A tank where the water level is 25.0 m above an arbitrary datum feeds a pipeline AB ending at B with a nozzle 4.0 cm diameter. Pipe AB is 15.0 cm diameter with point A being 20.0 m above datum and point B at datum. Find:

i) The discharge through the pipeline, the pressures and water velocities at A & B.

ii) If friction losses in the nozzle are 0.5 m, and between A & B are 5.0 m, resolve (i) and plot the hydraulic gradient and total energy lines.

Solution



i) Ideal flow

Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$25 + 0 + 0 = 0 + 0 + v_2^2 / 2g \rightarrow v_2 = 22.147$ m/sec

$Q = A_2 v_2 = \pi/4\ (0.04)^2 * 22.147 = 0.0278\ m^3/sec$

$Q = A_A v_A = A_B v_B = \pi/4\ (0.15)^2 * v_A = 0.0278\ m^3/sec$

$v_A = v_B = 1.575$ m/sec

Apply B.E bet. 1 & B

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_B + v_B^2 / 2g + P_B / \gamma$

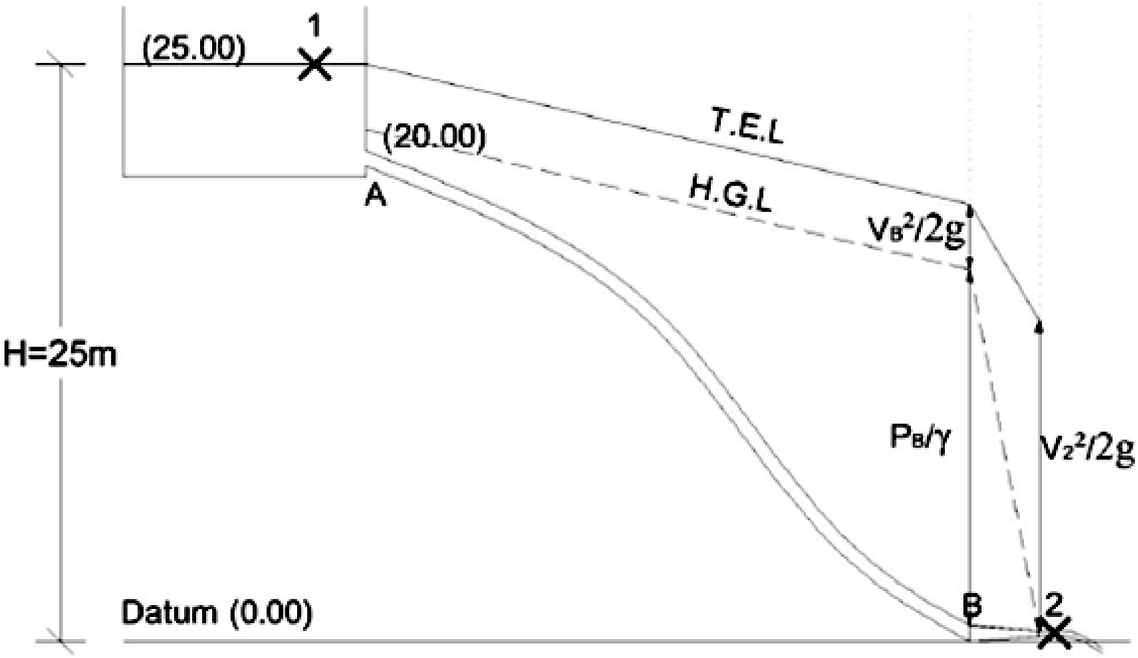
$25 + 0 + 0 = 0 + v_B^2 / 2g + P_B / \gamma \rightarrow P_B = 24.873$ m of water

Apply B.E bet. 1 & A

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_A + v_A^2 / 2g + P_A / \gamma$

$25 + 0 + 0 = 20 + v_A^2 / 2g + P_A / \gamma \rightarrow P_A = 4.873$ m of water

ii) Real flow



$hl_{nozzle} = 0.5$ m

$hl_{AB} = 5$ m

Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_2 + v_2^2 / 2g + P_2/\gamma + hl$

$25 + 0 + 0 = 0 + 0 + v_2^2 / 2g + 5.5 \rightarrow v_2 = 19.56$ m/sec

$Q = A_2 v_2 = \pi/4\ (0.04)^2 * 19.56 = 0.02458\ m^3/sec$

$Q = A_A v_A = A_B v_B = \pi/4\ (0.15)^2 * v_A = 0.02458\ m^3/sec$

$v_A = v_B = 1.391$ m/sec

Apply B.E bet. 1 & B

$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_B + v_B^2 / 2g + P_B/\gamma + hl$

$25 + 0 + 0 = 0 + v_B^2 / 2g + P_B/\gamma + 5 \rightarrow P_B = 19.9$ m of water

Apply B.E bet. 1 & A

$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_A + v_A^2 / 2g + P_A/\gamma$

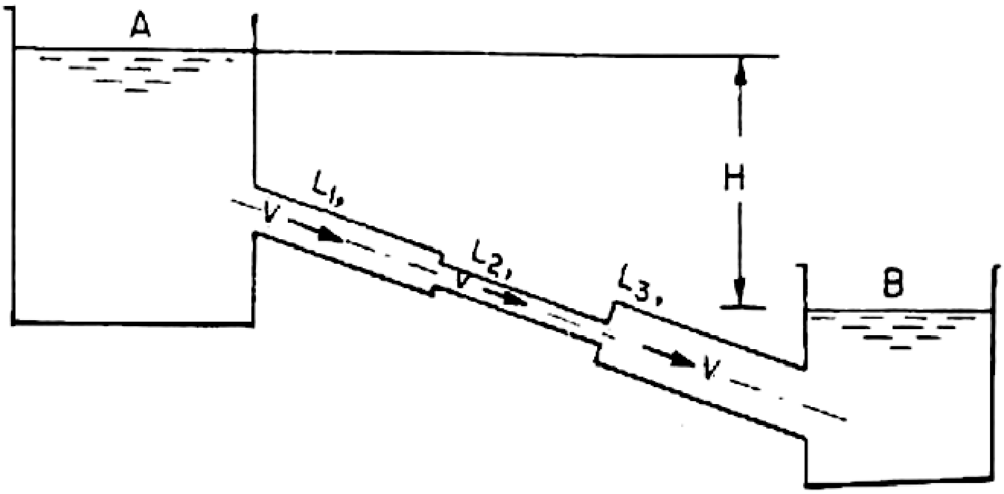
$25 + 0 + 0 = 20 + v_A^2 / 2g + P_A/\gamma \rightarrow P_A = 4.901$ m of water

1.8 Compound Pipe flow :

When two or more pipes with different diameters are connected together head to tail (***in series***) or connected to two common nodes (***in parallel***). The system is called "***compound pipe flow***"

1.8.1 Flow Through Pipes in Series توصيل المواسير على التوالى:

Pipes of different lengths and different diameters connected end to end (in series) to form a pipeline



- ***Discharge:*** The discharge through each pipe is the same

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

- ***Head loss:*** The difference in liquid surface levels is equal to the sum of the total head loss in the pipes:

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_L$$

$$z_A - z_B = h_L = H$$

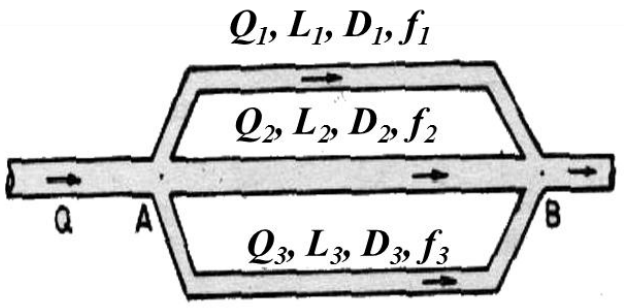
Where

$$h_L = \sum_{i=1}^{3} h_{fi} + \sum_{j=1}^{4} h_{mj}$$

$$h_L = \sum_{i=1}^{3} F_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + K_{ent} \frac{V_1^2}{2g} + K_c \frac{V_2^2}{2g} + K_{enl} \frac{V_2^2}{2g} + K_{exit} \frac{V_3^2}{2g}$$

1.8.2 Flow Through Parallel Pipe توصيل المواسير على التوازى

If a main pipe divides into two or more branches and again join together downstream to form a single pipe, then the branched pipes are said to be connected in parallel (compound pipes). Points A and B are called nodes.



➢ ***Discharge:***

$$Q = Q_1 + Q_2 + Q_3 = \sum_{i=1}^{3} Q_i$$

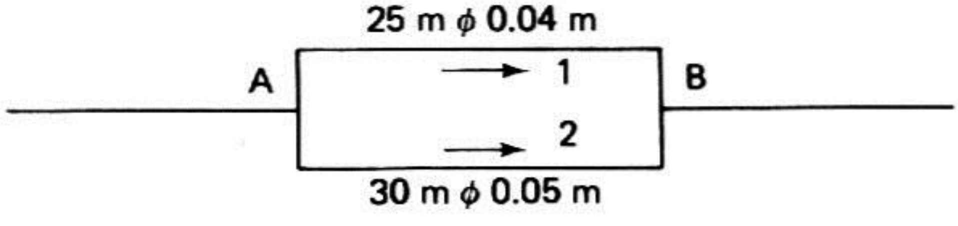
➢ ***Head loss:*** the head loss for each branch is the same

$$h_L = h_{f1} = h_{f2} = h_{f3}$$

$$F_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = F_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = F_3 \frac{L_3}{D_3}\frac{V_3^2}{2g}$$

Example 1-5

Determine the flow in each pipe and the flow in the main pipe if Head loss between A & B is 2m & F=0.01



Solution

$$h_{f1} = h_{f2} = 2$$

$$F\frac{L_1}{D_1}.\frac{V_1^2}{2g} = 2$$

$$0.01 \times \frac{25}{0.04} \times \frac{V_1^2}{2 \times 9.81} = 2$$

$$V_1 = 2.506\ m/s$$

$$Q_1 = V_1 A_1 = \frac{\pi}{4}(0.04)^2 \times 2.506 = 3.15 \times 10^{-3} m^3/s$$

$$F\frac{L_2}{D_2}.\frac{V_2^2}{2g} = 2$$

$$0.01 \times \frac{30}{0.05} \times \frac{V_2^2}{2 \times 9.81}$$

$$V_2 = 2.557\ m/s$$

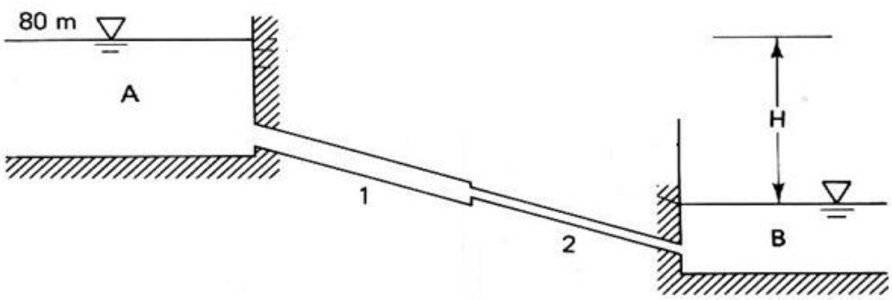
$$Q_2 = \frac{\pi}{4}(0.05)^2 \times 2.557 = 5.02 \times 10^{-3} m^3/s$$

$$Q = Q_1 + Q_2 = 8.17 \times 10^{-3} m^3/s$$

Example 1-6

In the figure shown two new cast iron pipes in series, D_1 = 0.6 m, D_2 = 0.4 m length of the two pipes is 300m, level at A =80 m, Q = 0.5 m^3/s (F_1 = 0.017, F_2 = 0.018). There are a sudden contraction between Pipe 1 and 2, and Sharp entrance at pipe 1, and Sharp outlet at pipe 2.

1- Find the water level at B, and draw T.E.L & H.G.L
2- If the two pipes are connected in parallel to each other, and the difference in elevation is 20 m. Find the total flow.



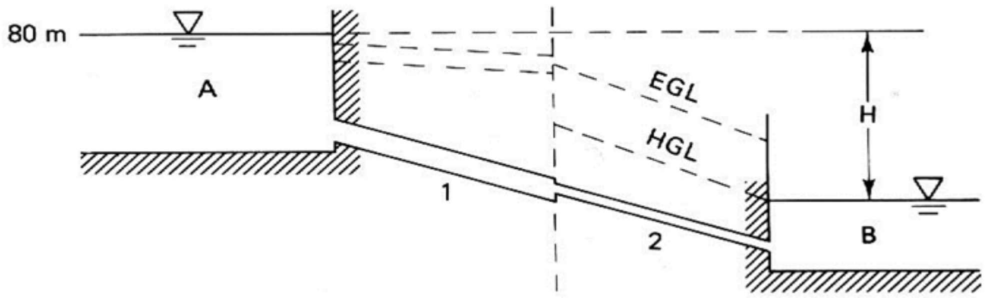
Solution

$$K_{ent} = 0.5, \qquad K_c = 0.27, \qquad K_{exit} = 1$$

$$h_L = F_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + F_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + k_{ent}\frac{V_1^2}{2g} + k_c \frac{V_2^2}{2g} + k_{exit}\frac{V_2^2}{2g}$$

$$h_f = 0.017 \left(\frac{300}{0.6}\right) . \frac{1.77^2}{2g} + 0.018 \left(\frac{300}{0.4}\right) . \frac{3.98^2}{2g} + 0.5\left(\frac{1.77^2}{2g}\right) + 0.27\left(\frac{3.98^2}{2g}\right) + \left(\frac{3.98^2}{2g}\right) = 13.36m$$

$Z_B = 80 – 13.36 = 66.64$ m



<u>Pipes are connected in parallel</u>

$$h_L = F\frac{L_1}{D_1}\frac{V_1^2}{2g} + k_{ent}\frac{V_1^2}{2g} + k_{exit}\frac{V_1^2}{2g}$$

$$20 = 0.017 \ \frac{300}{0.60}\frac{V_1^2}{2g} + 0.50\frac{V_1^2}{2g} + \frac{V_1^2}{2g}$$

$V_1 = 6.26 m/s$

$Q_1 = 1.77 m3/s$

$$20 = 0.018 \ \frac{300}{0.40}\frac{V_2^2}{2g} + 0.50\frac{V_2^2}{2g} + \frac{V_2^2}{2g}$$

$V_2 = 5.11 m/s$

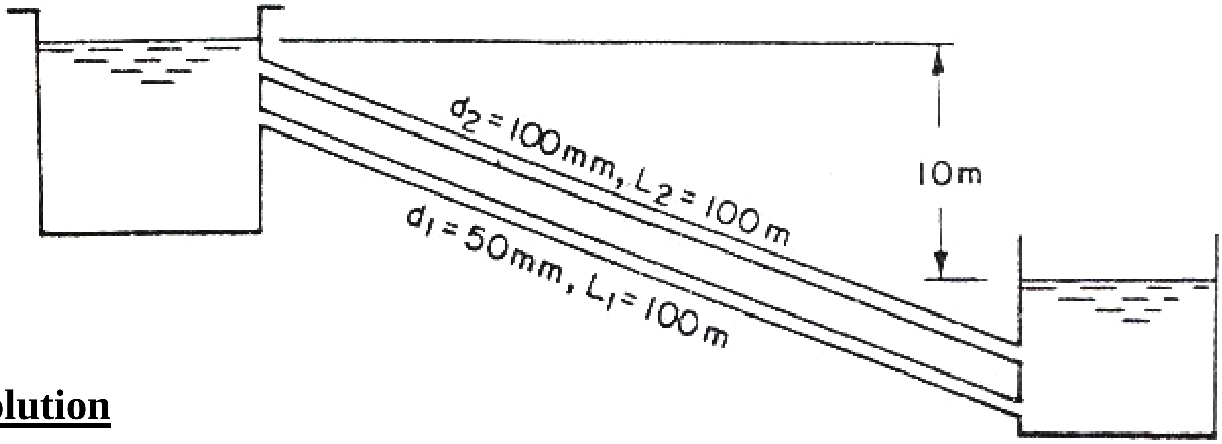
$Q_1 = 0.64 m3/s$

$$Q = 1.77 + 0.64 = 2.41 m3/s$$

Example 1-7

1- Determine the flow rate in each pipe (F=0.015)

2- Also, if the two pipes are replaced with one pipe of the same length determine the diameter which give the same flow.



Solution

$$H = h_{L1} = \frac{f \times L_1 \times V_1^2}{D_1 \times 2g} = \frac{0.032 \times 100 \times V_1^2}{0.05 \times 2 \times 9.81}$$

$$10 = 3.2619\, V_1^2$$

$$V_1 = \sqrt{\frac{10}{3.2619}} = 1.75 \text{ m/s}$$

$$\therefore Q_1 = V_1 \times A_1 = 1.75 \times \frac{\pi}{4}(d_1)^2$$

$$= 1.75 \times \frac{\pi}{4}(0.05)^2 = 0.00344 \text{ m}^3/\text{s}$$

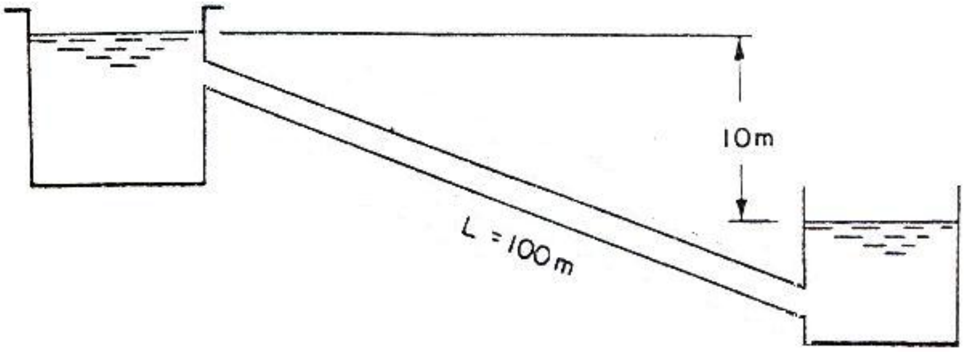
$$H = h_{L2} = \frac{f \times L_2 \times V_2^2}{d_2 \times 2g} = 10$$

$$V_2 = \sqrt{\frac{10 \times 0.10 \times 2 \times 9.81}{0.032 \times 100}} = 2.48 \text{ m/s}$$

$$\therefore Q_2 = V_2 \times A_2 = 2.48 \times \frac{\pi}{4}(d_2)^2$$

$$= 2.48 \times \frac{\pi}{4}(0.10)^2 = 0.0195 \text{ m}^3/\text{s}$$

<u>If the two pipes are replaced with one pipe of the same length</u>



$$Q = Q_1 + Q_2 = 0.00344 + 0.0195 = 0.02294$$

$$V = \frac{Q}{Area} = \frac{0.02294}{\frac{\pi}{4}D^2} = \frac{0.0292}{D^2}\ m/s$$

$$\mathrm{H} = h_L = \frac{f \times L \times V^2}{d \times 2g} = \frac{0.032 \times 100 \times \left(\frac{0.0292}{D^2}\right)^2}{D \times 2 \times 9.81}$$

$$10 = \frac{0.032 \times 100 \times (0.0292)^2}{D^5 \times 2 \times 9.81} = \frac{0.000139}{D^5}$$

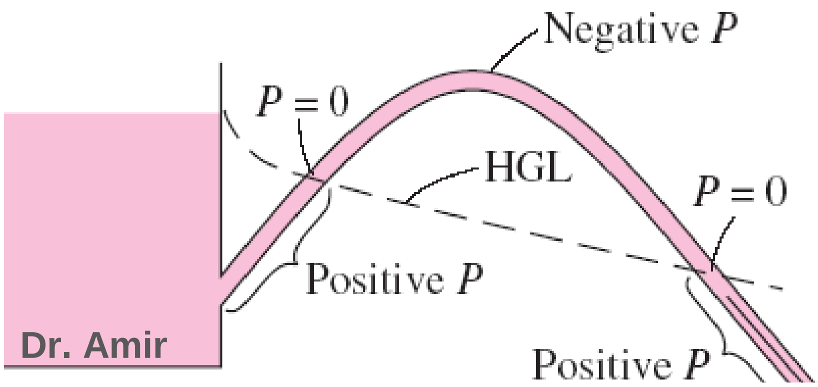
$$\mathrm{D}^5 = \frac{0.000139}{10} = 0.0000139$$

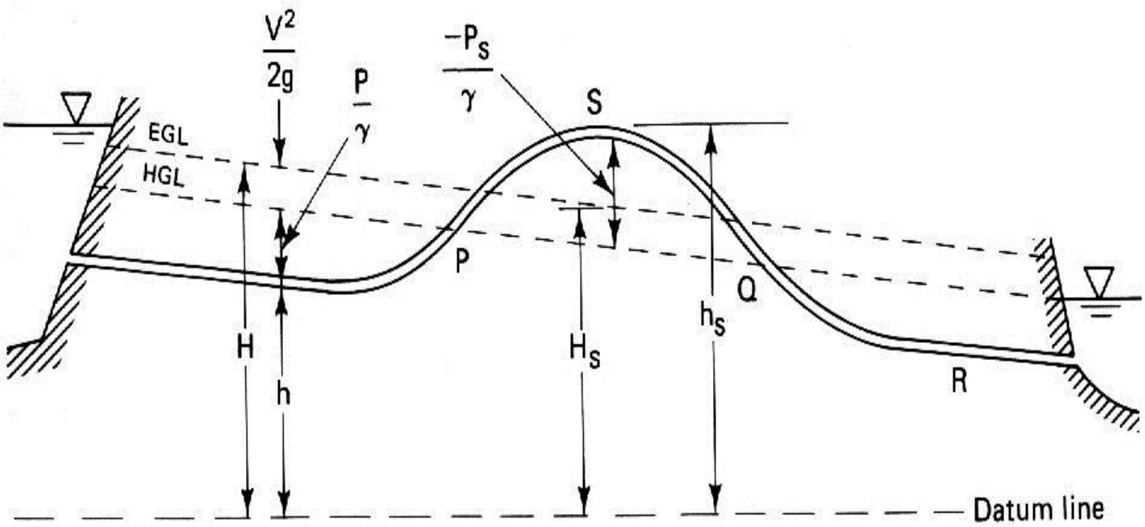
$$\therefore D = (0.0000139)^{\frac{1}{5}} = 0.1068\ \mathrm{m} = \mathbf{106.8\ mm.}$$

1.9 Pipe line with negative Pressure (syphon phenomena) ضغط سالب على الماسورة:

It is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high ground

Occasionally, a section of the pipeline may be raised to an elevation that is above ***the local HGL***.





$$\frac{V_p^{\ 2}}{2g}+\frac{P_p}{\gamma}+Z_p=\frac{V_S^{\ 2}}{2g}+\frac{P_S}{\gamma}+Z_S+h_L$$

$$Z_p-Z_S=\frac{V_S^{\ 2}}{2g}+\frac{P_S}{\gamma}+h_L$$

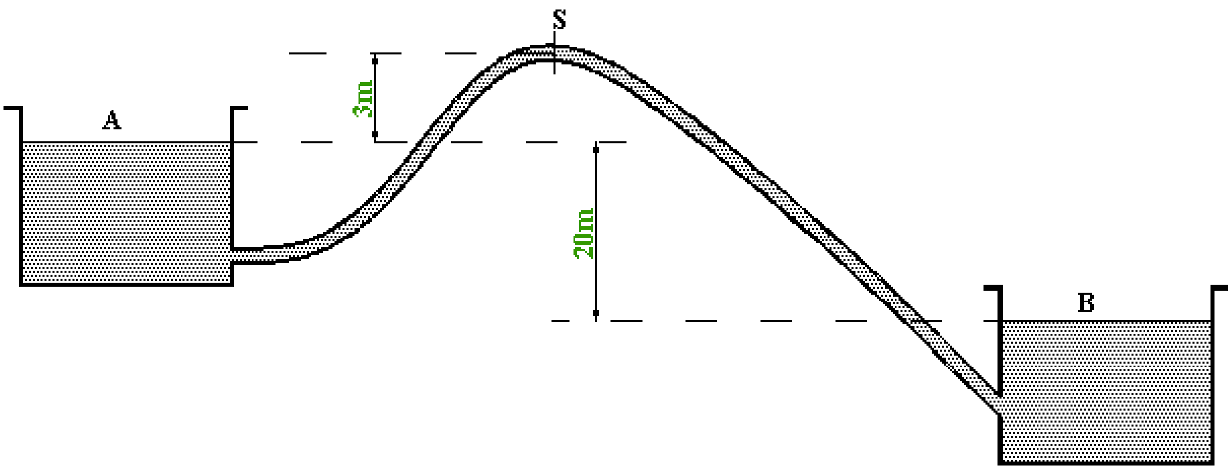
-ve value → Must be -ve value (below the atmospheric pressure)

Negative pressure exists in the pipelines wherever the pipe line is raised above the hydraulic gradient line (between P & Q)

The negative pressure at the summit point can reach theoretically -10.33 m water head (gauge pressure) and zero (absolute pressure). But in the practice water contains dissolved gasses that will vaporize before -10.33 m water head which reduces the pipe flow cross section.

Example 1-8

Syphon pipe between two pipe has diameter of 20cm and length 500m as shown. The difference between reservoir levels is 20m. The distance between reservoir A and summit point S is 100m. Calculate the flow in the system and the pressure head at summit. F=0.02



Solution

D = 0.2 m, Z_A-Z_B = 20m, L = 500m, L_{AS} = 100m, f = 0.02

$$Z_A - Z_B = h_f = f\ \frac{L}{D}\frac{V^2}{2g}$$

$$20 = 0.02 \times \frac{500}{0.2} \times \frac{V^2}{2\times 9.81}$$

V = 2.8 m/s

$Q = VA = 0.08796\ m^3/s$

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + Z_A = \frac{V_S^2}{2g} + \frac{P_S}{\gamma} + Z_S + h_L$$

$$Z_A - Z_S = \frac{V_S^{\ 2}}{2g} + \frac{P_S}{\gamma} + h_L$$

$$0 - 3 = \frac{2.8^2}{2g} + 0.02\frac{100}{0.2}\times\frac{2.8^2}{2g} + \frac{P_S}{\gamma}$$

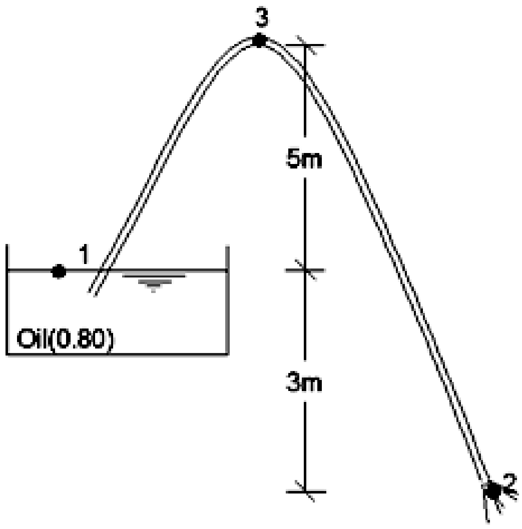
$$\frac{P_S}{\gamma} = \text{-7.396 m } \textit{of water}$$

Best regards,
Dr. Amir Mobasher

Example 1-9

A syphon filled with oil of specific gravity 0.8 discharges 220 lit/s to the atmosphere at an elevation of 3.0 m below oil level. The siphon is 0.2 m in diameter and its invert is 5.0 m above oil level. Find the losses in the siphon in terms of the velocity head. Find the pressure at the invert if two thirds of the losses occur in the first leg.

Solution



$v = Q / A = 0.22/ (\pi/4\ (0.2)2) = 7$ m/sec

Apply B.E bet. 1 & 2

$Z1 + v12 / 2g + P1 / \gamma = Z2 + v22 / 2g + P2 / \gamma + hl$

$5 + 0 + 0 = 0 + 0 + (7)2/2g + hl$

$hl = 0.50$ m

Apply B.E bet. 1 & 3

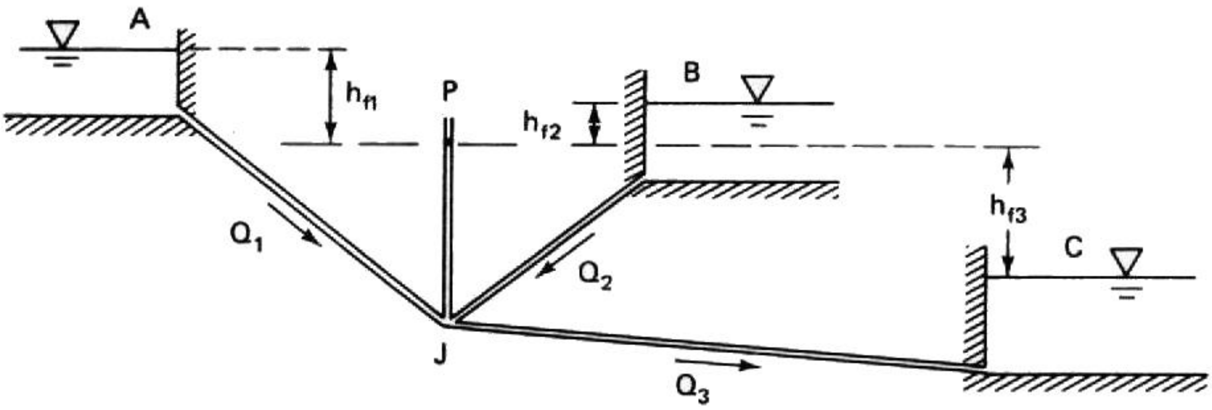
$Z1 + v12 / 2g + P1 / \gamma = Z3 + v32 / 2g + P3 / \gamma + hl$

$5 + 0 + 0 = 8 + (7)2/2g + P3 / \gamma + 2/3 * 0.5$

$P3 / \gamma = -7.833$ m

$P3 = -7.833$ m of oil $= -61474.6\ N/m^2$

1.10 Three-reservoirs problem حالة الخزانات الثلاثة:



- This system must satisfy:
  - The quantity of water brought to junction "J" is equal to the quantity of water taken away from the junction:

$$Q_3 = Q_1 + Q_2$$ **Flow Direction????**

  - All pipes that meet at junction "J" must share the same pressure at the junction.

1.10.1 Types of three-reservoirs problem:

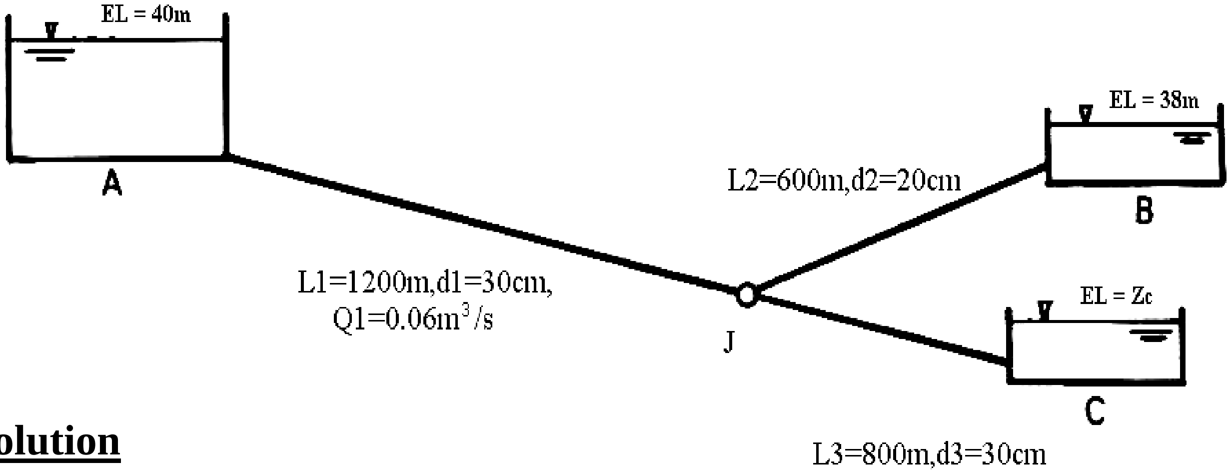
- **Type 1:**

- Given the lengths , diameters, and materials of all pipes involved;
  D_1 , D_2 , D_3 , L_1 , L_2 , L_3 , *and e or F*
- Given the water elevation in any two reservoirs,
  Z_1 *and* Z_2 *(for example)*
- Given the flow rate from any one of the reservoirs,
  Q_1 or Q_2 or Q_3
- Determine the elevation of the third reservoir Z_3 (for example) and the rest of Q's.

This types of problems can be solved by simply using:
- **Bernoulli's equation** for each pipe
- **Continuity equation** at the junction.

Example 1-10

In the following figure determine the flow in pipe BJ & pipe CJ. Also, determine the water elevation in tank C



Solution

Applying Bernoulli Equation between A , J :

$$V_1 = \frac{Q_1}{A_1} = \frac{0.06}{\frac{\pi}{4}(0.3)^2} = 0.849\ m/s$$

$$Z_A - Z_P = F_1 \frac{L_1}{D_1}.\frac{V_1^2}{2g} \longrightarrow 40 - Z_P = 0.024 \times \frac{1200}{0.3} \times \frac{0.849^2}{2 \times 9.81}$$

$$Z_P = 36.475\ m$$

Applying Bernoulli Equation between B , J :

$$Z_B - Z_P = F_2 \frac{L_2}{D_2}.\frac{V_2^2}{2g} \longrightarrow 38 - 36.475 = 0.024 \times \frac{600}{0.2} \times \frac{V_2^2}{2 \times 9.81}$$

$$V_2 = 0.645 m/s \longrightarrow Q_2 = 0.0203\ m^3/s$$

Applying Bernoulli Equation between C , J :

$$\sum Q = Q_1 + Q_2 + Q_3 = 0$$

$$Q_3 = -Q_1 - Q_2 = -0.06 - 0.0203 = -0.0803\ m^3 / s$$

$$V_3 = \frac{Q_3}{A_3} = \frac{0.0803}{\frac{\pi}{4}(0.3)^2} = 1.136\ m/s$$

$$Z_P - Z_C = f_3 \frac{L_3}{D_3}.\frac{V_3^2}{2g} \longrightarrow 36.475 - Z_c = 0.024 \times \frac{800}{0.3} \times \frac{1.136^2}{2g}$$

$$Z_c = 32.265\ m$$

❑ Type 2:

- given the lengths , diameters, and materials of all pipes involved;

  $D_1 , D_2 , D_3 , L_1 , L_2 , L_3$, *and e or F*

- given the water elevation in each of the three reservoirs,

  Z_1 , Z_2 , Z_3

  determine the discharges to or from each reservoir,

  Q_1 , Q_2 ,*and* Q_3 .

1- First assume a piezometric surface elevation, *P*, at the junction.

2- This assumed elevation gives the head losses hf_1, hf_2, and hf_3

3- From this set of head losses and the given pipe diameters, lengths, and material, the trail computation gives a set of values for discharges Q_1 , Q_2 ,*and* Q_3 .

4- If the assumed elevation *P* is correct, the computed *Q's* should satisfy:

5- Otherwise, a new elevation *P* is assumed for the second trail.

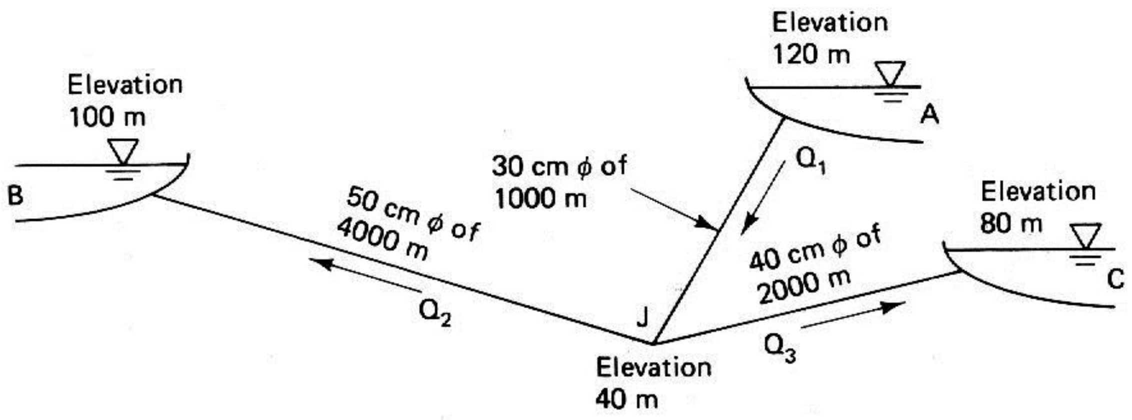
The computation of another set of *Q's* is performed until the above condition is satisfied

This types of problems are ***most conveniently*** solved by **trail and error**

Example 1-11

In the following figure determine the flow in each pipe.

Pipe	**CJ**	**BJ**	**AJ**
Length m	2000	4000	1000
Diameter cm	40	50	30
F	0.022	0.021	0.024



Solution

Trial 1

$Z_P = 110m$

Applying Bernoulli Equation between A , J :

$$Z_A - Z_P = F_1 \frac{L_1}{D_1} \cdot \frac{V_1^2}{2g} \longrightarrow 120 - 110 = 0.024 \times \frac{1000}{0.3} \times \frac{V_1^2}{2g}$$

V1 = 1.57 m/s , Q1 = 0.111 m3/s

Applying Bernoulli Equation between B , J :

$$Z_P - Z_B = F_2 \frac{L_2}{D_2} \cdot \frac{V_2^2}{2g} \longrightarrow 110 - 100 = 0.021 \times \frac{4000}{0.5} \times \frac{V_2^2}{2g}$$

V2 = 1.08 m/s , Q2 = - 0.212 m3/s

Applying Bernoulli Equation between C , J :

$$Z_P - Z_C = F_3 \frac{L_3}{D_3} \cdot \frac{V_3^2}{2g} \longrightarrow 110 - 80 = 0.022 \times \frac{2000}{0.4} \times \frac{V_3^2}{2g}$$

V3 = 2.313 m/s , Q2 = - 0.291 m3/s

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.111 - 0.212 - 0.291 = -0.392 \neq 0$$

Trial 2

$Z_P = 100m$

Q1 = 0.157
Q2 = 0
Q3 = -0.237

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.157 + 0 - 0.237 = -0.08\, m^3/s \neq 0$$

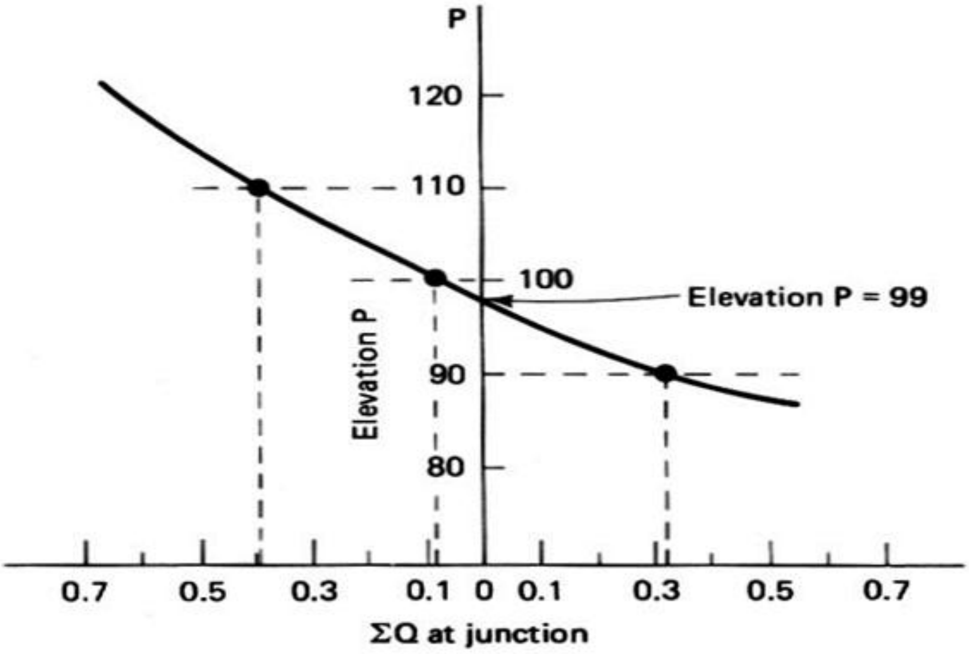
Trial 3

$Z_P = 90m$

Q1 = 0.192
Q2 = 0.3
Q3 = -0.168

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.192 + 0.3 - 0.168 = 0.324\, m^3/s \neq 0$$

Draw the relationship between $\sum Q$ and P



$$\therefore \sum Q = 0 \Rightarrow \text{at } P = 99\text{m}$$

Q1 = 0.160
Q2 = 0.067
Q3 = -0.231

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.16 + 0.067 - 0.0231 = 0.004\, m^3/s \approx 0$$

CHAPTER 2

PIPE NETWORKS

2.1 Introduction

To deliver water to individual consumers with appropriate quality, quantity, and pressure in a community setting requires an extensive system of:

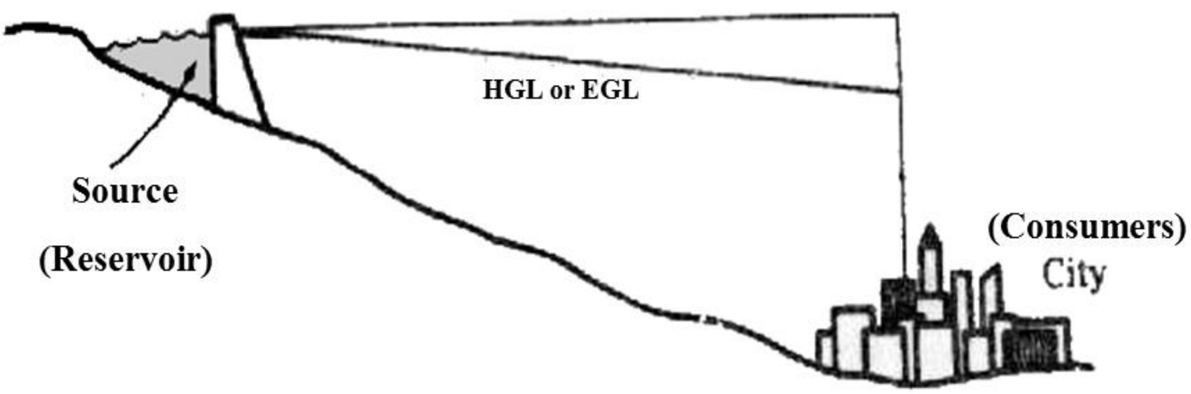
- Pipes.
- Storage reservoirs.
- Pumps.
- Other related accessories.

Distribution system: is used to describe collectively the facilities used to supply water from its source to the point of usage.

2.2 Methods of Supplying Water

2.2.1 Gravity Supply

The source of supply is at a sufficient elevation above the distribution area (consumers).



❑ Advantages of Gravity supply

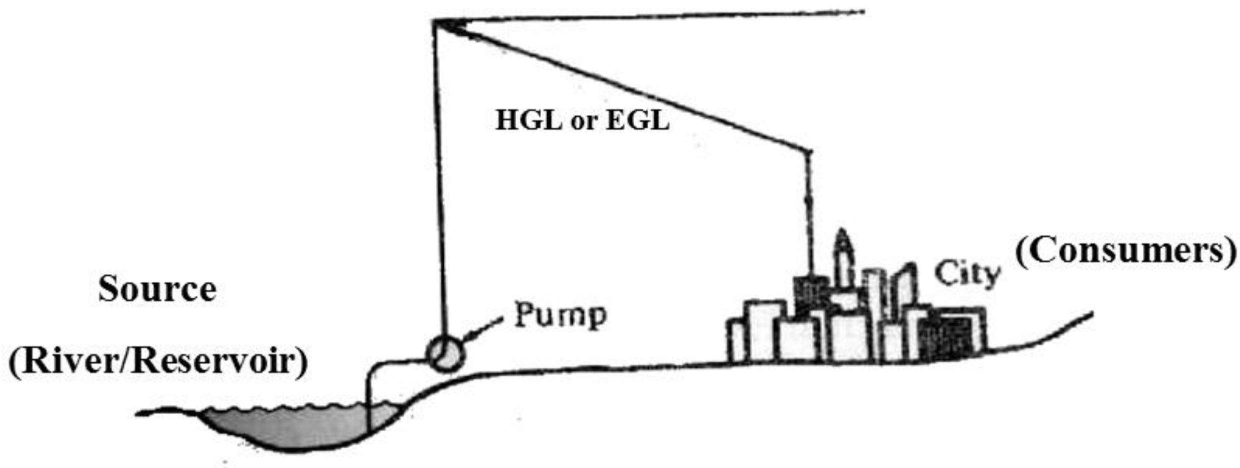
- No energy costs.
- Simple operation (fewer mechanical parts, independence of power supply, ….)
- Low maintenance costs.
- No sudden pressure changes

2.2.2 Pumped Supply

Used whenever:

- The source of water is lower than the area to which we need to distribute water to (consumers)
- The source cannot maintain minimum pressure required.

➔ Pumps are used to develop the necessary head (pressure) to distribute water to the consumer and storage reservoirs.



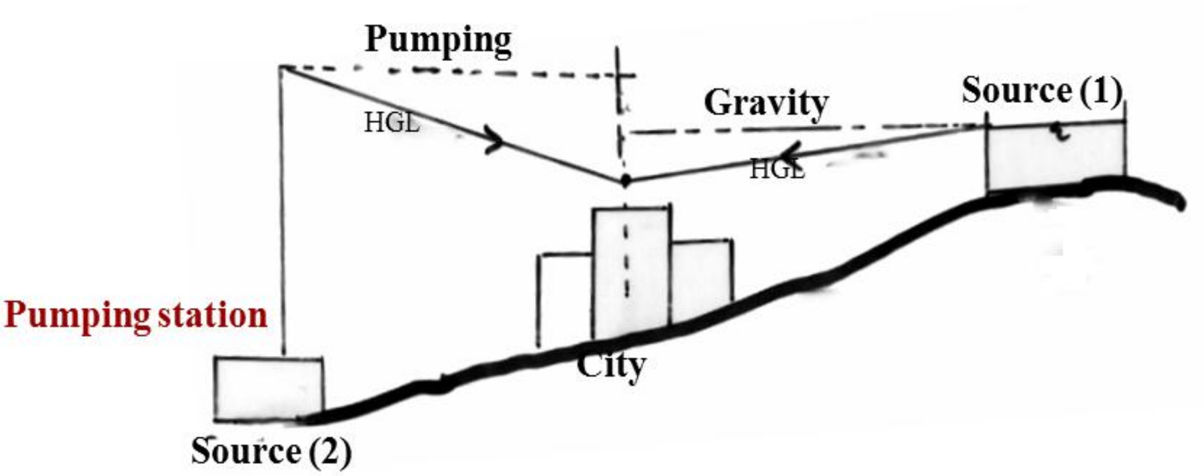
❑ Disadvantages of pumped supply

- Complicated operation and maintenance.
- Dependent on reliable power supply.
- Precautions have to be taken in order to enable permanent supply:
  - Stock with spare parts
  - Alternative source of power supply ….

2.2.3 Combined Supply (pumped-storage supply)

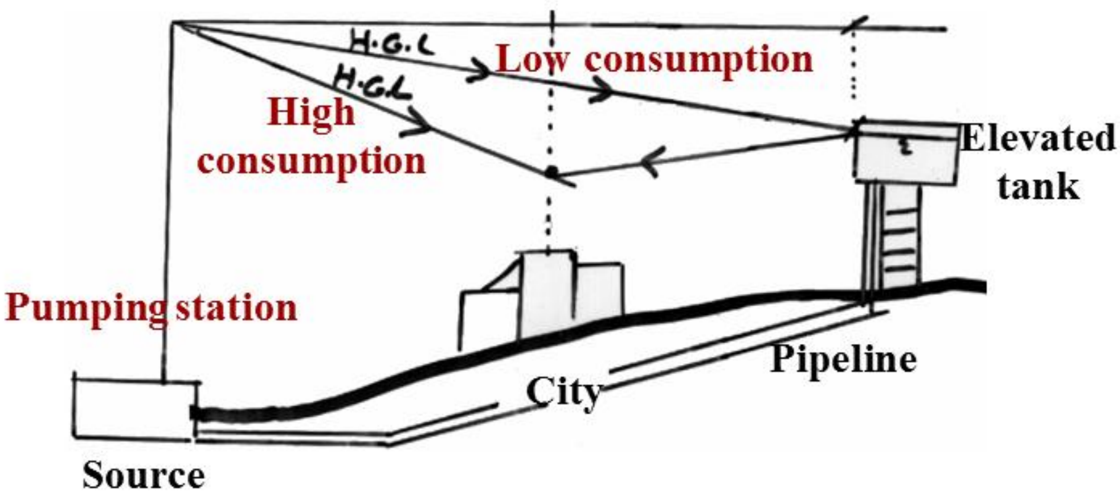
Both pumps and storage reservoirs are used. This system is usually used in the following cases:

3-When two sources of water are used to supply water:



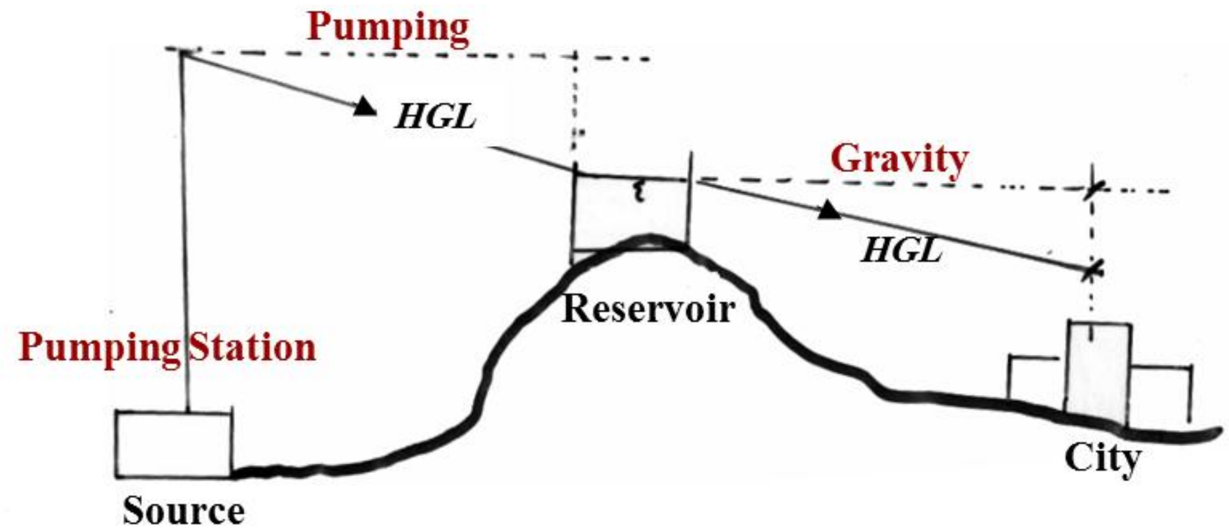
4-In the pumped system sometimes a storage (elevated) tank is connected to the system.

- When the water consumption is low, the residual water is pumped to the tank.
- When the consumption is high the water flows back to the consumer area by gravity.



5-When the source is lower than the consumer area

- A tank is constructed above the highest point in the area,
- Then the water is pumped from the source to the storage tank (reservoir).
- And the hence the water is distributed from the reservoir by gravity.

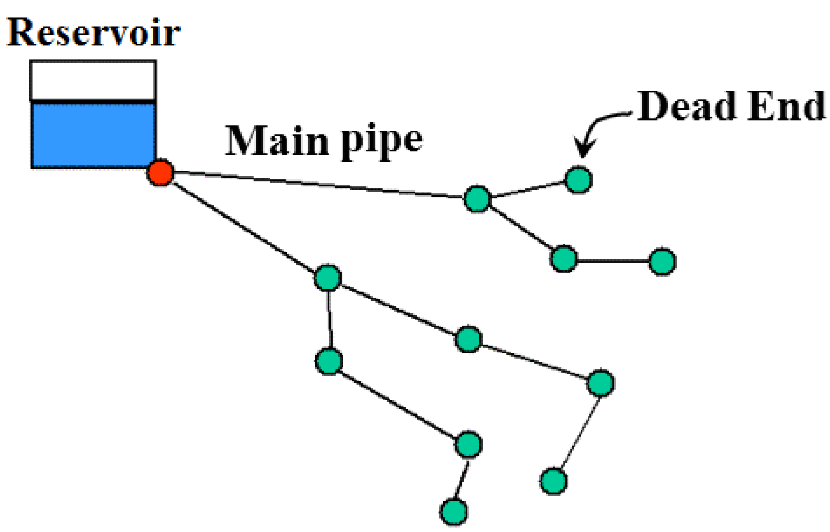


2.3 Distribution Systems (Network Configurations):

In laying the pipes through the distribution area, the following configuration can be distinguished:

- Branching system (Tree)
- Grid system (Looped)
- Combined system

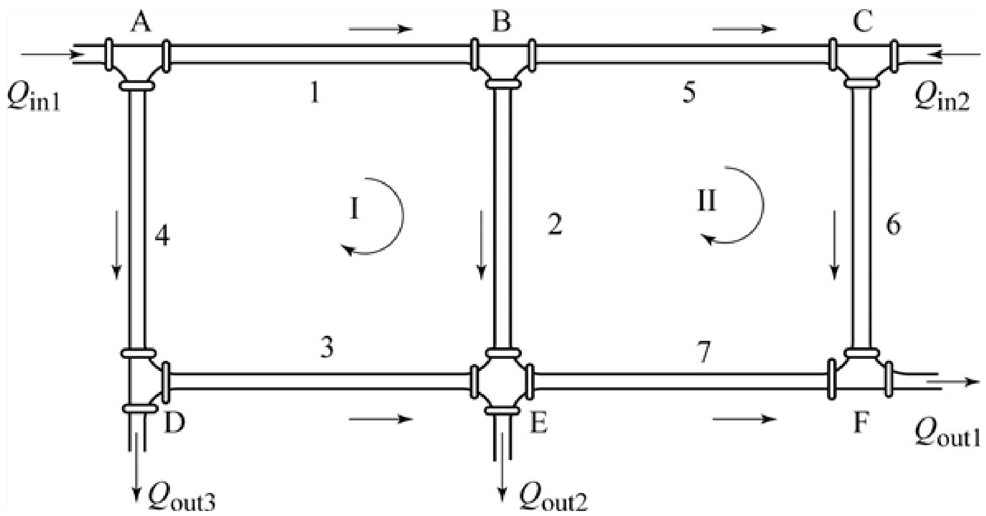
2.3.1 Branching System (Tree System)



- **Advantages:**
  - Simple to design and build.
  - Less expensive than other systems.

- **Disadvantages:**
  - The large number of dead ends which results in sedimentation and bacterial growths.
  - When repairs must be made to an individual line, service connections beyond the point of repair will be without water until the repairs are made.
  - The pressure at the end of the line may become undesirably low as additional extensions are made.

2.3.2 Grid System (Looped system)



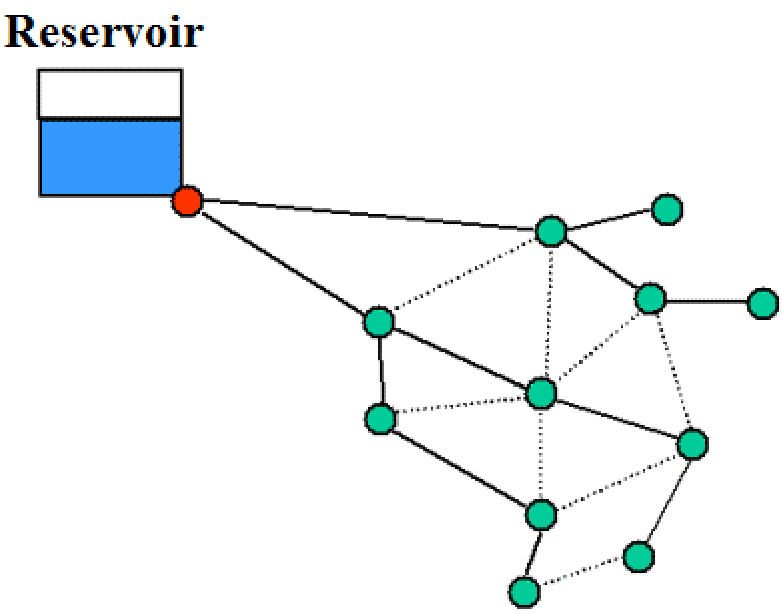
- **Advantages:**
  - The grid system overcomes all of the difficulties of the branching system discussed before.
  - No dead ends. (All of the pipes are interconnected).
  - Water can reach a given point of withdrawal from several directions.

- **Disadvantages:**
  - Hydraulically far more complicated than branching system (Determination of the pipe sizes is somewhat more complicated).
  - Expensive (consists of a large number of loops).

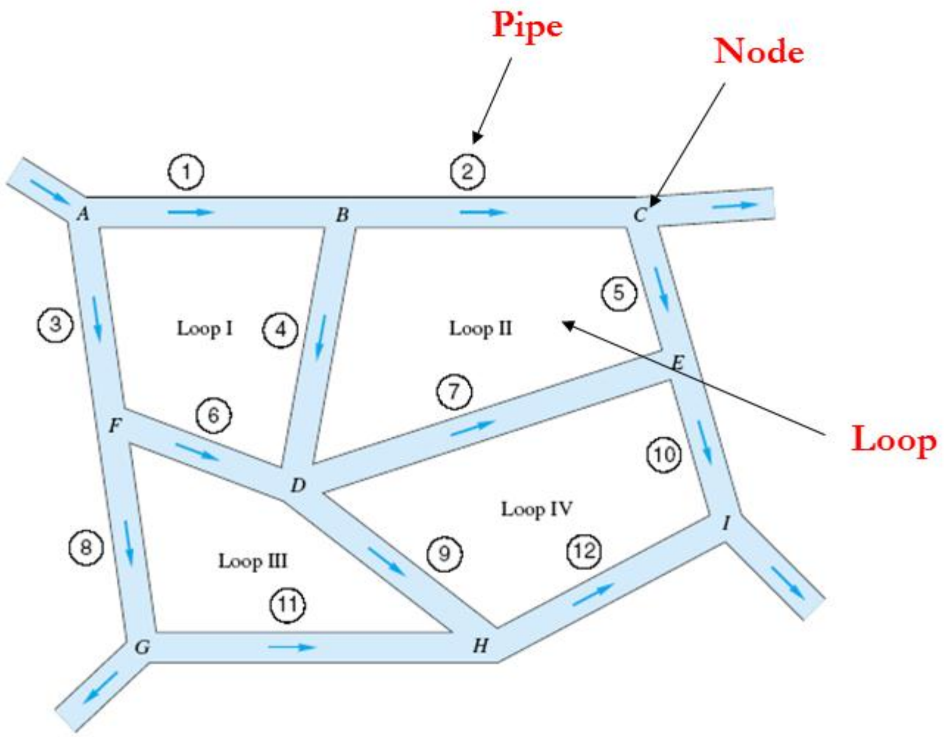
But, it is the most reliable and used system.

2.3.3 Combined System

- It is a combination of both Grid and Branching systems.
- This type is widely used all over the world.



2.4 Pipe Networks



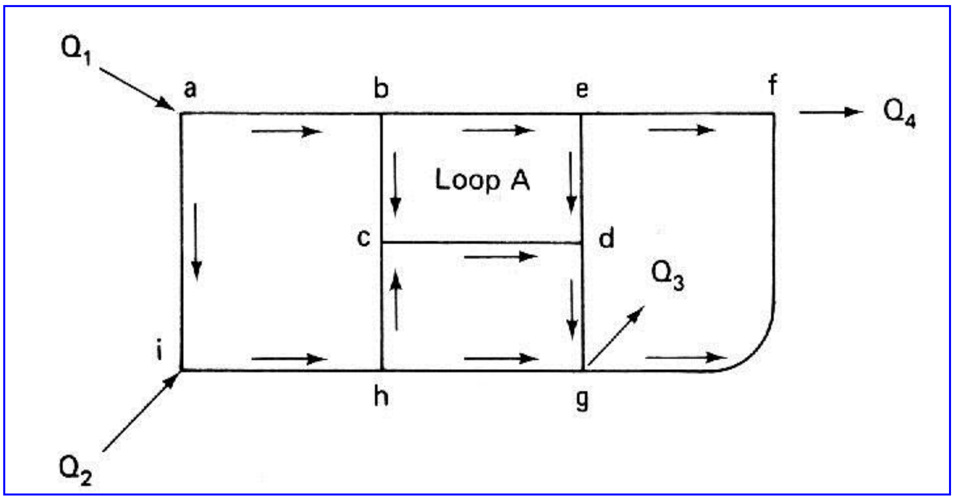
The equations to solve Pipe network must satisfy the following condition:

- The net flow into any junction must be zero.

$$\sum Q = 0$$

- The net head loss a round any closed loop must be zero. The HGL at each junction must have one and only one elevation
- All head losses must satisfy the Moody and minor-loss friction correlation

2.4.1 Hardy Cross Method



- This method is applicable to closed-loop pipe networks (a complex set of pipes in parallel).
- It depends on the idea of head balance method
- Was originally devised by professor Hardy Cross.

Assumptions / Steps of this method:

1- Assume that the water is withdrawn from nodes only; not directly from pipes.
2- The discharge, Q , entering the system will have (+) value, and the discharge, Q , leaving the system will have (-) value.
3- Usually neglect minor losses since these will be small with respect to those in long pipes, i.e.; Or could be included as equivalent lengths in each pipe.
4- Assume flows for each individual pipe in the network.
5- At any junction (node), as done for pipes in parallel,

$$\sum Q_{in} = \sum Q_{out} \quad \textbf{or} \quad \sum Q = 0$$

6. Around any loop in the grid, the sum of head losses must equal to zero:

$$\sum_{loop} h_f = 0$$

- The probability of initially guessing all flow rates correctly is virtually null.
- Therefore, to balance the head around each loop, a flow rate correction (Δ) for each loop in the network should be computed, and hence some iteration scheme is needed.

How to find the correction value (Δ)

$$h_F = kQ^n \longrightarrow (1)$$
$$n = 2 \Rightarrow Darcy, Manning$$
$$n = 1.85 \Rightarrow Hazen \quad William$$

$$Q = Q_o + \Delta \longrightarrow (2)$$

from 1 & 2

$$h_f = kQ^n = k(Q_o + \Delta)^n = k\left[Q_o^n + nQ_o^{n-1}\Delta + \frac{n(n-1)}{2}Q_o^{n-2}\Delta^2 + \ldots\right]$$

Neglect terms contains Δ^2 $\quad h_f = kQ^n = k\left(Q_o^n + nQ_o^{n-1}\Delta\right)$

For each loop

$$\sum_{loop} h_F = \sum_{loop} kQ^n = 0$$

$$\therefore \sum kQ^n = \sum kQ_o^n + \sum nkQ^{(n-1)}\Delta = 0$$

$$\Delta = \frac{-\sum kQ_o^n}{\sum nkQ_o^{(n-1)}} = \frac{-\sum h_F}{n\sum \frac{h_F}{Q_o}}$$

- Note that if Hazen Williams (which is generally used in this method) is used to find the head losses, then

  $h_f = k\,Q^{1.85}$ $\quad$ ($n = 1.85$) , then

$$\Delta = \frac{-\sum h_f}{1.85\sum \frac{h_f}{Q}}$$

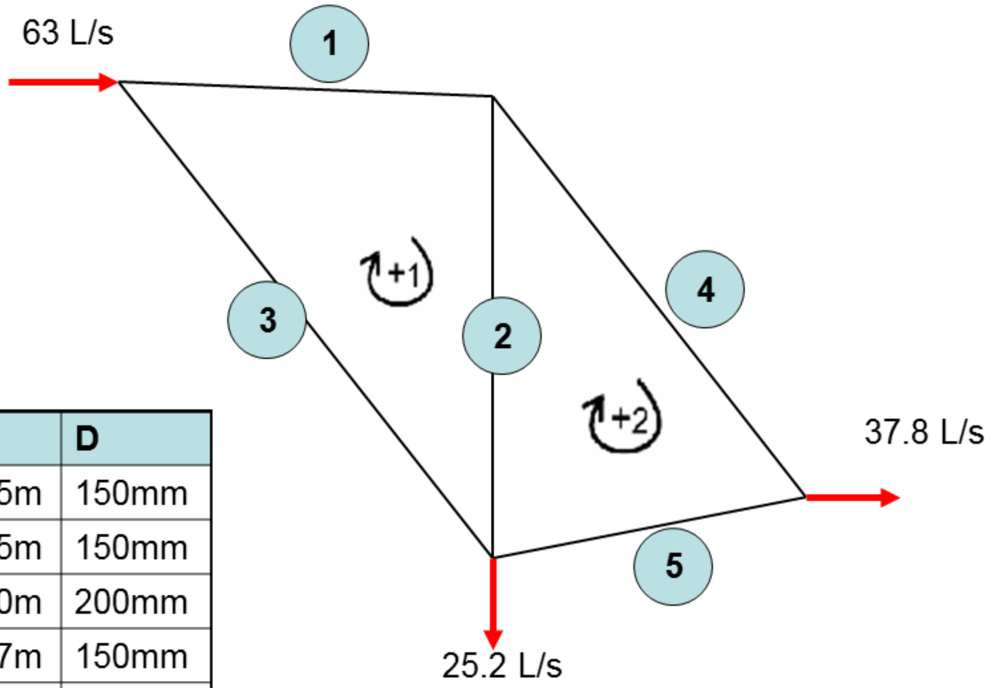
- If Darcy-Wiesbach is used to find the head losses, then

  $h_f = k\,Q^2$ $\quad$ ($n = 2$) , then

$$\Delta = \frac{-\sum h_f}{2\sum \frac{h_f}{Q}}$$

Example 2-1

Solve the following pipe network using Hazen William Method C_{HW} =100



pipe	L	D
1	305m	150mm
2	305m	150mm
3	610m	200mm
4	457m	150mm
5	153m	200mm

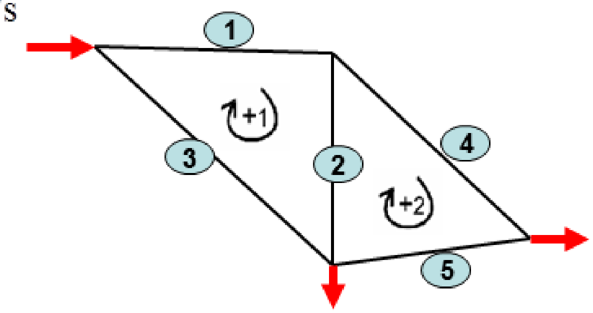
Solution

$$h_f = \frac{10.7\,L}{C_{HW}^{\;1.852}\,D^{4.87}}\;Q^{1.852} \Rightarrow C_{HW} = 100, Q = in \;\; \text{L/s}$$

$$h_f = \frac{10.7\,L}{C_{HW}^{\;1.852}\,D^{4.87}}\left(\frac{Q}{1000}\right)^{1.852}$$

$$h_f = \left\{6.02\times10^{-9}\,\frac{L}{D^{4.87}}\right\}Q^{1.852}$$

$$h_f = \{K\}Q^{1.852}$$



1st Iteration

Loop	Pipe	Dia (m)	L (m)	*K*	Q∘ (L/s)	h_f (m)	h_f/Q∘ (m/L/s)	Correction L/s	Q L/s
1	1	0.150	305	0.0187	+24.0	+6.68	0.28	-0.24	+23.76
	2	0.150	305	0.0187	+11.4	+1.69	0.15	-0.24+0.57	+11.73
	3	0.200	610	0.0092	-39.0	-8.09	0.21	-0.24	-39.24
						+0.28	0.64		
2	2	0.150	305	0.0187	-11.4	-1.69	0.15	-0.57+0.24	-11.73
	4	0.150	457	0.0280	+12.6	+3.04	0.24	-0.57	+12.03
	5	0.200	153	0.0023	-25.2	-0.90	0.04	-0.57	-25.77
						+0.45	0.43		

$$\Delta_1 = \frac{-\sum h_F}{n\sum \frac{h_F}{Q_o}} = \frac{-0.28}{1.85(0.64)} = -0.24$$

$$\Delta_2 = \frac{-\sum h_F}{n\sum \frac{h_F}{Q_o}} = \frac{-0.45}{1.85(0.43)} = -0.57$$

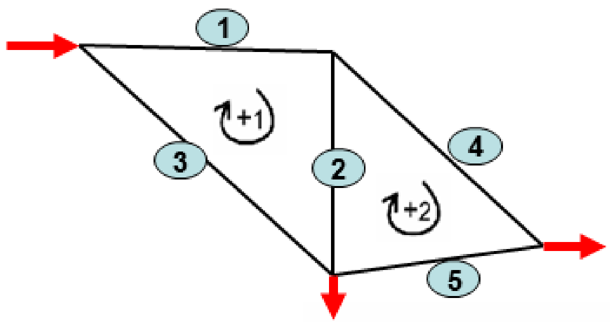
for pipe 2 in loop 1

$\Delta = \Delta_1 - \Delta_2$

for pipe 2 in loop 2

$\Delta = \Delta_2 - \Delta_1$

Dr. Amir Mobasher



2nd Iteration

Loop	Pipe	Dia (m)	L (m)	*K*	Q∘ (L/s)	h_f (m)	h_f/Q∘ (m/L/s)	Correction L/s	Q L/s
1	1	0.150	305	0.0187	+23.76	+6.56	0.28	-0.15	+23.61
	2	0.150	305	0.0187	+11.73	+1.79	0.15	-0.15+0.09	+11.67
	3	0.200	610	0.0092	-39.24	-8.17	0.21	-0.15	-39.39
						+0.18	0.64		
2	2	0.150	305	0.0187	-11.73	-1.78	0.15	-0.09+0.15	-11.67
	4	0.150	457	0.0280	+12.03	+2.79	0.23	-0.09	+11.94
	5	0.200	153	0.0023	-25.77	-0.94	0.04	-0.09	-25.86
						+0.07	0.42		

$$\Delta_1 = \frac{-\sum h_F}{n\sum \frac{h_F}{Q_o}} = \frac{-0.18}{1.85(0.64)} = -0.15$$

$$\Delta_1 = \frac{-\sum h_F}{n\sum \frac{h_F}{Q_o}} = \frac{-(0.07)}{1.85(0.42)} = -0.09$$

for pipe 2 in loop 1

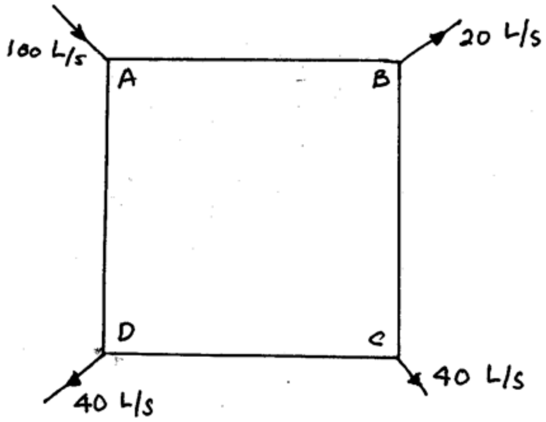
$$\Delta = \Delta_1 - \Delta_2$$

for pipe 2 in loop 2

$$\Delta = \Delta_2 - \Delta_1$$

Example 2-2

For the square loop shown, find the discharge in all the pipes. All pipes are 1 km long and 300 mm in diameter, with a friction factor of 0.0163. Assume that minor losses can be neglected.



Solution

$$H_L = h_f = F\frac{L}{D}\frac{V^2}{2g} = \frac{8FL}{g\pi^2 D^5}Q^2$$

$$H_L = \frac{8x0.0163x1000}{9.81x\pi^2x0.30^5}Q^2$$

$$H_L = 554Q^2$$

$$H_L = K'Q^2$$

$$\therefore \quad K' = 554$$

1. **First trial**

Pipe	Q (L/s)	H_L (m)	H_L/Q
AB	60	2.0	0.033
BC	40	0.886	0.0222
CD	0	0	0
AD	-40	-0.886	0.0222
Σ		2.00	0.0774

Since $\sum H_L$ > 0.01 m, then correction has to be applied.

$$\Delta Q = -\frac{\sum H_L}{2\sum H_L/Q} = -\frac{2}{2x0.0774} = -12.92\, L/s$$

2. **Second trial**

Pipe	Q (L/s)	H_L (m)	H_L/Q
AB	47.08	1.23	0.0261
BC	27.08	0.407	0.015
CD	-12.92	-0.092	0.007
AD	-52.92	-1.555	0.0294
Σ		-0.0107	0.07775

Since $\sum H_L \approx 0.01$ m, then it is OK.

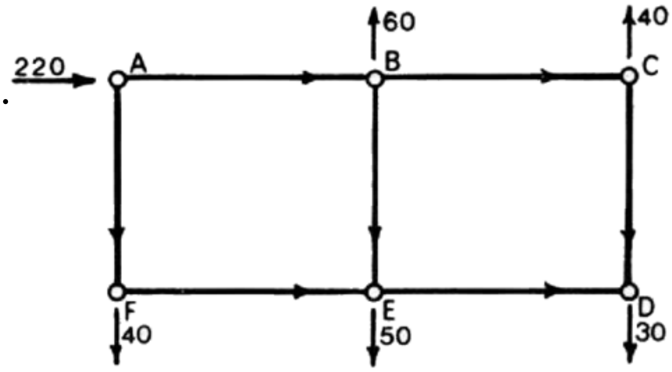
Thus, the discharge in each pipe is as follows (to the nearest integer).

Pipe	Discharge (L/s)
AB	47
BC	27
CD	-13
AD	-53

Example 2-3

The following example contains nodes with different elevations and pressure heads. Neglecting minor loses in the pipes, determine:

- The flows in the pipes.
- The pressure heads at the nodes.



Pipe	AB	BC	CD	DE	EF	AF	BE
Length (m)	600	600	200	600	600	200	200
Diameter (mm)	250	150	100	150	150	200	100

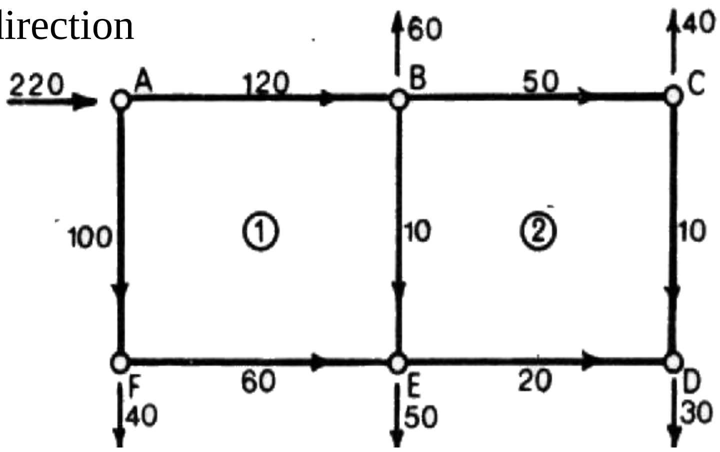
Roughness size of all pipes = 0·06 mm
Pressure head elevation at A = 70 m o.d.

Elevation of pipe nodes

Node	A	B	C	D	E	F
Elevation (m o.d.)	30	25	20	20	22	25

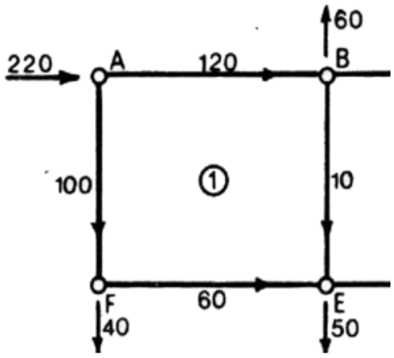
Solution

Assume flows magnitude and direction



First Iteration

- Loop (1)

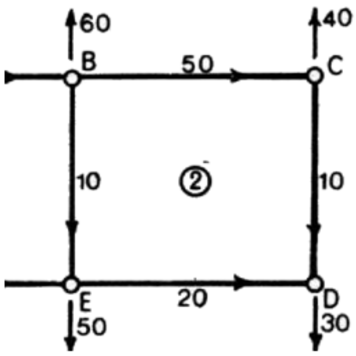


Pipe	*L* (m)	*D* (m)	*Q* (m^3/s)	*f*	h_f (m)	h_f/Q ($m/m^3/s$)
AB	600	0.25	0.12	0.0157	11.48	95.64
BE	200	0.10	0.01	0.0205	3.38	338.06
EF	600	0.15	-0.06	0.0171	-40.25	670.77
FA	200	0.20	-0.10	0.0162	-8.34	83.42
				Σ	-33.73	1187.89

$$\Delta = -\frac{-33.73}{2(1187.89)} = 0.01419\ m^3/s = 14.20\ L/s$$

First Iteration

- Loop (2)

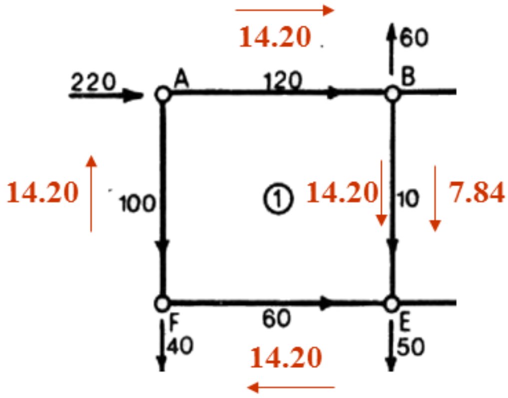


Pipe	*L* (m)	*D* (m)	*Q* (m^3/s)	*f*	h_f (m)	h_f/Q ($m/m^3/s$)
BC	600	0.15	0.05	0.0173	28.29	565.81
CD	200	0.10	0.01	0.0205	3.38	338.05
DE	600	0.15	-0.02	0.0189	-4.94	246.78
EB	200	0.10	-0.01	0.0205	-3.38	338.05
				Σ	23.35	1488.7

$$\Delta = -\frac{23.35}{2(1488.7)} = -0.00784\ m^3/s = -7.842\ L/s$$

Second Iteration

- Loop (1)

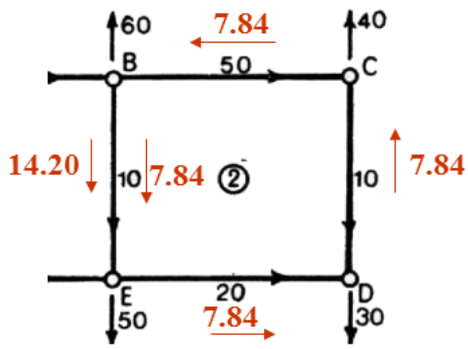


Pipe	*L* (m)	*D* (m)	*Q* (m³/s)	*f*	h_f (m)	h_f/Q (m/m³/s)
AB	600	0.25	0.1342	0.0156	14.27	106.08
BE	200	0.10	0.03204	0.0186	31.48	982.60
EF	600	0.15	-0.0458	0.0174	-23.89	521.61
FA	200	0.20	-0.0858	0.0163	-6.21	72.33
				Σ	15.65	1682.62

$$\Delta = -\frac{15.65}{2(1682.62)} = -0.00465\ \text{m}^3/\text{s} = -4.65\ \text{L/s}$$

Second Iteration

- Loop (2)

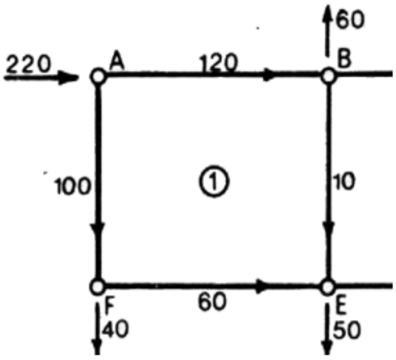


Pipe	*L* (m)	*D* (m)	*Q* (m³/s)	*f*	h_f (m)	h_f/Q (m/m³/s)
BC	600	0.15	0.04216	0.0176	20.37	483.24
CD	200	0.10	0.00216	0.0261	0.20	93.23
DE	600	0.15	-0.02784	0.0182	-9.22	331.23
EB	200	0.10	-0.03204	0.0186	-31.48	982.60
				Σ	-20.13	1890.60

$$\Delta = -\frac{-20.13}{2(1890.3)} = 0.00532\ \text{m}^3/\text{s} = 5.32\ \text{L/s}$$

Third Iteration

- Loop (1)

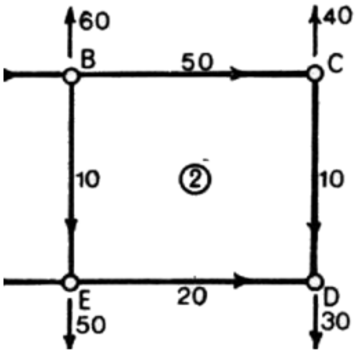


Pipe	*L* (m)	*D* (m)	*Q* (m^3/s)	*f*	h_f (m)	h_f/Q ($m/m^3/s$)
AB	600	0.25	0.1296	0.0156	13.30	102.67
BE	200	0.10	0.02207	0.0190	15.30	693.08
EF	600	0.15	-0.05045	0.0173	-28.78	570.54
FA	200	0.20	-0.09045	0.0163	-6.87	75.97
				Σ	-7.05	1442.26

$$\Delta = -\frac{-7.05}{2(1442.26)} = 0.00244 \text{ m}^3/\text{s} = 2.44 \text{ L/s}$$

Third Iteration

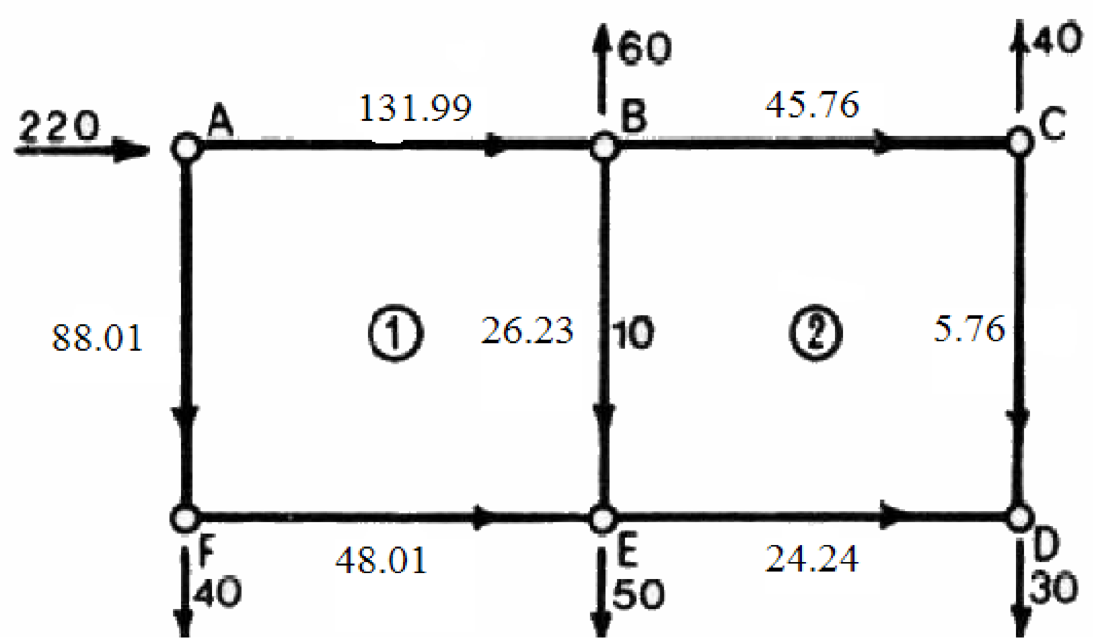
- Loop (2)



Pipe	*L* (m)	*D* (m)	*Q* (m^3/s)	*f*	h_f (m)	h_f/Q ($m/m^3/s$)
BC	600	0.15	0.04748	0.0174	25.61	539.30
CD	200	0.10	0.00748	0.0212	1.96	262.11
DE	600	0.15	-0.02252	0.0186	-6.17	274.07
EB	200	0.10	-0.02207	0.0190	-15.30	693.08
				Σ	6.1	1768.56

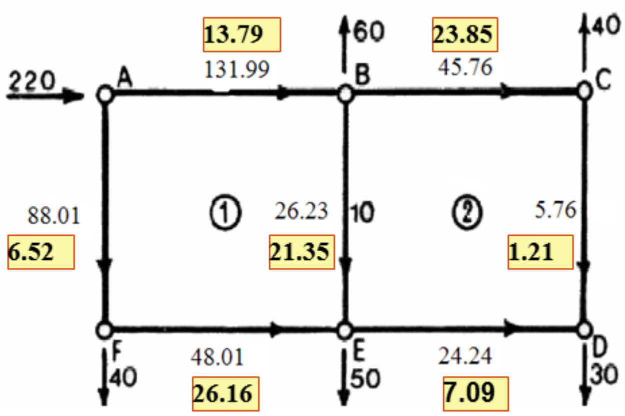
$$\Delta = -\frac{6.1}{2(1768.56)} = -0.00172 \text{ m}^3/\text{s} = -1.72 \text{ L/s}$$

After applying Third

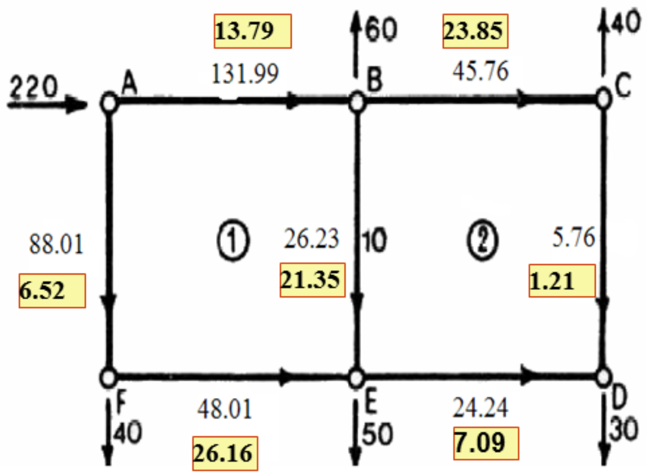


Velocity and Pressure Heads:

pipe	Q (l/s)	V (m/s)	h_f (m)
AB	131.99	2.689	13.79
BE	26.23	3.340	21.35
FE	48.01	2.717	26.16
AF	88.01	2.801	6.52
BC	45.76	2.589	23.85
CD	5.76	0.733	1.21
ED	24.24	1.372	7.09



Node	$p/\gamma+Z$ (m)	Z (m)	P/γ (m)
A	**70**	**30**	**40**
B	**56.21**	**25**	**31.21**
C	**32.36**	**20**	**12.36**
D	**31.15**	**20**	**11.15**
E	**37.32**	**22**	**15.32**
F	**63.48**	**25**	**38.48**



Best regards,
Dr. Amir Mobasher

CHAPTER 3

PUMPS & TURBINES

3.1 Introduction

Pumps are devices designed to convert mechanical energy to hydraulic energy. They are used to move water from lower points to higher points with a required discharge and pressure head. This chapter will deal with the basic hydraulic concepts of water pumps

3.2 Pump Classification

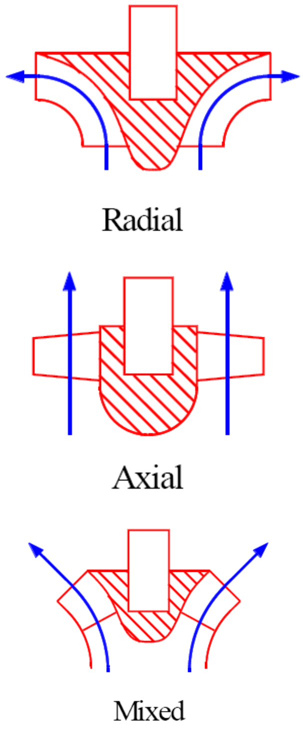
- ❑ **Turbo-hydraulic (kinetic) pumps**
  - ➢ Centrifugal pumps (radial-flow pumps)
  - ➢ Propeller pumps (axial-flow pumps)
  - ➢ Jet pumps (mixed-flow pumps)
- ❑ **Positive-displacement pumps**
  - ➢ Screw pumps
  - ➢ Reciprocating pumps

This classification is based on the way by which the water leaves the rotating part of the pump.

In radial-flow pump the water leaves the impeller in radial direction,

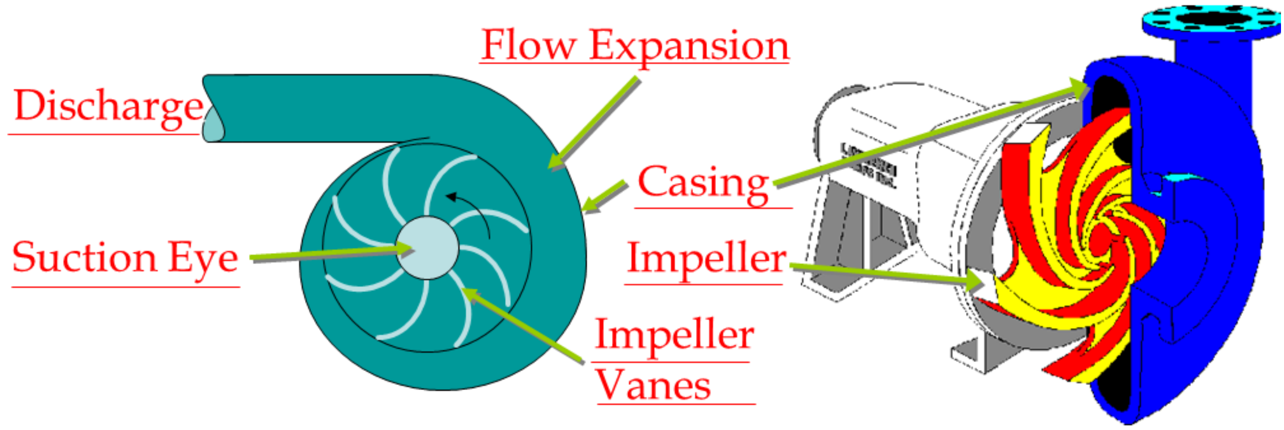
while in the axial-flow pump the water leaves the propeller in the axial direction.

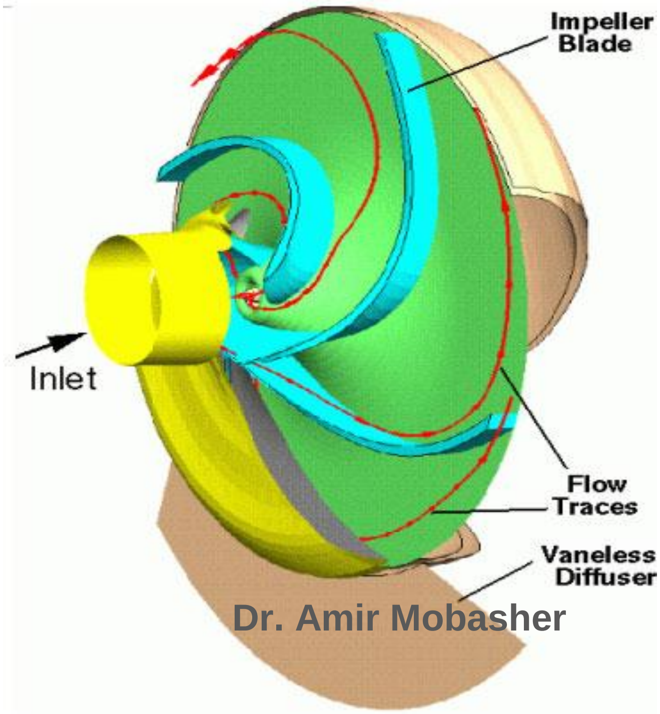
In the mixed-flow pump the water leaves the impeller in an inclined direction having both radial and axial components



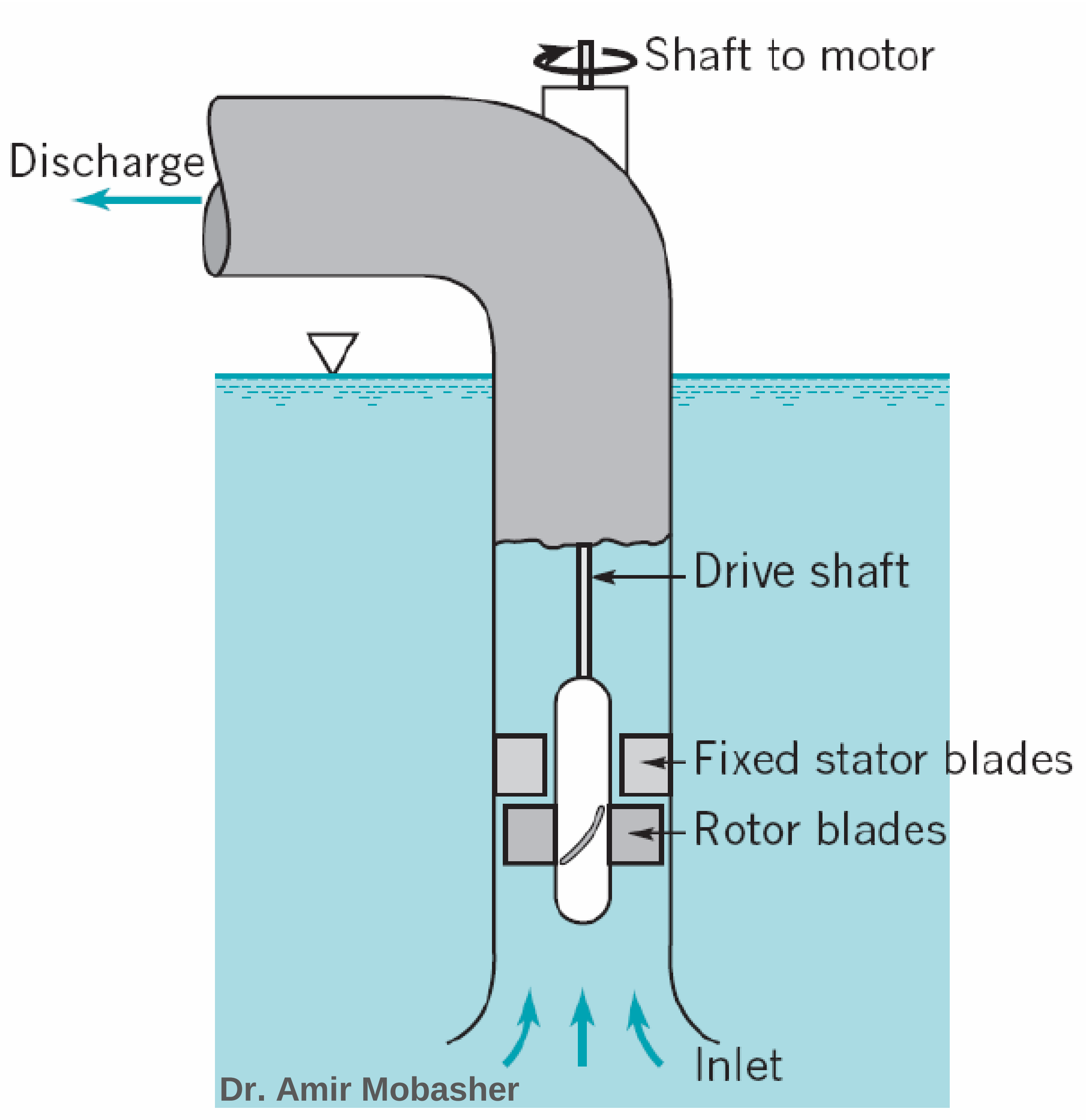
3.2.1 <u>Centrifugal Pumps</u>

- Broad range of applicable flows and heads
- Higher heads can be achieved by increasing the diameter or the rotational speed of the impeller



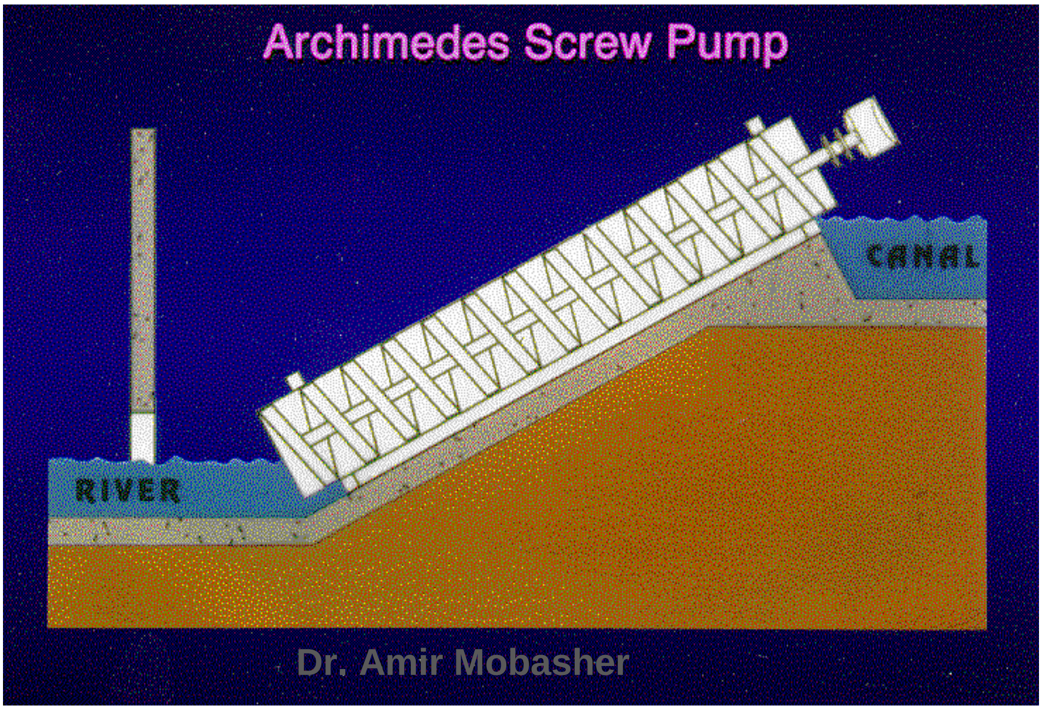


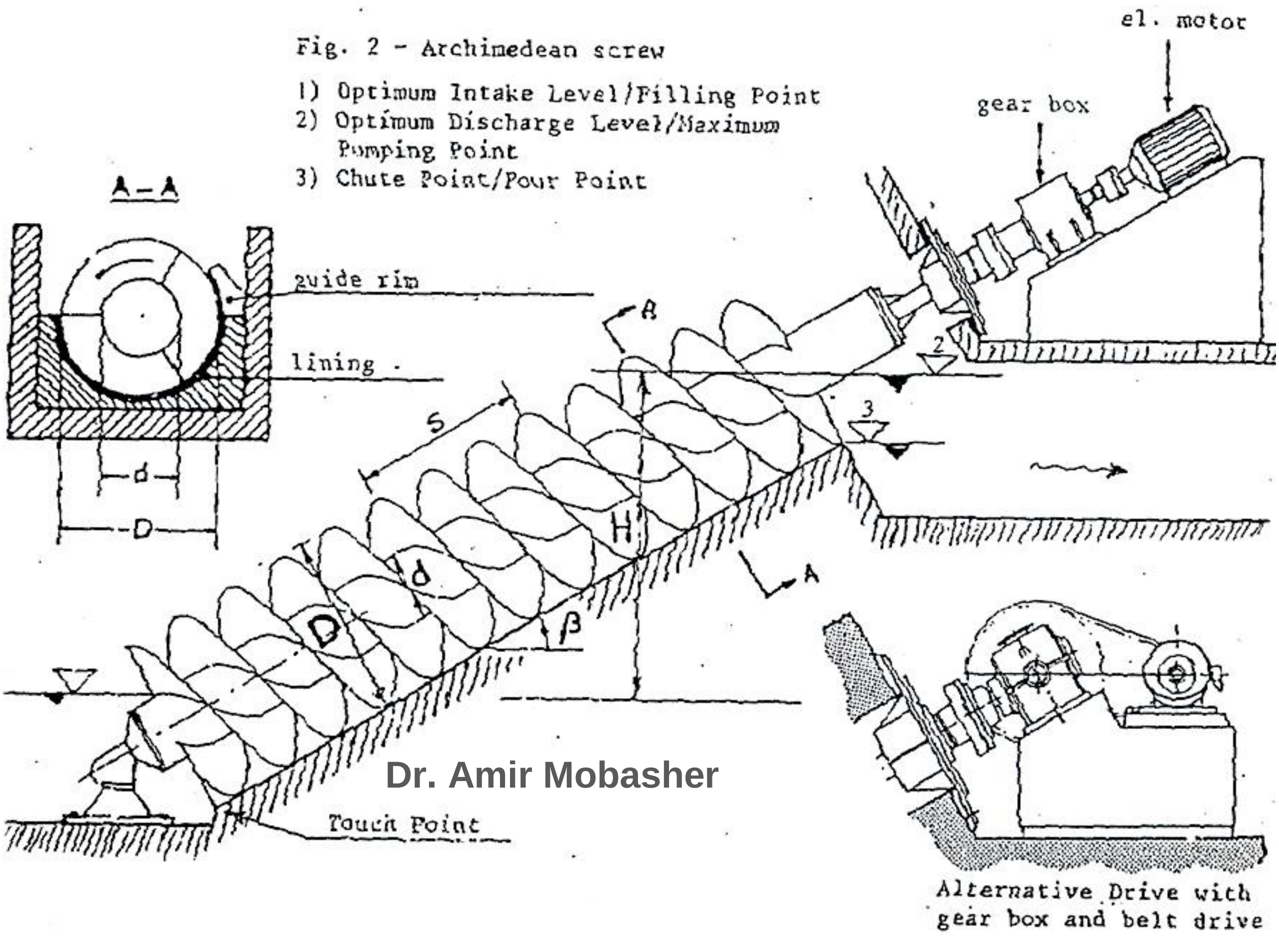
3.2.2 Axial-flow Pump



3.2.3 Screw Pumps

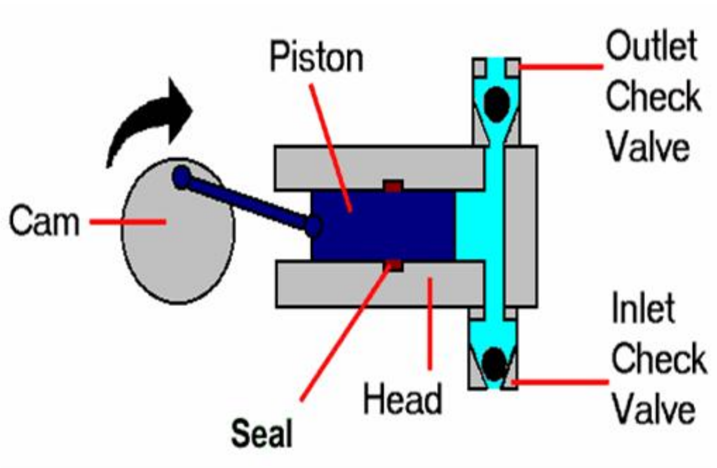
In the screw pump a revolving shaft fitted with blades rotates in an inclined trough and pushes the water up the trough.

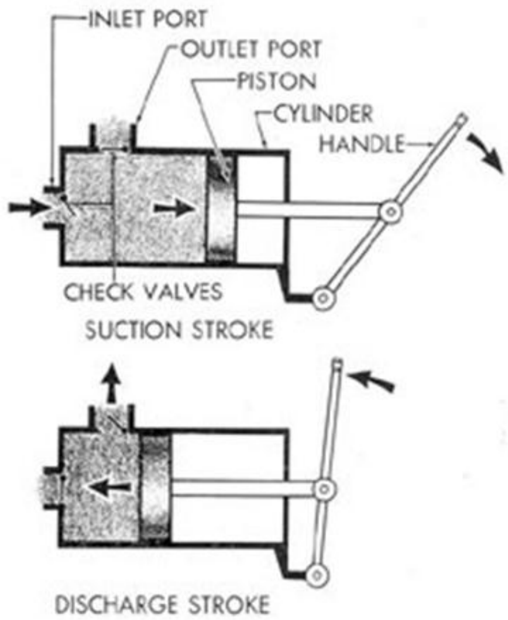




3.2.4 Reciprocating Pumps

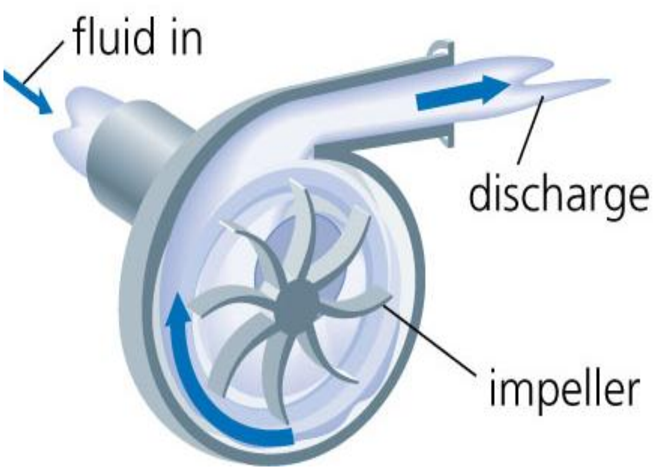
In the reciprocating pump a piston sucks the fluid into a cylinder then pushes it up causing the water to rise.

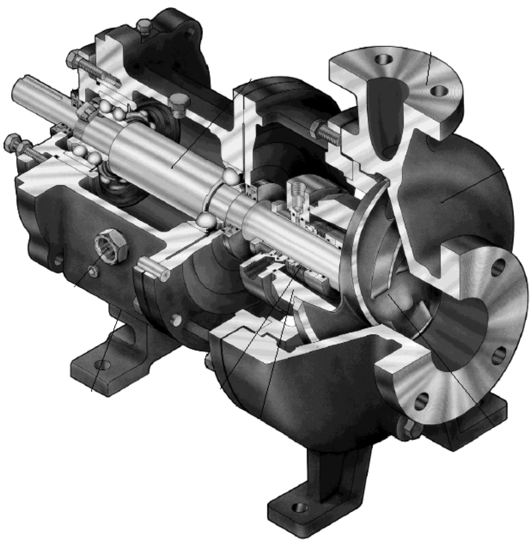




3.3 Centrifugal Pumps

Centrifugal pumps (radial-flow pumps) are the most used pumps for hydraulic purposes. For this reason, their hydraulics will be studied in the following sections.





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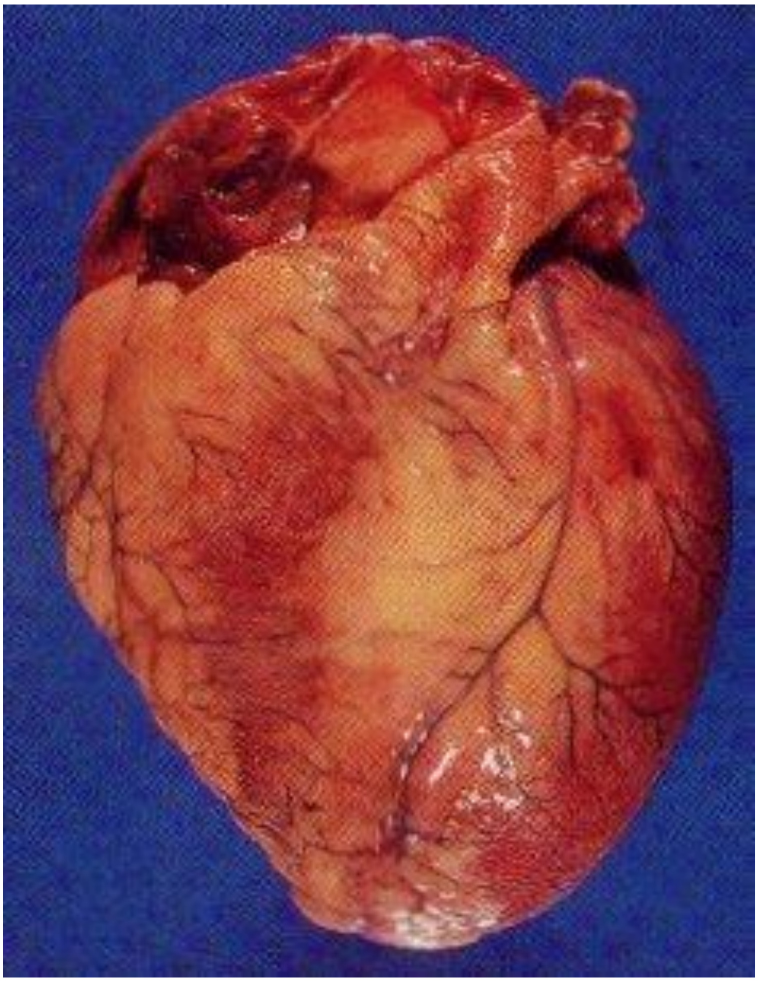
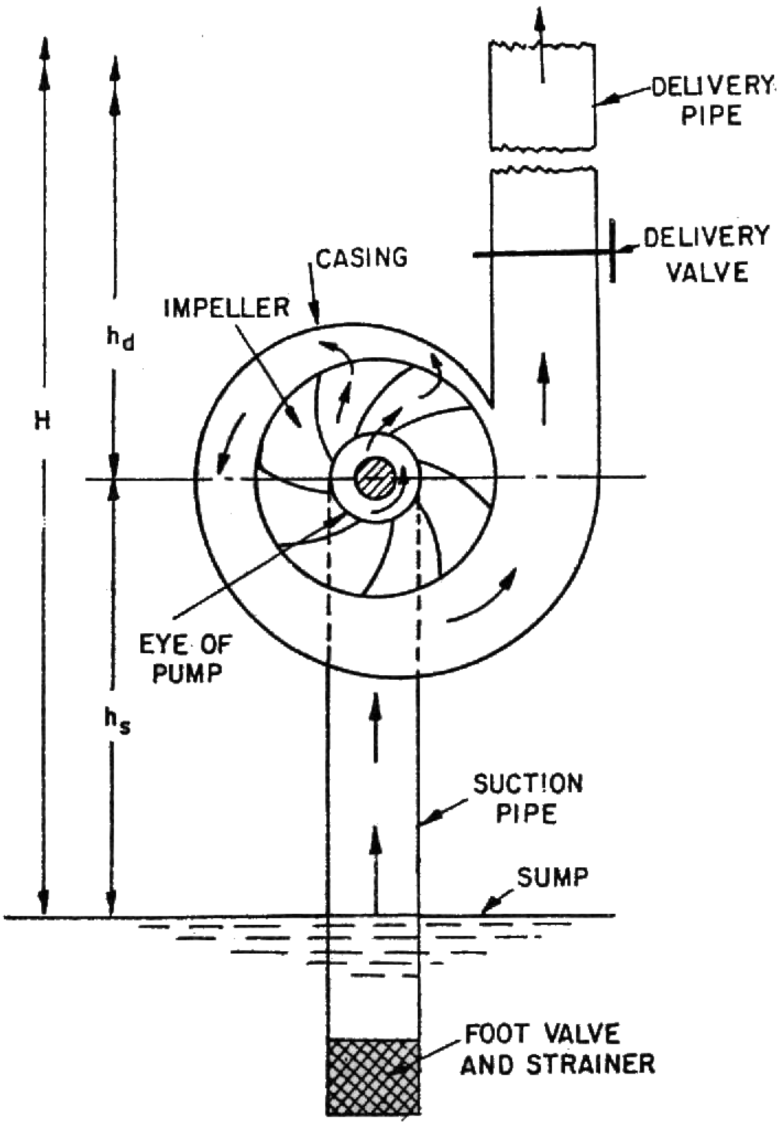


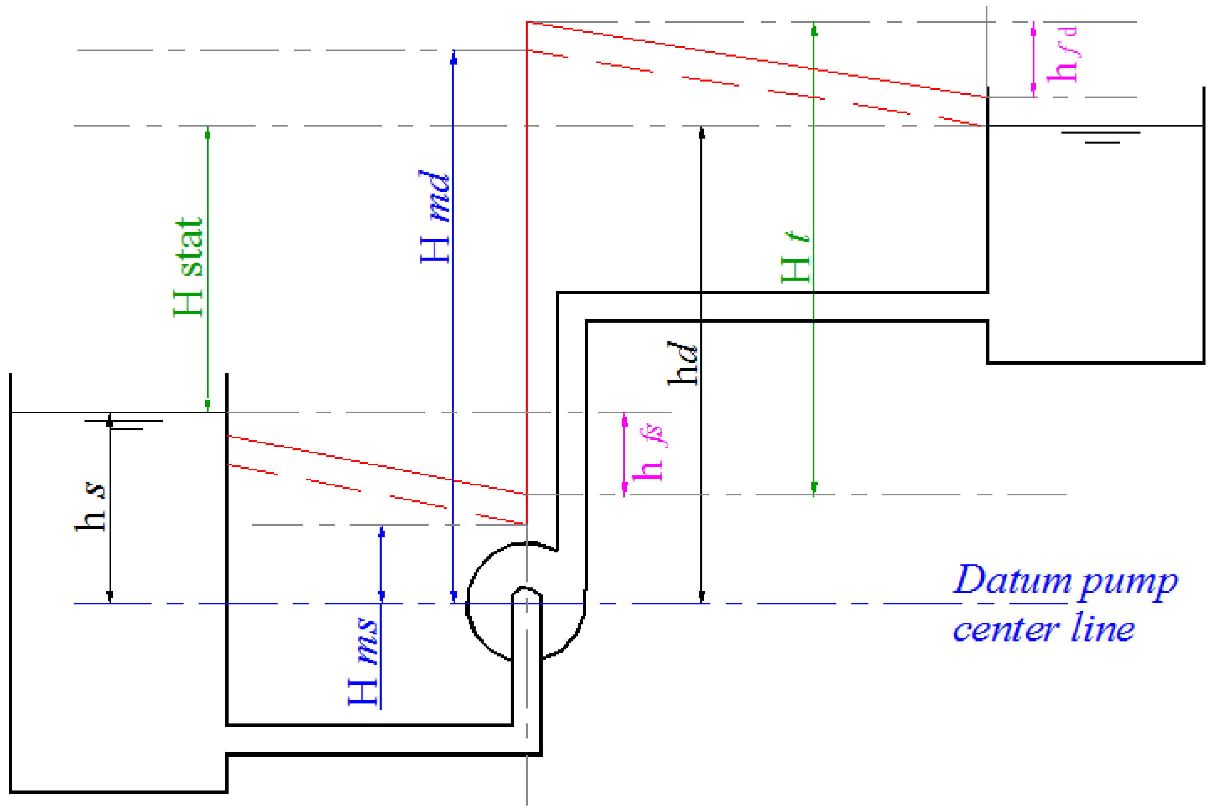
3.3.1 Main Parts of Centrifugal Pumps

- **Impeller**
  - Which is the rotating part of the centrifugal pump.
  - It consists of a series of backwards curved vanes (blades).
  - The impeller is driven by a shaft which is connected to the shaft of an electric motor.
- **Suction Pipe**
- **Delivery Pipe**
- **The Shaft:** which is the bar by which the power is transmitted from the motor drive to the impeller.
- **The driving motor:** which is responsible for rotating the shaft. It can be mounted directly on the pump, above it, or adjacent to it.

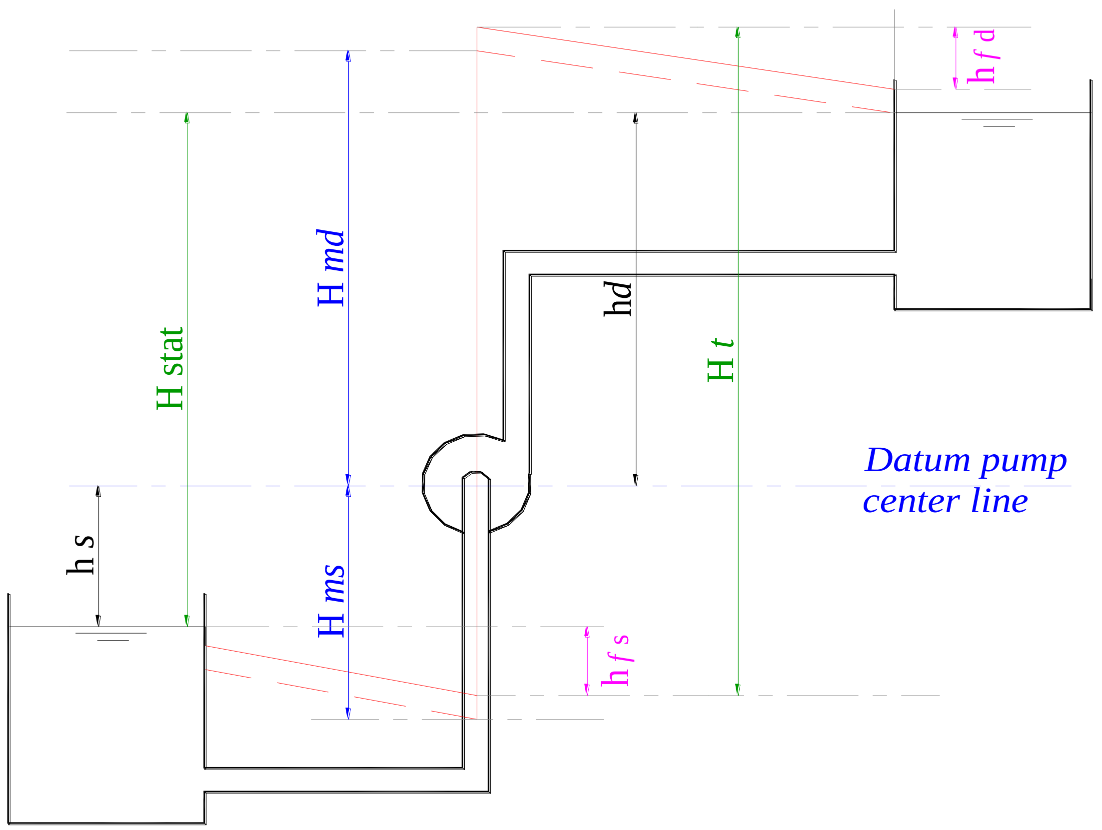


3.3.2 Hydraulic Analysis of Pumps and Piping Systems

❑ **Case 1**



❑ **Case 2**



❑ The following terms can be defined

- h_s ***(static suction head)*****:** it is the difference in elevation between the suction liquid level and the centerline of the pump impeller.
- h_d ***(static discharge head)*****:** it is the difference in elevation between the discharge liquid level and the centerline of the pump impeller.
- H_{stat} ***(static head)*****:** it is the difference (or sum) in elevation between the static discharge and the static suction heads:

$$H_{stat} = h_d \pm h_s$$

- H_{ms} ***(manometric suction head)*****:** it is the suction gage reading (if a manometer is installed just at the inlet of the pump, then H_{ms} is the height to which the water will rise in the manometer).
- H_{md} ***(manometric discharge head)*****:** it is the discharge gage reading (if a manometer is installed just at the outlet of the pump, then H_{md} is the height to which the water will rise in the manometer).
- H_m ***(manometric head)*****:** it is the increase of pressure head generated by the pump:

$$H_m = H_{md} \pm H_{ms}$$

- H_t ***(total dynamic head)*****:** it is the total head delivered by the pump:

$$H_t = H_{md} + \frac{V_d^2}{2g} - \left(H_{ms} + \frac{V_s^2}{2g}\right)$$

Case 1 ***Eq.(1)***

$$H_t = H_{md} + \frac{V_d^2}{2g} + \left(H_{ms} - \frac{V_s^2}{2g}\right)$$

Case 2 ***Eq.(2)***

- H_t can be written in another form as follows:

$$H_{md} = h_d + h_{fd} + \sum h_{md}$$

$$H_{ms} = h_s - h_{fs} - \sum h_{ms} - \frac{V_s^2}{2g}$$ **Case 1**

$$H_{ms} = h_s + h_{fs} + \sum h_{ms} + \frac{V_s^2}{2g}$$ **Case 2**

Substitute ino eq. (1)

$$H_t = h_d + h_{fd} + \sum h_{md} + \frac{V_d^2}{2g} - \left[h_s - h_{fs} - \sum h_{ms} - \frac{V_s^2}{2g} + \frac{V_s^2}{2g} \right]$$

but

$$H_{stat} = h_d - h_s$$

$$H_t = H_{stat} + h_{fd} + \sum h_{md} + h_{fs} + \sum h_{ms} + \frac{V_d^2}{2g}$$

Eq.(3) **Case 1**

- Equation (3) can be applied to Case 2 with the exception that :

$$H_{stat} = h_d + h_s$$

In the above equations; we define:

h_{fs} : is the friction losses in the suction pipe.

h_{fd} : is the friction losses in the discharge (delivery) pipe.

h_{ms} : is the minor losses in the suction pipe.

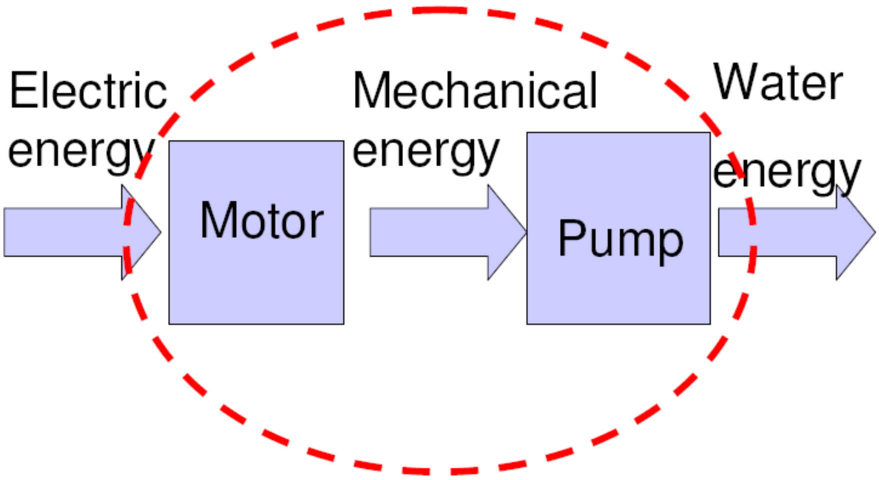
h_{md}: is the minor losses in the discharge (delivery) pipe.

- ***Bernoulli's equation can also be applied to find H_t***

$$H_t = \frac{P_d}{\gamma} + \frac{V_d^2}{2g} + Z_d \pm \left(\frac{P_s}{\gamma} + \frac{V_s^2}{2g} + Z_s \right)$$

Eq.(4)

3.3.3 Pump Efficiency



Motor efficiency x Pump efficiency = Overall efficiency

$\eta_m \times \eta_p = \eta_t$

- **Pump efficiency** η_p

$$\eta_p = \frac{Power\ output}{Power\ input} = \frac{P_o}{P_i} = \frac{\gamma\, Q\, H_t}{P_i}$$

or

$$P_i = \frac{\gamma\, Q\, H_t}{\eta_p}$$

Which is the power input delivered from the motor to the impeller of the pump.

- **Motor efficiency** η_m

$$\eta_m = \frac{P_i}{P_m}$$

$$P_m = \frac{P_i}{\eta_m}$$

which is the power input delivered to the motor.

❑ Overall efficiency of the motor-pump system η_t

$$\eta_t = \eta_p \eta_m$$

$$\eta_t = \frac{P_o}{P_m}$$

➢ ***Imprtant Units***

Hydraulic water power $= \gamma\ (kg/m^3)\ \times Q\ (m^3/s) \times H_{req}\ (m)$

Hydraulic Horsepower (HP) = $\gamma\ (kg/m^3)\ \times Q\ (m^3/s) \times H_{req}\ (m) / 75$

(1 HP = 75 kg.m/sec)

Electrical input power (HP) = Hydraulic water power / η_t
Electrical input power (KW) = (Hydraulic water power / η_t) x 0.746
1 KW = 0.746 HP

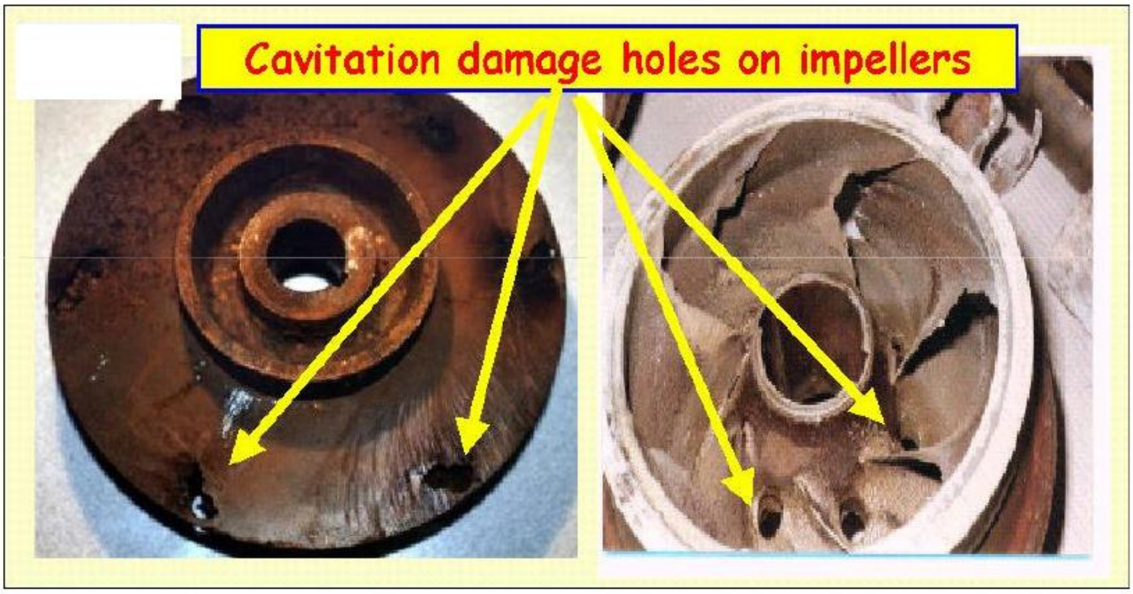
Required Pump Motor Power ≥ 1.20 (factor of safety) x Electrical input power

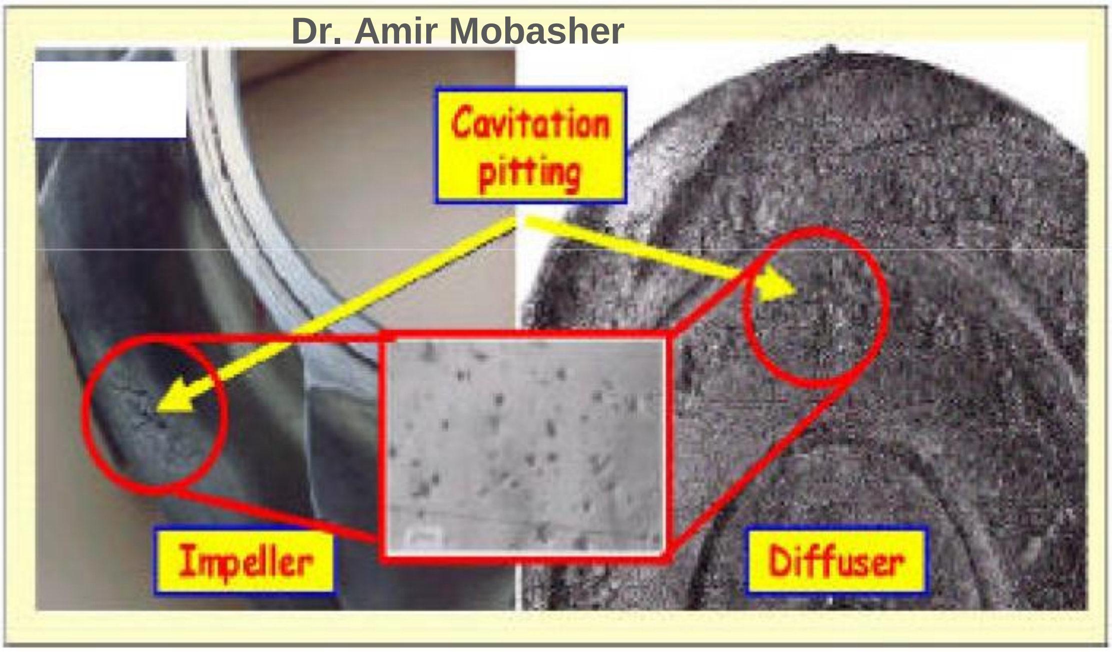
3.3.4 Cavitation of Pumps and NPSH

- In general, **cavitation** occurs when the liquid pressure at a given location is reduced to the vapor pressure of the liquid.
- For a piping system that includes a pump, **cavitation** occurs when the absolute pressure at the inlet falls below the vapor pressure of the water.
- This phenomenon may occur at the inlet to a pump and on the impeller blades, particularly if the pump is mounted above the level in the suction reservoir.

- Under this condition, vapor bubbles form (water starts to boil) at the impeller inlet and when these bubbles are carried into a zone of higher pressure, they collapse abruptly and hit the vanes of the impeller (near the tips of the impeller vanes). **causing:**

- Damage to the pump (pump impeller)
- Violet vibrations (and noise).
- Reduce pump capacity.
- Reduce pump efficiency





❑ **How we avoid Cavitation ??**

- To avoid cavitation, the pressure head at the inlet should not fall below a certain minimum which is influenced by the further reduction in pressure within the pump impeller.
- To accomplish this, we use the difference between the total head at the inlet $\frac{P_s}{\gamma}+\frac{V_s^2}{2g}$, and the water vapor pressure head $\frac{P_{vapor}}{\gamma}$

Where we take the datum through the centerline of the pump impeller inlet (eye). This difference is called the ***Net Positive Suction Head*** (NPSH), so that

$$NPSH=\frac{P_s}{\gamma}+\frac{V_s^2}{2g}-\frac{P_{vapor}}{\gamma}$$

There are two values of NPSH of interest. The first is the required NPSH, denoted $(NPSH)_R$, that must be maintained or exceeded so that cavitation will not occur and usually determined experimentally and provided by the manufacturer.

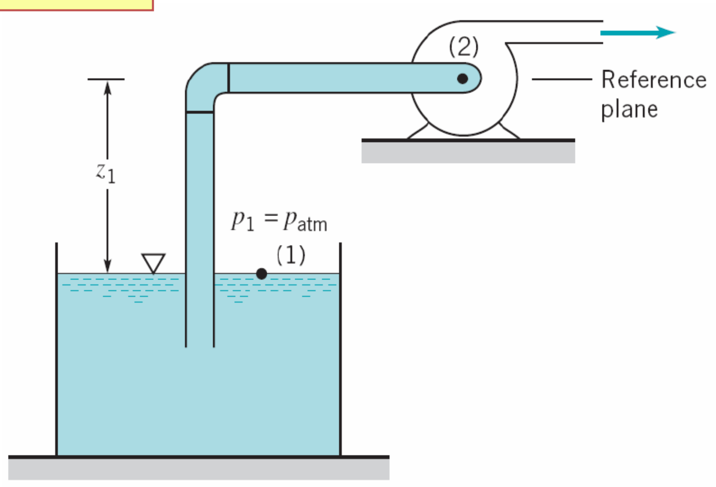
The second value for NPSH of concern is the available NPSH, denoted $(NPSH)_A$, which represents the head that actually occurs for the particular piping system. This value can be determined experimentally, or calculated if the system parameters are known.

- For proper pump operation (no cavitation) :

$$(NPSH)_A > (NPSH)_R$$

Determination of $(NPSH)_A$

Applying the energy equation between point (1) and (2), datum at pump center line



$$\frac{P_{atm}}{\gamma} - h_S = \frac{P_S}{\gamma} + \frac{V_S^2}{2g} + \sum h_L$$

$$\frac{P_S}{\gamma} + \frac{V_S^2}{2g} = \frac{P_{atm}}{\gamma} - h_S - \sum h_L$$

$$\frac{P_S}{\gamma} + \frac{V_S^2}{2g} - \frac{P_{Vapor}}{\gamma} = \frac{P_{atm}}{\gamma} - h_S - \sum h_L - \frac{P_{Vapor}}{\gamma}$$

$$(NPSH)_A = \frac{P_{atm}}{\gamma} - h_S - \sum h_L - \frac{P_{Vapor}}{\gamma}$$

$$(NPSH)_A = \mp h_s - h_{fs} - \sum h_{ms} + \frac{P_{atm}}{\gamma} - \frac{P_{vapor}}{\gamma}$$

Note that (+) is used if h_s is above the pump centerline (datum).

at T = 20°

$P_{atm} = 10.14\ kN/m^2$

$P_{Vapor} = 2.335\ kN/m^2$

- **Thoma's cavitation constant**

The cavitation constant: is the ratio of $(NPSH)_R$ to the total dynamic head (H_t) is known as the Thoma's cavitation constant (σ)

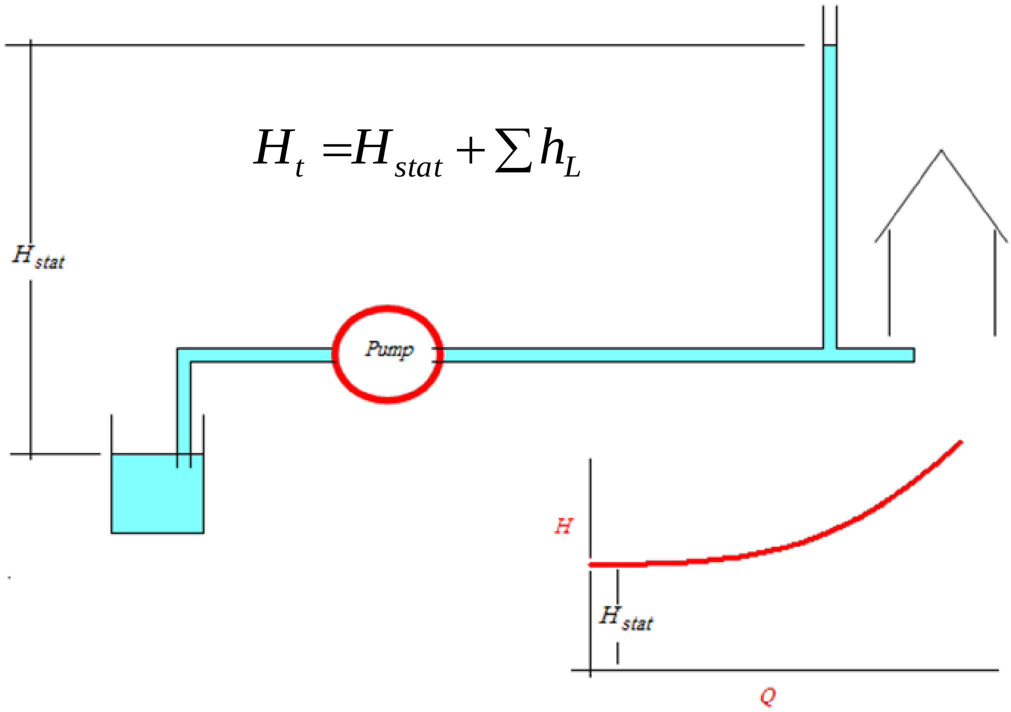
$$\sigma = \frac{(NPSH)_R}{H_t}$$

3.3.5 Selection of A Pump

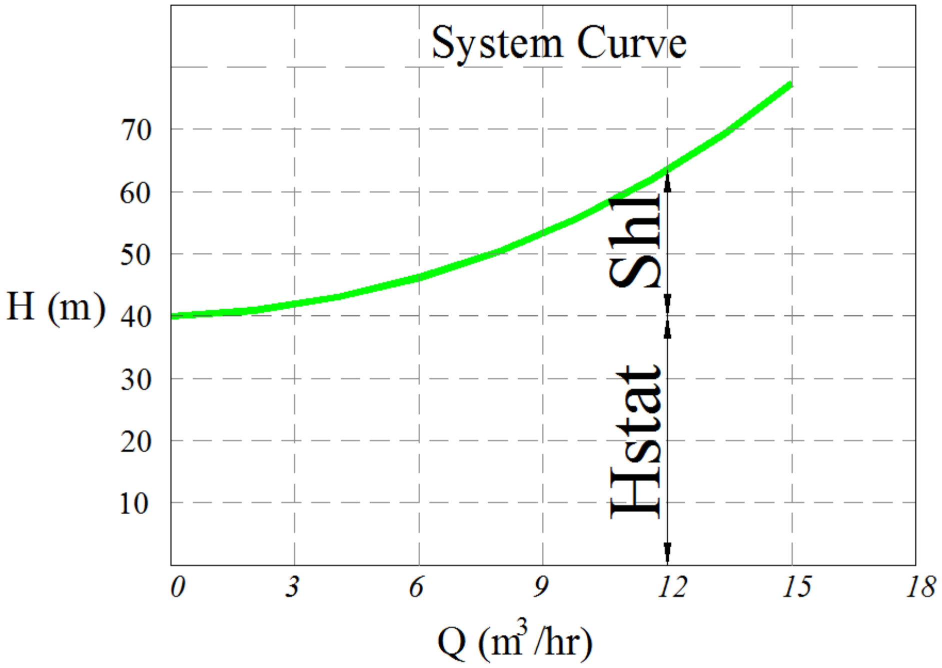
- The design conditions are specified and a pump is selected for the range of applications.
- A system characteristic curve (H-Q) is then prepared.
- The H-Q curve is then matched to the pump characteristics chart which is provided by the manufacturer.
- The matching point (operating point) indicates the actual working conditions.

❑ System Characteristic Curve

- The total head, H_t, that the pump delivers includes the elevation head and the head losses incurred in the system. The friction loss and other minor losses in the pipeline depend on the velocity of the water in the pipe, and hence the total head loss can be related to the discharge rate
- For a given pipeline system (including a pump or a group of pumps), a unique system head-capacity (H-Q) curve can be plotted. This curve is usually referred to as a system characteristic curve or simply system curve. It is a graphic representation of the system head and is developed by plotting the total head, over a range of flow rates starting from zero to the maximum expected value of Q.



$$H_t = H_{stat} + \sum h_L$$

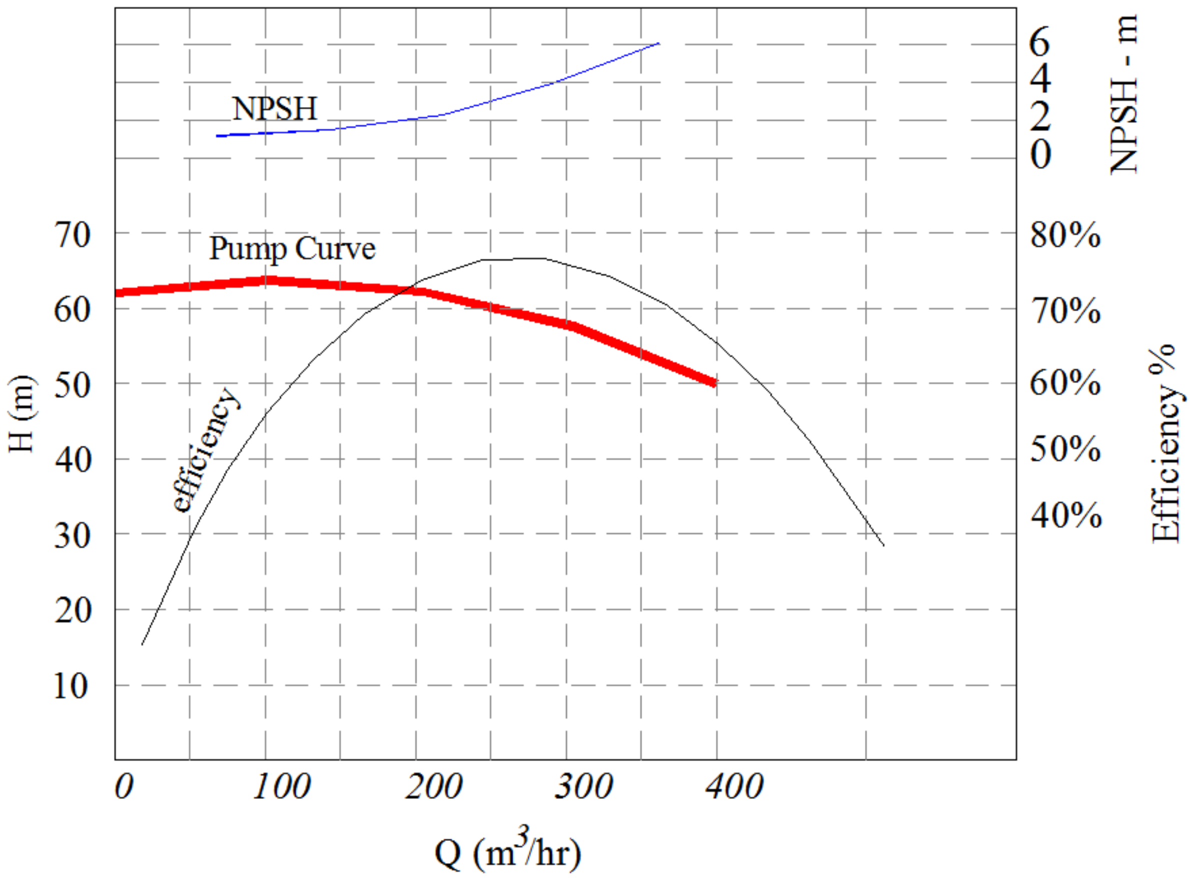


❑ Pump Characteristic Curves

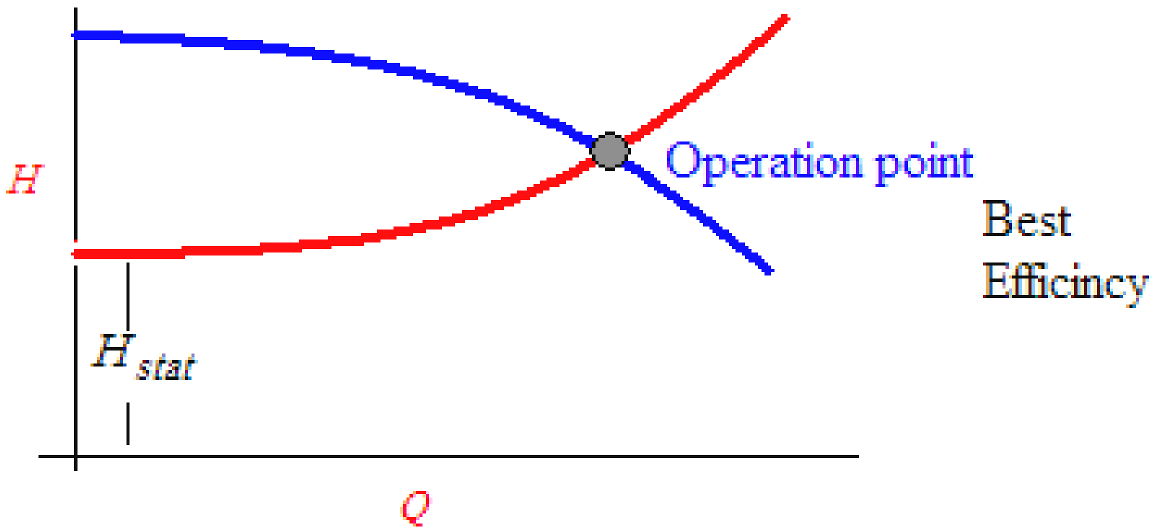
➢ Pump manufacturers provide information on the performance of their pumps in the form of curves, commonly called pump characteristic curves (or simply pump curves).

➢ In pump curves the following information may be given:

- the discharge on the x-axis,
- the head on the left y-axis,
- the pump power input on the right y-axis,
- the pump efficiency as a percentage,
- the speed of the pump (rpm = revolutions/min).
- the NPSH of the pump.

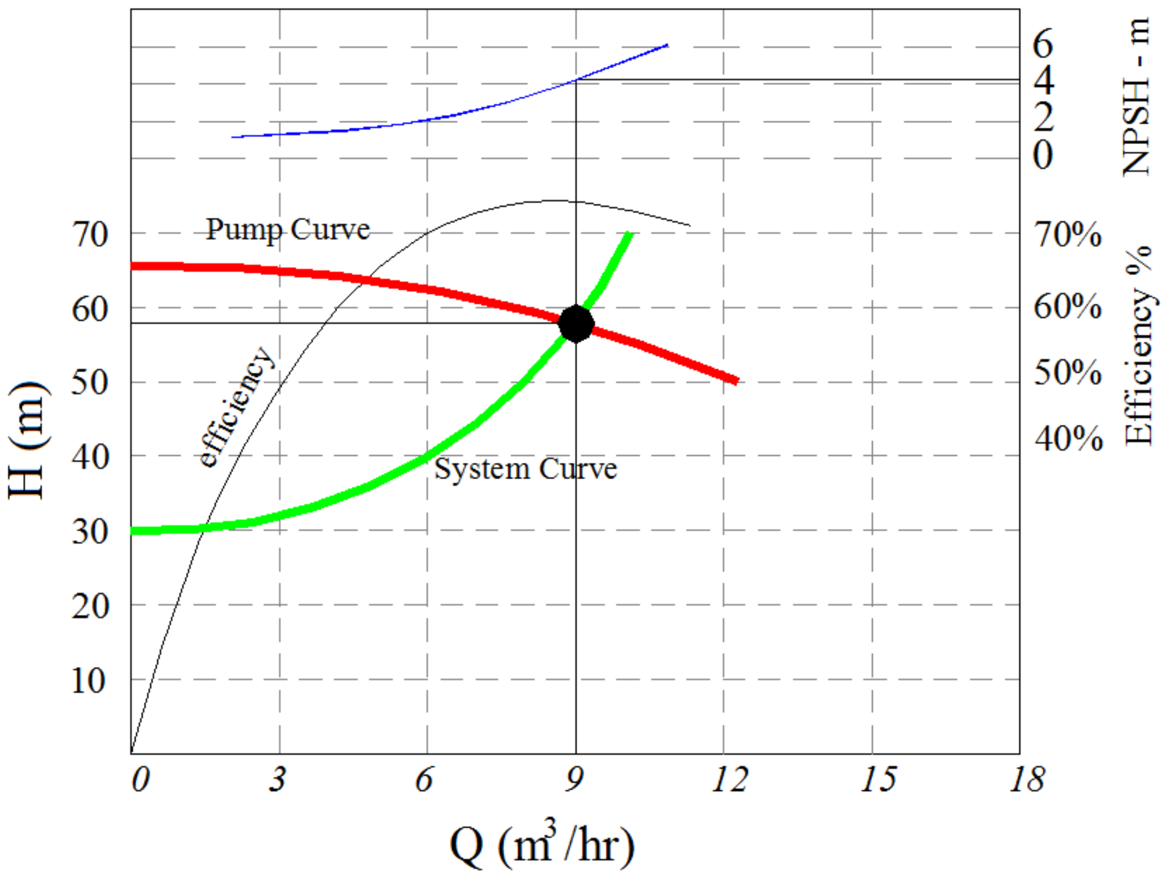


- The pump characteristic curves are very important to help select the required pump for the specified conditions.
- If the system curve is plotted on the pump curves in we may produce the following Figure:



Matching the system and pump curves.

- The point of intersection is called the operating point.
- This matching point indicates the actual working conditions, and therefore the proper pump that satisfy all required performance characteristic is selected.



Example 3-1

A Pump has a cavitation constant = 0.12, this pump was instructed on well using UPVC pipe of 10 m length and 200 mm diameter, there are elbow (k_e=1) and valve (k_e=4.5) in the system. The flow is 35 m[3] and the total Dynamic Head H_t = 25 m (from pump curve) f = 0.0167

Calculate the maximum suction head

$$\text{atm. pressure head} = 9.69\ m$$
$$\text{Vapour pressure head} = 0.2m$$

Solution

$$\sigma = 0.12$$
$$NPSH_R = \sigma \times H_t = 0.12 \times 25 = 3$$
$$(NPSH)_A = \pm h_S - h_{fS} - \sum h_{mS} + \frac{P_{atm}}{\gamma} - \frac{P_{Vapor}}{\gamma}$$

$$V_S = \frac{Q}{A} = \frac{0.035}{\pi/4 \times (0.2)^2} = 1.11\ m/s$$

$$h_e = \frac{V_S^2}{2g} = \frac{1.11^2}{2g} = 0.063 \qquad h_V = 4.5\frac{V_S^2}{2g} = 4.5 \times \frac{1.11^2}{2g} = 0.283m$$

$$h_{fS} = f\frac{L}{D} \times \frac{V^2}{2g} = 0.0167 \times \frac{10}{0.2} \times \frac{1.11^2}{2g} = 0.053m$$

$$(NPSH)_A = \pm h_S - h_{fS} - \sum h_{mS} + \frac{P_{atm}}{\gamma} - \frac{P_{Vapor}}{\gamma}$$
$$3 = h_S - 0.053 - (0.283 + 0.063) + (9.69) - 0.2$$
$$h_S = -6.088m$$

Example 3-2

For the following pump, determine the required pipes diameter to pump 60 L/s and also calculate the needed power.
Minor losses 10 $v^2/2g$
Pipe length 10 km
roughness = 0.15 mm
H_s = 20 m

Q *L/s*	70	60	50	40	30	20	10	0
H_t	31	35	38	40.6	42.5	43.7	44.7	45
	40	53	60	60	57	50	35	–

Solution

To get 60 L/s from the pump $H_s + h_L$ must be < 35 m

Assume the diameter = 300mm Then:

$$A = 0.070 m^2, V = 0.85 m/s$$

$$R_e = 2.25 \times 10^5, K_S / D = 0.0005, f = 0.019$$

$$h_f = \frac{0.019 \times 10000 \times (0.85)^2}{0.3 \times 19.62} = 23.32 m$$

$$h_m = \frac{10 \times V^2}{2g} = \frac{10 \times (0.85)^2}{2g} = 0.37 m$$

$$h_s + h_f + h_m = 43.69\ m > 35\ m$$

Assume the diameter = 350mm Then:

$$A = 0.0962m^2, V = 0.624m/s$$

$$R_e = 1.93\times10^5, K_S/D = 0.00043, f = 0.0185$$

$$h_f = 10.48m,$$

$$h_m = \frac{10\times V^2}{2g} = \frac{10\times(0.624)^2}{2g} = 0.2m$$

$$\therefore h_s + h_f + h_m = 30.68m < 35m$$

The pump would deliver approximately 70 l/s through the 350 mm pipe and to regulate the flow to 60 l/s an additional head loss of 4.32 m by valve closure would be required.

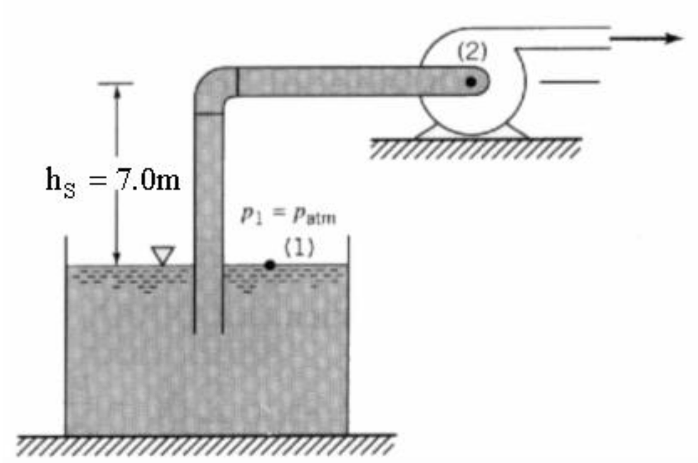
$$P_i = \frac{\gamma Q H_t}{\eta_p} = \frac{1000\times9.81\times\frac{60}{1000}\times35}{0.53} = 38869.8W = 38.87kW$$

Example 3-3

A pump was designed to satisfy the following system:

Q (m^3/hr)	3	6	9
h_f (m)	12	20	38

Check whether the pump is suitable or not



$$\text{atm. pressure head} = 10.3\,m$$
$$\text{Vapour pressure head} = 0.25m$$

Pipe diameter is 50mm

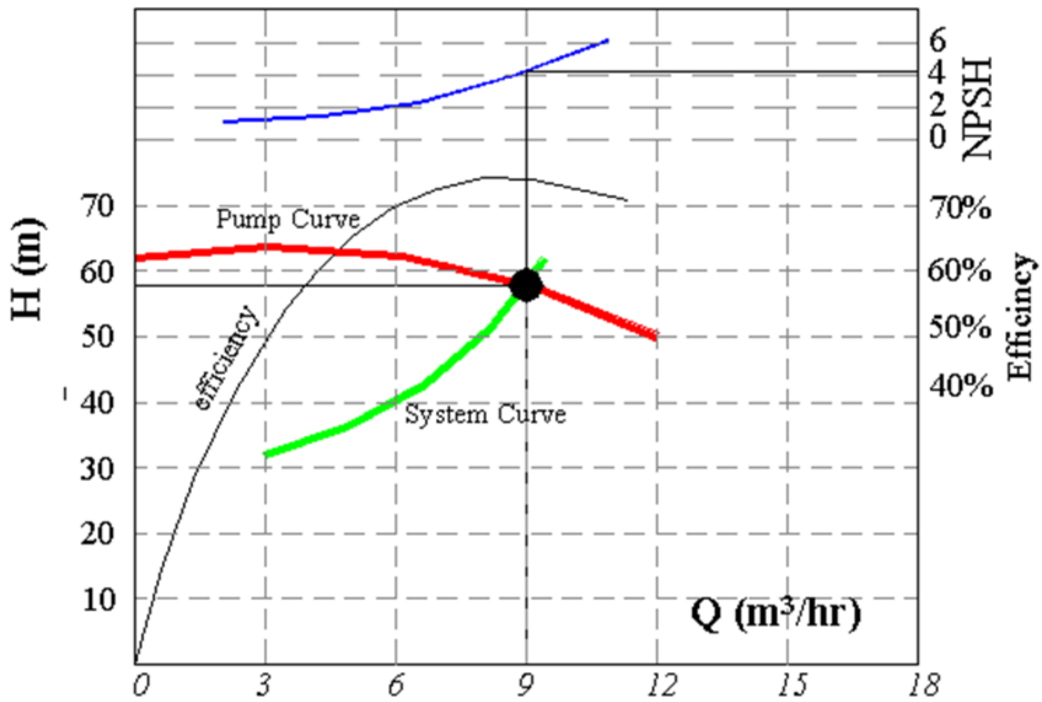
$$h_d = 13m$$

$$(\text{suction Part})h_L = \frac{24\times V^2}{2g}$$

Solution

1- Draw the system curve and check the operation point

$$H_{STAT} = h_d + h_S = 13 + 7 = 20m$$



There are an operation point at:

$Q = 9\ m^3/hr$ $H = 58m$

$NPSH_R = 4.1$
Then Check $NPSH_A$

$$V = \frac{Q}{A} = \frac{9/3600}{\frac{\pi}{4}\times(0.05)^2} = 1.27 m/s$$

$$h_L = \frac{24\times(1.27)^2}{2g} = 2.0m$$

$$(NPSH)_A = \pm h_S - h_{fS} - \sum h_{mS} + \frac{P_{atm}}{\gamma} - \frac{P_{Vapor}}{\gamma}$$

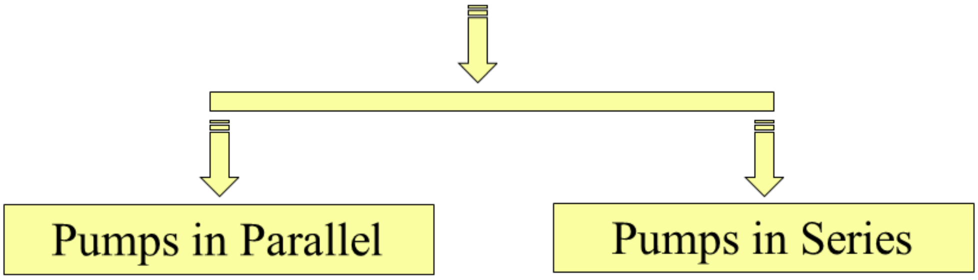
$$(NPSH)_A = -7 - 2 + 10.3 - 0.25$$

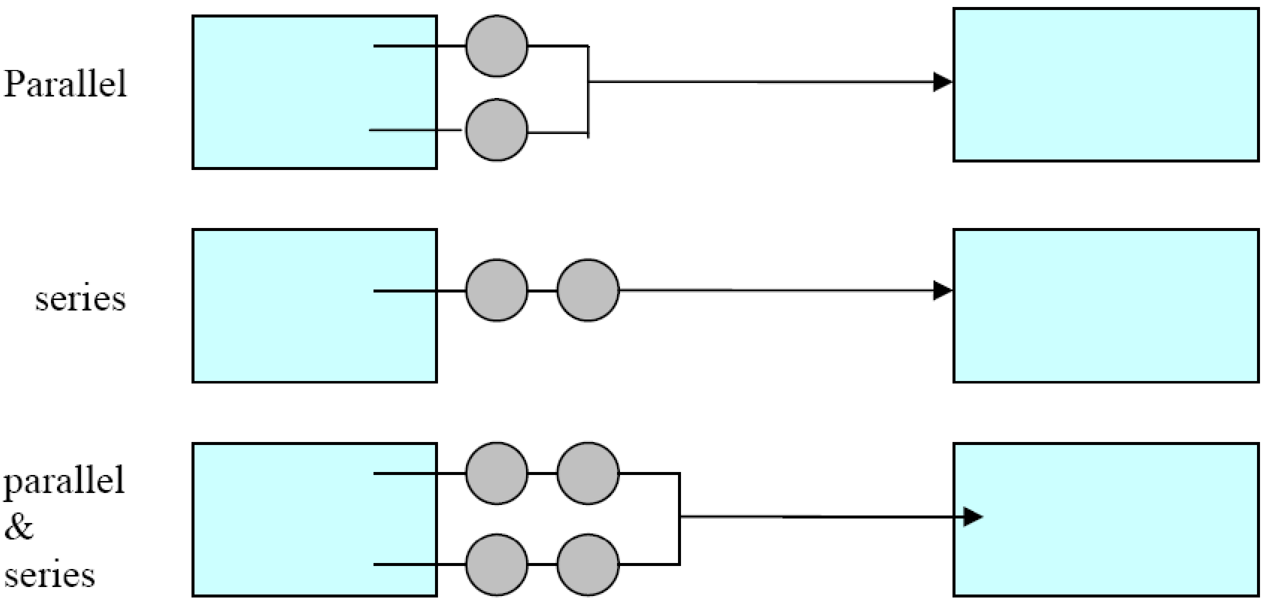
$$(NPSH)_A = 1.05 < 4.1$$

pump is not suitable, the cavitation will occur

3.3.6 <u>Multiple-Pump Operation</u>

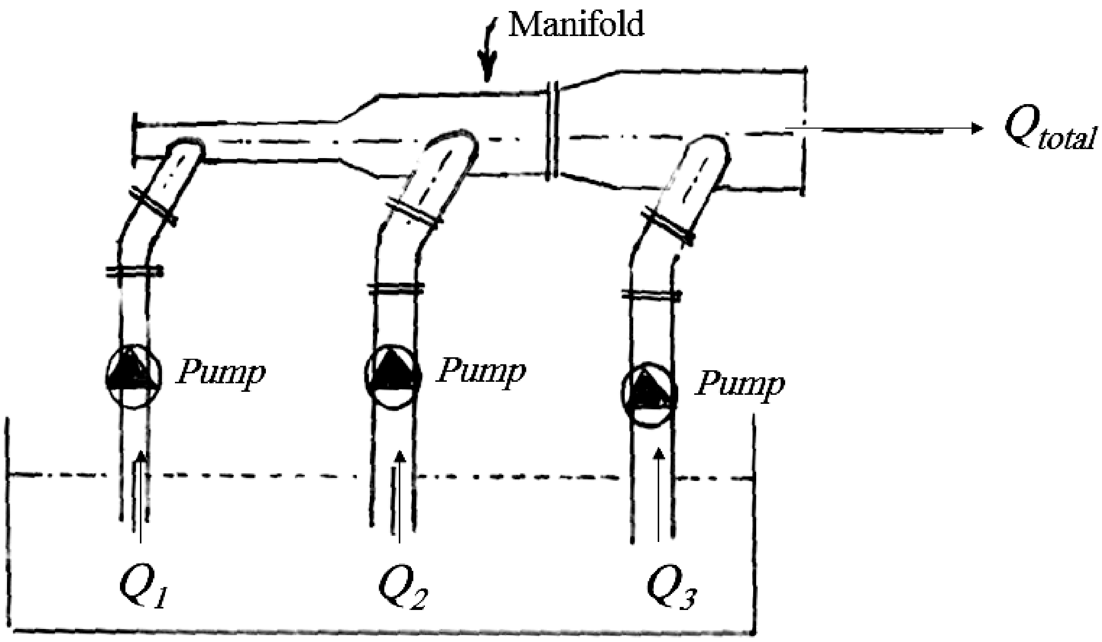
To install a pumping station that can be effectively operated over a large range of fluctuations in both discharge and pressure head, it may be advantageous to install several identical pumps at the station.





❑ Parallel Operation

- Pumping stations frequently contain several (two or more) pumps in a parallel arrangement.
- The objective being to deliver a range of discharges, i.e.; the discharge is increased but the pressure head remains the same as with a single pump.



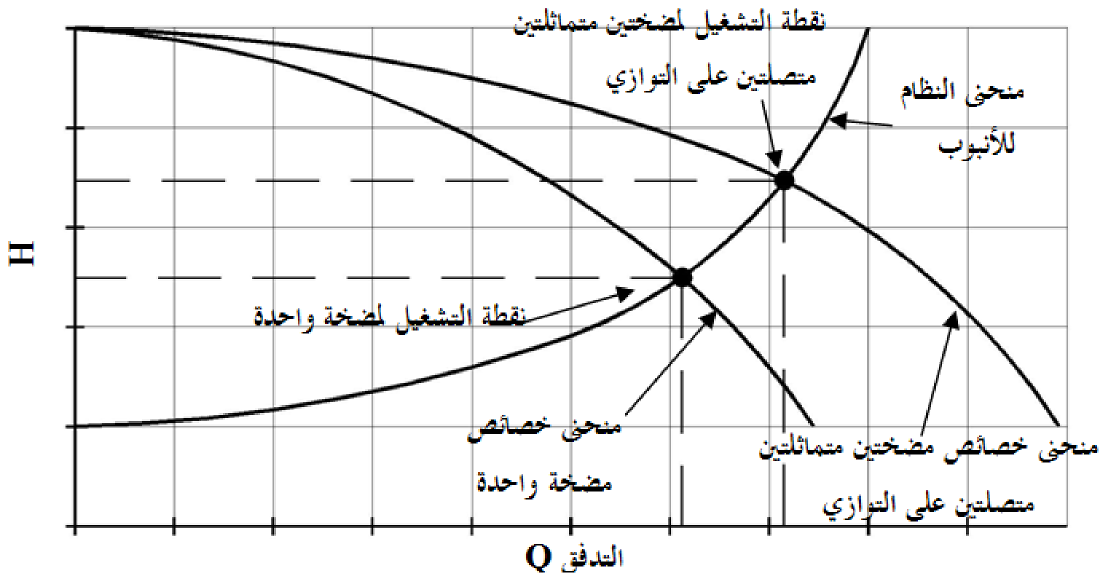
$$Q_{total} = Q_1 + Q_2 + Q_3$$

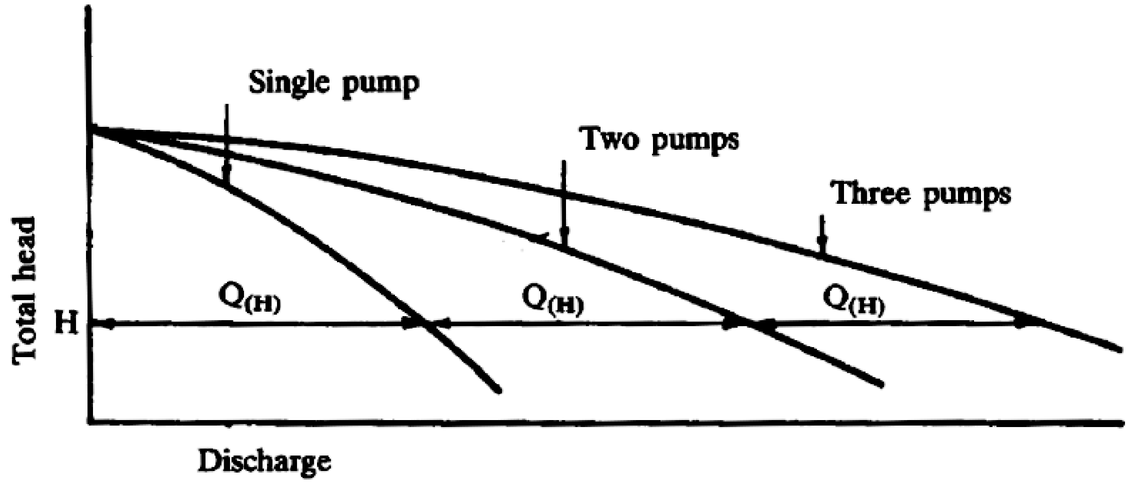
How to draw the pump curve for pumps in parallel???

- The manufacturer gives the pump curve for a single pump operation only.
- If two or pumps are in operation, the pumps curve should be calculated and drawn using the single pump curve.
- For pumps in parallel, the curve of two pumps, for example, is produced by adding the discharges of the two pumps at the same head (assuming identical pumps).

$$Q = Q_1 + Q_2 + Q_3 + \cdots\cdots\cdots = Q_n = \sum_{j=1}^{j=n} Q$$

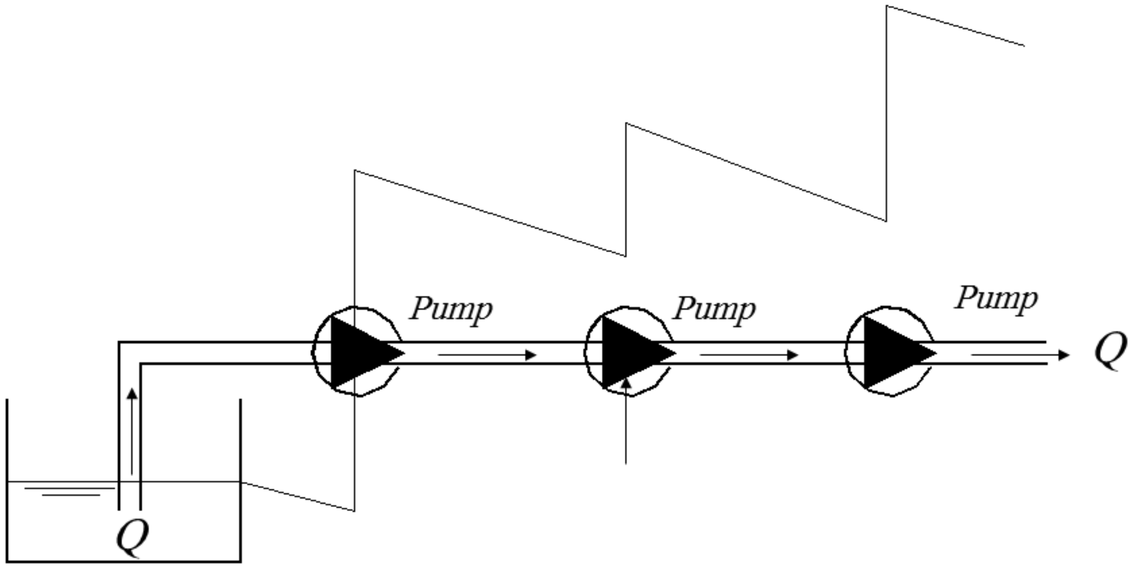
$$H_m = H_{m1} = H_{m2} = H_{m3} = \cdots\cdots\cdots = H_{mn}$$





❑ Series Operation

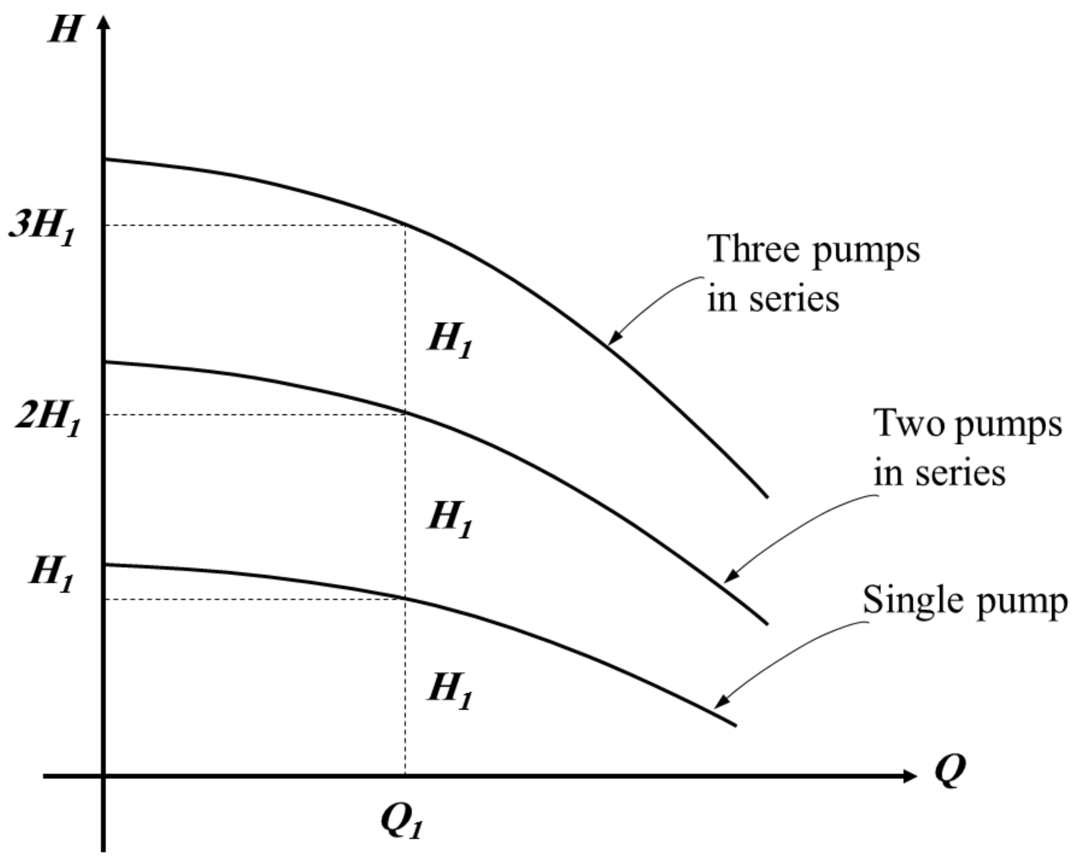
- The series configuration which is used whenever we need to increase the pressure head and keep the discharge approximately the same as that of a single pump.
- This configuration is the basis of multistage pumps; the discharge from the first pump (or stage) is delivered to the inlet of the second pump, and so on.
- The same discharge passes through each pump receiving a pressure boost in doing so.



$$H_{total} = H_1 + H_2 + H_3$$

How to draw the pump curve for pumps in series???

- the manufacturer gives the pump curve for a single pump operation only.
- For pumps in series, the curve of two pumps, for example, is produced by adding the heads of the two pumps at the same discharge.
- Note that, of course, all pumps in a series system must be operating simultaneously



Example 3-4

A centrifugal pump running at 1000 rpm gave the following relation between head and discharge:

Discharge (m^3/min)	0	4.5	9.0	13.5	18.0	22.5
Head (m)	22.5	22.2	21.6	19.5	14.1	0

The pump is connected to a 300 mm suction and delivery pipe the total length of which is 69 m and the discharge to atmosphere is 15 m above sump level. The entrance loss is equivalent to an additional 6m of pipe and f is assumed as 0.024. ***Calculate the discharge in m^3 per minute.***

Solution

1) System curve:

- The head required from pump =

  static + friction + velocity head

$$H_t = H_{stat} + h_{fd} + \sum h_{md} + h_{fs} + \sum h_{ms} + \frac{V_d^2}{2g}$$

- $H_{stat} = 15$ m
- Friction losses (including equivalent entrance losses) =

$$\sum h_{fs} + h_{ms} + \sum h_{fd} + h_{md} = \frac{8fLQ^2}{\pi^2 g D^5}$$

$$= \frac{8 \times 0.024 \times (69+6)}{\pi^2 g\, (0.3)^5} Q^2$$

$$= 61.21 Q^2 \qquad \text{where } Q \text{ in m}^3/\text{s}$$

- Velocity head in delivery pipe = $\frac{V_d^2}{2g} = \frac{1}{2g}\left(\frac{Q}{A}\right)^2 = 10.2Q^2$

where Q in m³/s

Thus:

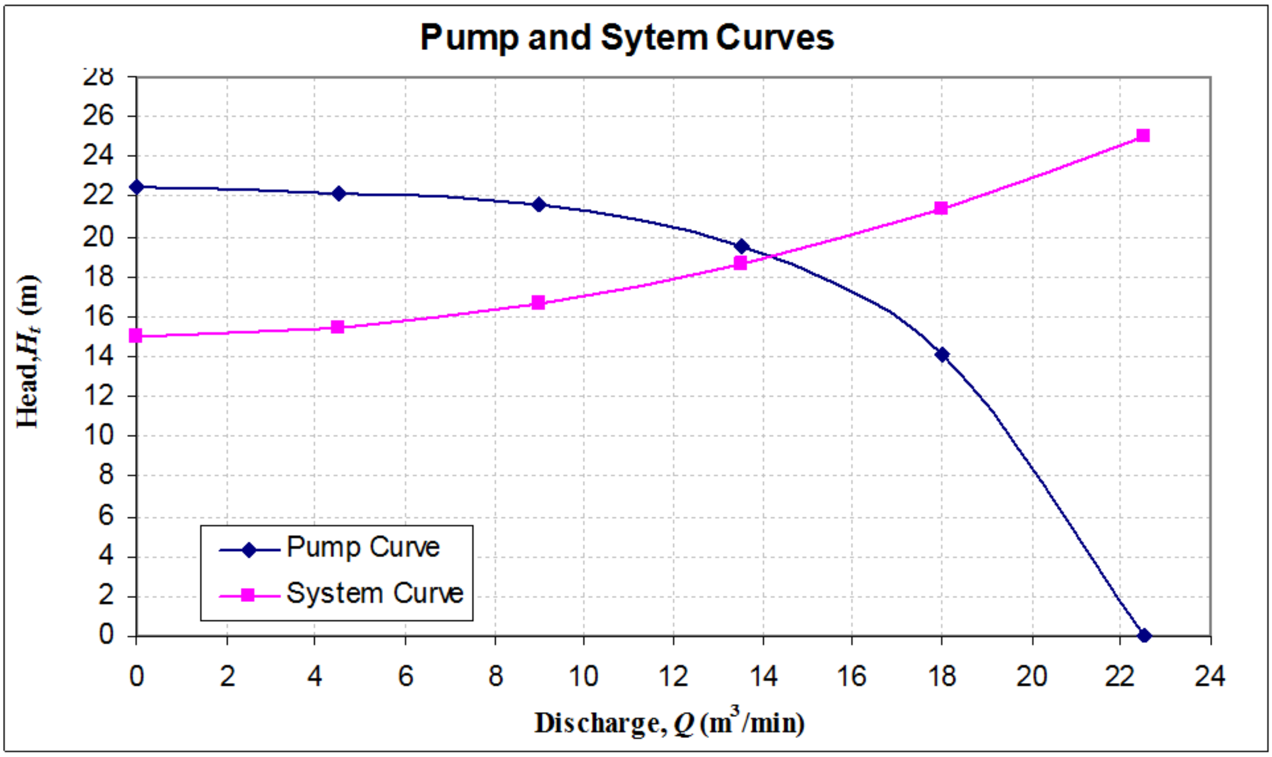
- $H_t = 15 + 71.41Q^2$ where Q in m³/s

or

- $H_t = 15 + 19.83 \times 10^{-3} Q^2$ where Q in m³/min

- From this equation and the figures given in the problem the following table is compiled:

Discharge (m3/min)	0	4.5	9.0	13.5	18.0	22.5
Head available (m)	22.5	22.2	21.6	19.5	14.1	0
Head required (m)	15.0	15.4	16.6	18.6	21.4	25.0



From the previous Figure, The operating point is:

- $Q_A = 14$ m³/min $H_A = 19$ m

Example 3-5

A centrifugal pump is used to deliver water against a static lift of 10.50 m. The head loss due to friction = 150000 Q^2, head loss is in m and the flow is in Q in m^3/s. The pump characteristic is given in the following table. Deduce operating head and flow.

H (m)	30	27	24	18	12	6
Q (L/s)	0	6.9	11.4	15.8	18.9	21.5
η	0	60	70	65	40	20

- Sketch the pump's characteristic curves for the case of two pumps connected in parallel
- Sketch the pump's characteristic curves for the case of two pumps connected in series

Solution

❑ **Pump curve**

H (m)	30	27	24	18	12	6
Q (L/s)	0	6.9	11.4	15.8	18.9	21.5
η	0	60	70	65	40	20

❑ **System curve**

- $H_t = 10.50 + 150000Q^2$ where Q in m^3/s

or

- $H_t = 10.50 + 0.15Q^2$ where Q in l/s

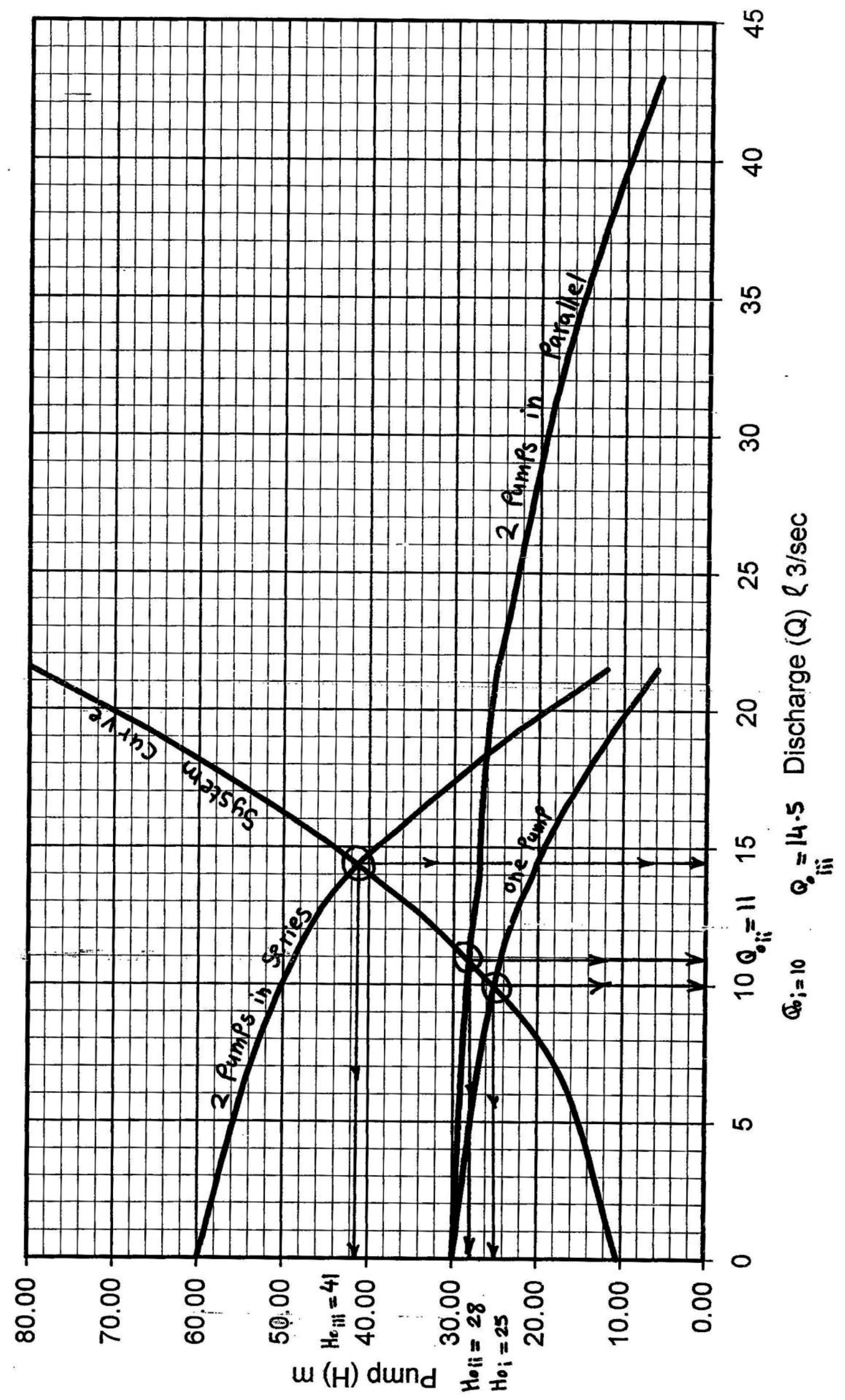
Q (L/s)	0	6.9	11.4	15.8	18.9	21.5
H_{system} (m)	10.5	17.64	30	47.95	64.08	79.84

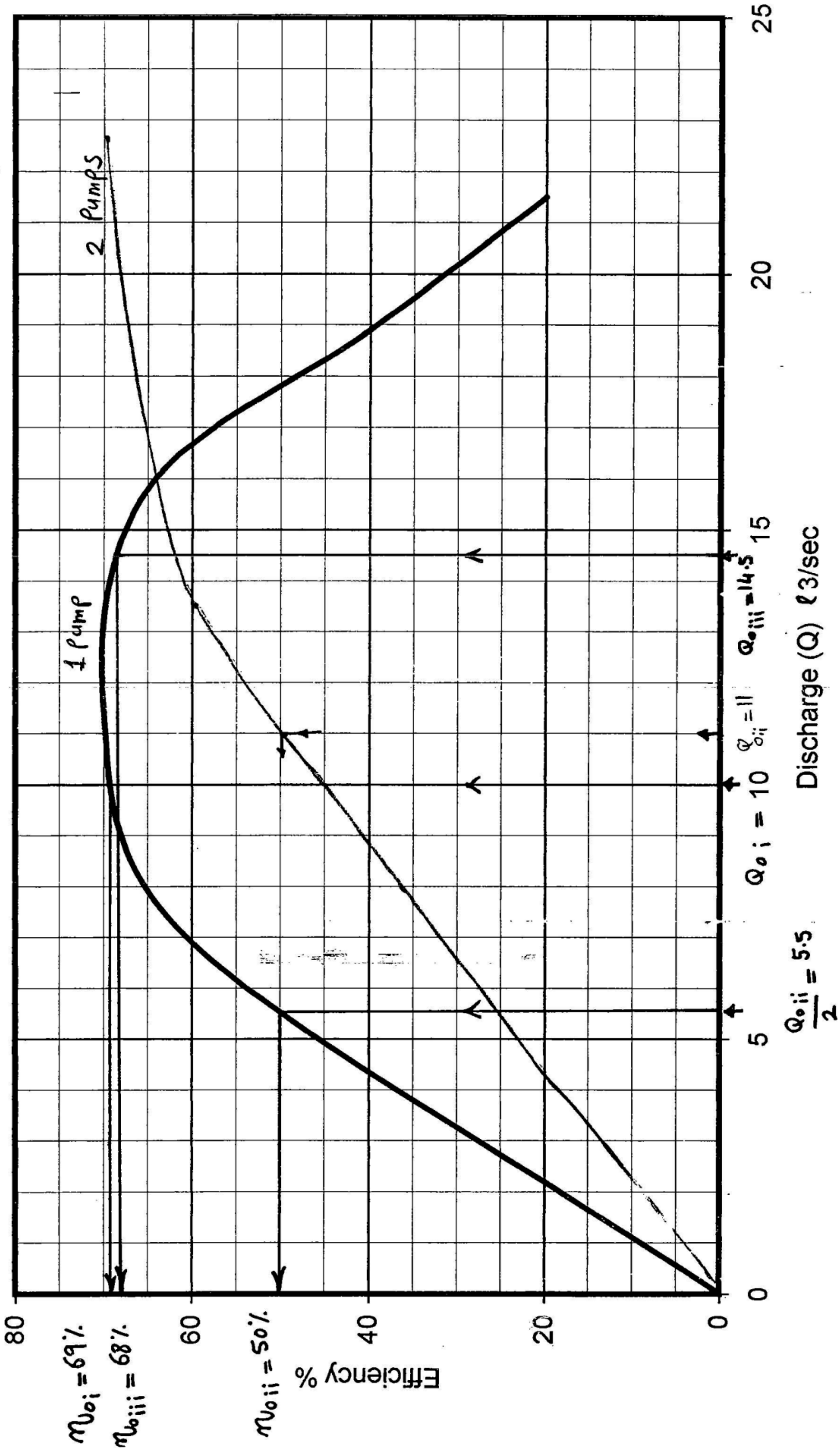
❑ **2 pumps in parallel**

H	30	27	24	18	12	6
2 Q	0	13.8	22.8	31.6	37.8	43

❑ **2 pumps in series**

2H	60	54	48	36	24	12
Q	0	6.9	11.4	15.8	18.9	21.5


Discharge (Q) ℓ 3/sec
Pump (H) m
0
5
10
15
20
25
30
35
40
45
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
System Curve
2 Pumps in Series
2 Pumps in Parallel
one Pump
Qo i = 10
Qo ii = 11
Qo iii = 14.5
Ho iii = 41
Ho ii = 28
Ho i = 25


2 Pumps
1 Pump
Efficiency %
Discharge (Q) ℓ3/sec
Qₒi = 10
Qₒii = 11
Qₒiii = 14.5
Qₒii/2 = 5.5
ηₒi = 69%
ηₒiii = 68%
ηₒii = 50%
0
5
10
15
20
25
20
40
60
80

❑ **<u>1 pump</u>**

Q **= 10 l/s** ***H*** **= 25 m** ***η*** **= 0.69 m**

$$P_i = \frac{\gamma Q H_t}{\eta_p} = \frac{9810 \times 0.01 \times 25}{0.69} = 3554.35W = 3.55kW$$

❑ **<u>2 pumps in parallel</u>**

Q **= 11 l/s** ***H*** **= 28 m** ***η*** **= 0.5 m**

<u>Power of one pump:</u>

$$P_i = \frac{\gamma Q H_t}{\eta_p} = \frac{9810 \times \frac{0.011}{2} \times 28}{0.50} = 3021.48W = 3.02kW$$

<u>Total motor Power:</u>

$$P_i = \frac{\gamma Q H_t}{\eta_p} = \frac{9810 \times 0.011 \times 28}{0.50} = 6042.96W = 6.04kW$$

❑ **<u>2 pumps in series</u>**

Q **= 14.50 l/s** ***H*** **= 41 m** ***η*** **= 0.68 m**

<u>Power of one pump:</u>

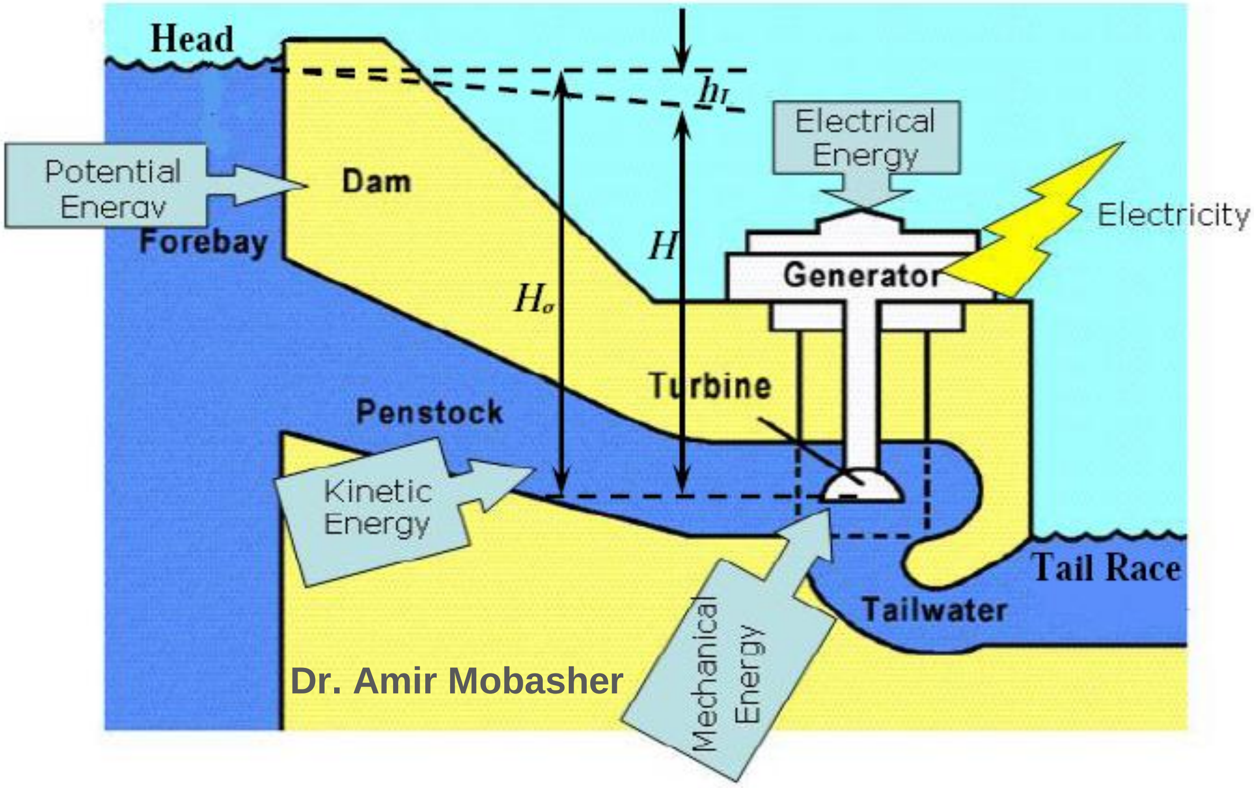
$$P_i = \frac{\gamma Q H_t}{\eta_p} = \frac{9810 \times 0.0145 \times \frac{41}{2}}{0.68} = 4288.27W = 4.29kW$$

<u>Total motor Power:</u>

$$P_i = \frac{\gamma Q H_t}{\eta_p} = \frac{9810 \times 0.0145 \times 41}{0.68} = 8576.54W = 8.58kW$$

3.4 Hydraulic Turbines

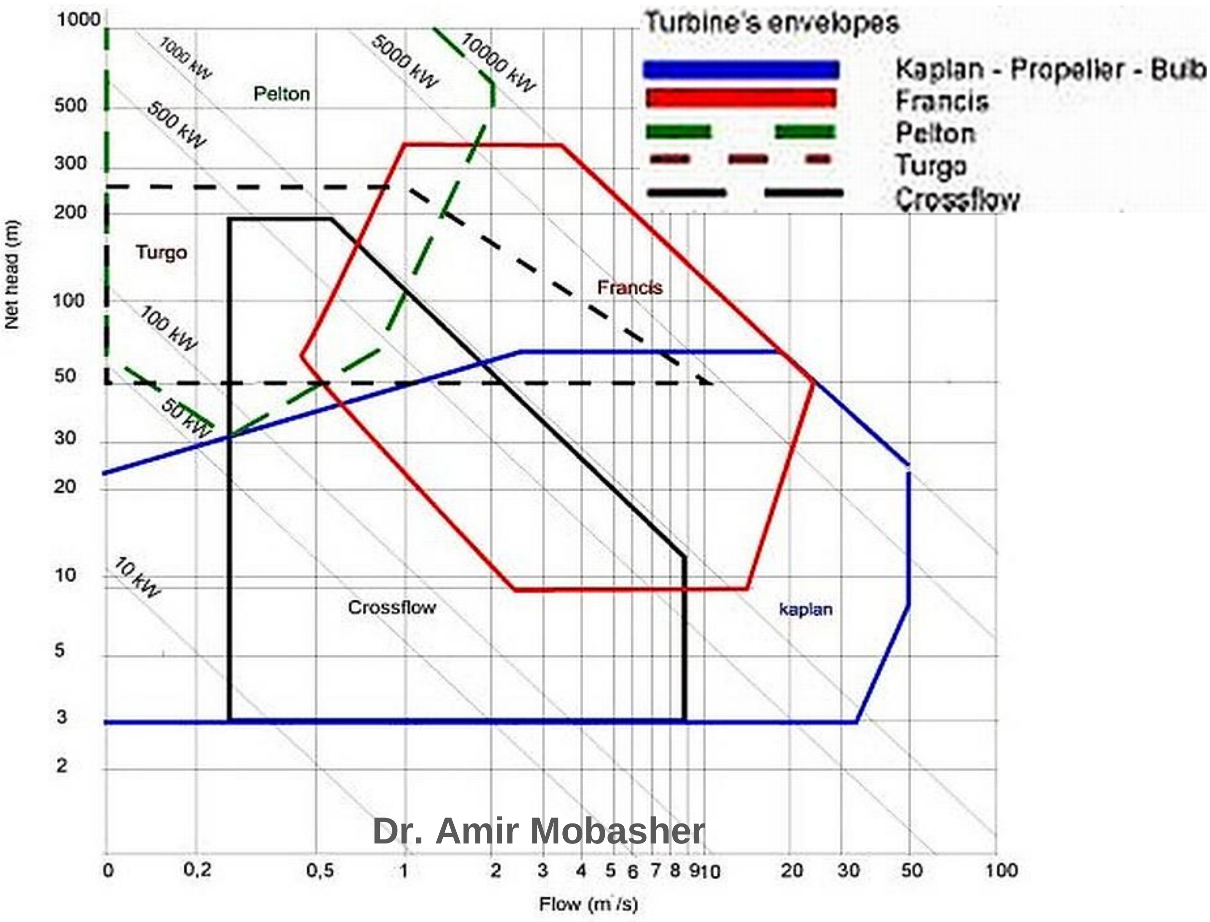
- Hydraulic Turbines (water wheels) have been in use for centuries.
- Hydraulic Turbines convert the potential energy of water into work.
- Basic Turbines are either Reaction or Impulse.
- First developed in the mid 1800's.
- Power outputs range up to 1,000 Mw.
- Included are Tidal and Wind Turbines.



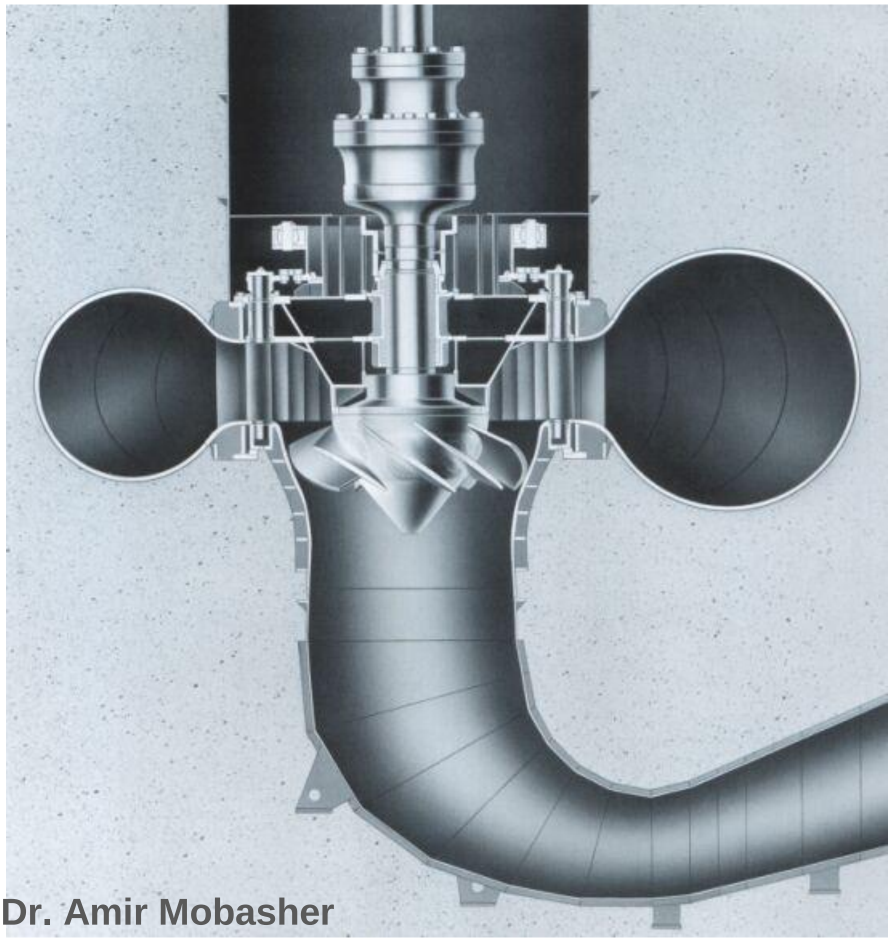
3.4.1 Early Hydraulic Turbines

- Amount of power depended on wheel diameter and height of water.

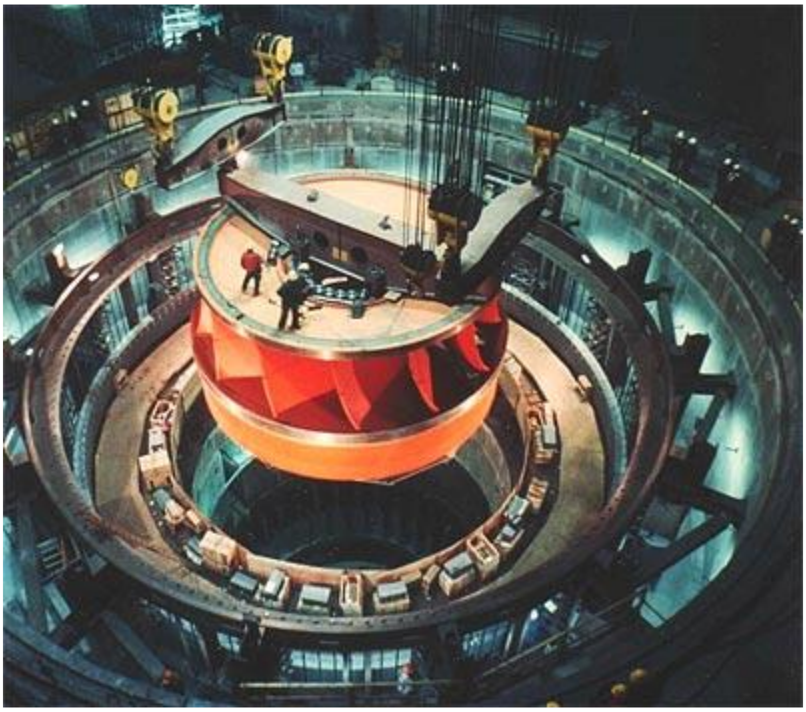

3.4.2 Hydraulic Turbine Types

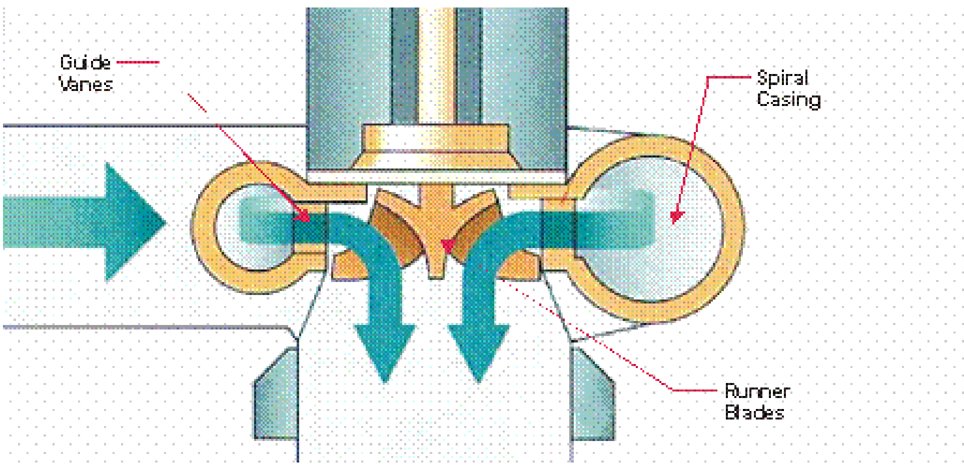


❑ Hydraulic Turbine – Kaplan – Reaction

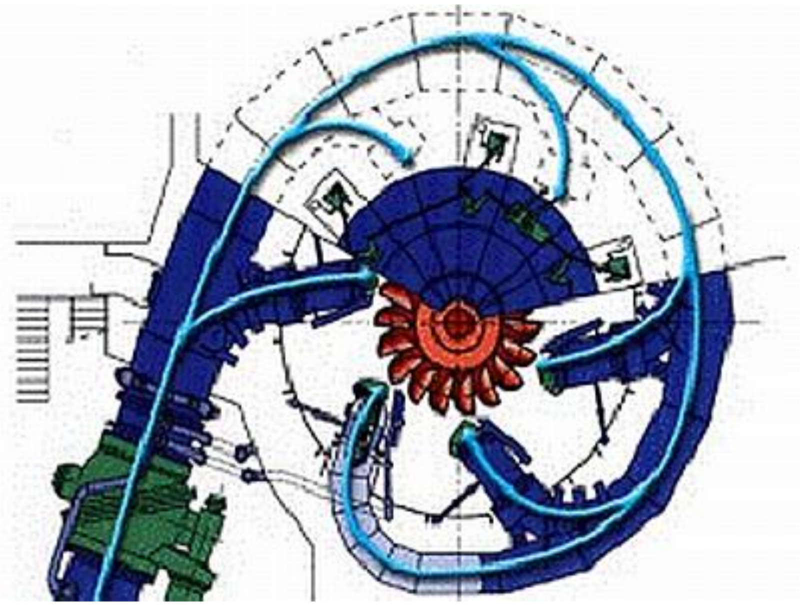


❑ Hydraulic Turbine – Francis – Reaction

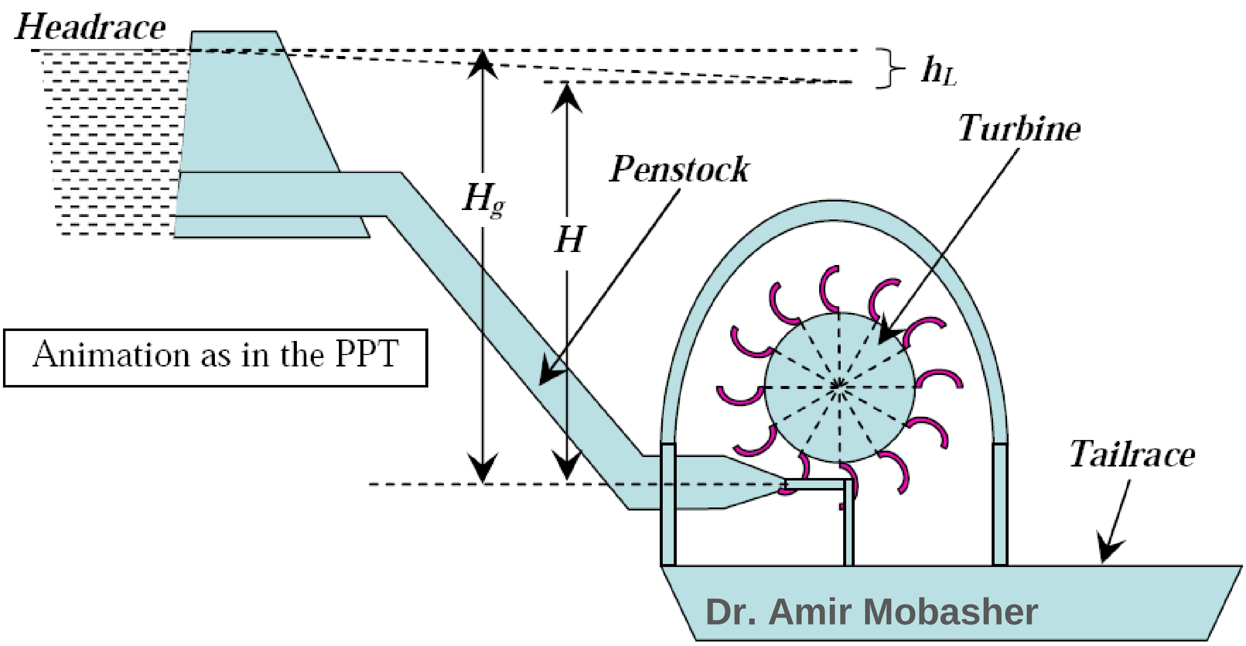




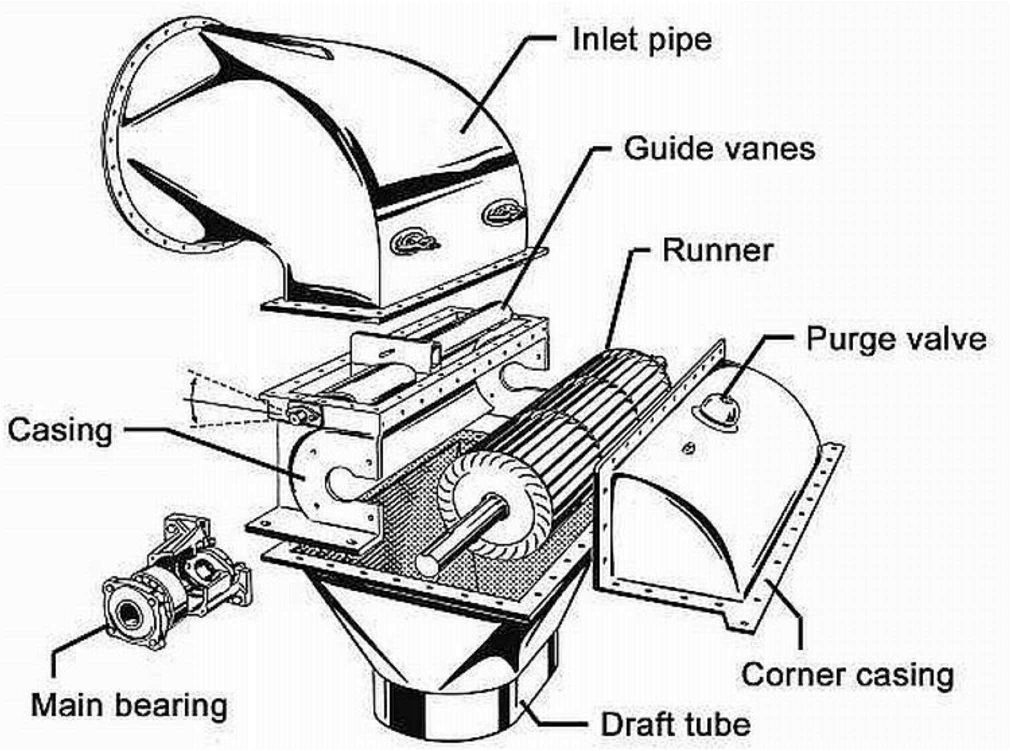
❑ Hydraulic Turbine – Pelton – Impulse



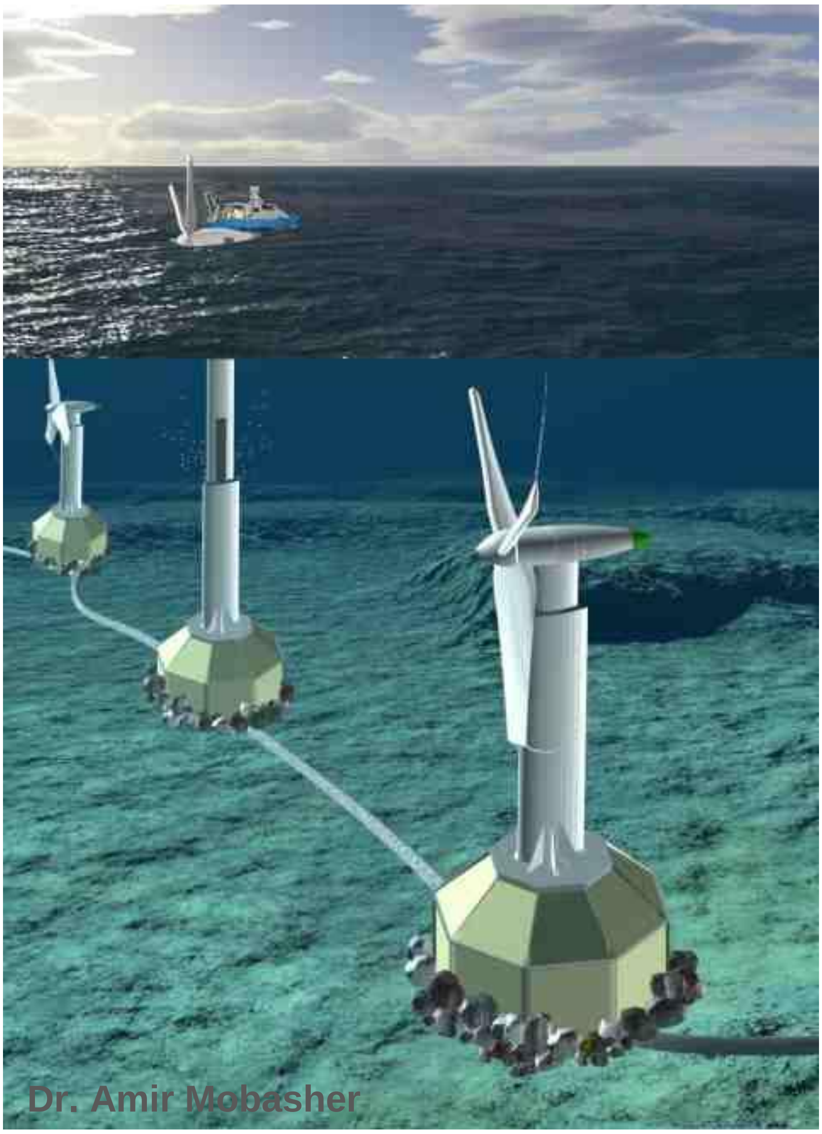
❑ Hydraulic Turbine – Turgo - Reaction



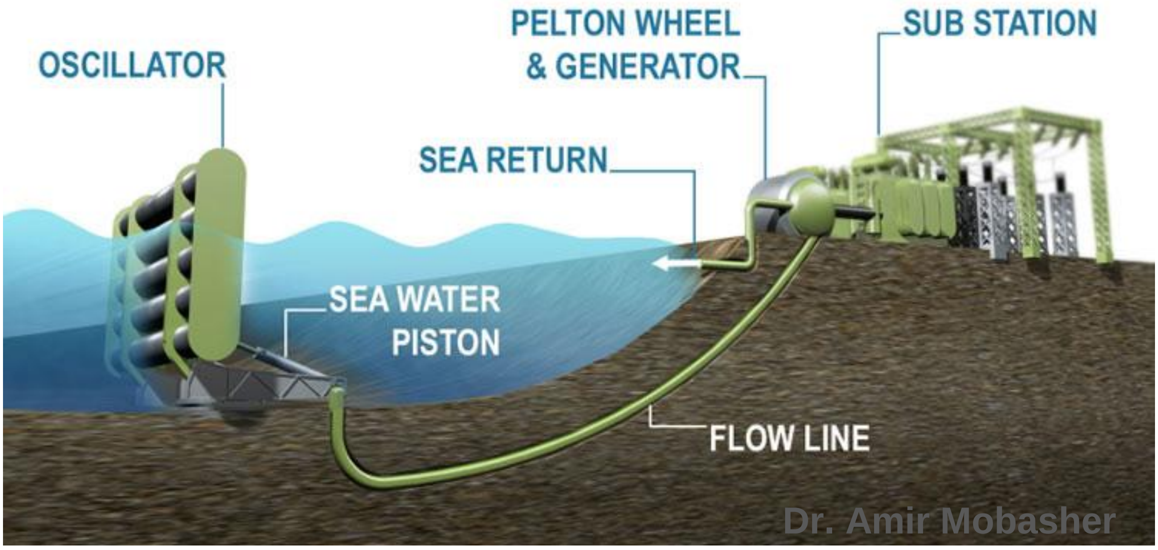
❑ Hydraulic Turbine – Cross-flow - Impulse



❑ Hydraulic Turbine – Tidal Power



❑ Hydraulic Turbine – Wave Power



3.4.3 Efficiency

Input Electric → **PUMP** → **Output Water**

$$\eta = \frac{Outpot}{Input} = \frac{\gamma QH}{Input}$$

Input Water → **Turbine** → **Output Electric**

$$\eta = \frac{Outpot}{Input} = \frac{Outpot}{\gamma QH}$$

- **Pump's Power**

$$P = \frac{\gamma Q H}{\eta} = ...Watt$$

$$HP = \frac{\gamma Q H}{\eta k} = ...Horsepower$$

- **Turbine's Power**

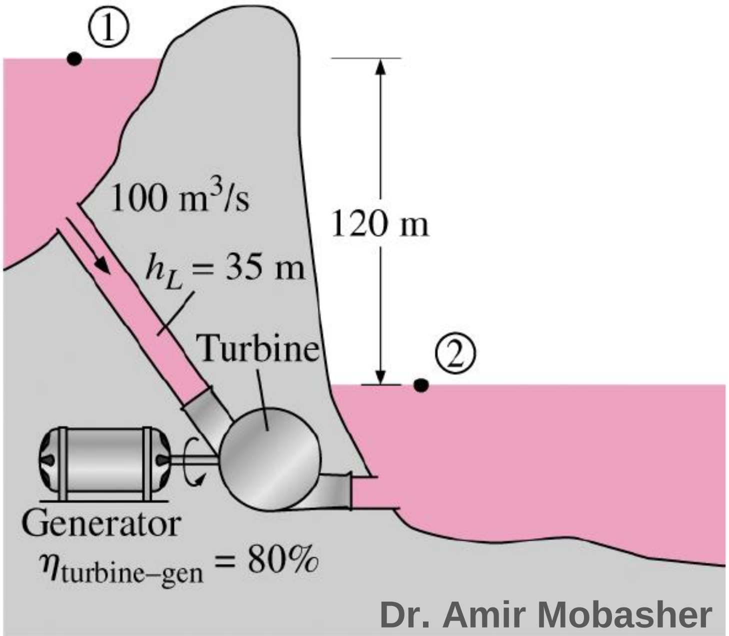
$$P = \gamma Q H \eta = ...Watt$$

$$HP = \frac{\gamma Q H \eta}{k} = ...Horsepower$$

γ	**1000 kg/m^3**	**9810 N/m^3**	**62.42 Ib/ft^3**
k	**75**	**735**	**550**

Example 3-6

In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.



Solution

Use the steady head form of the energy equation for a single stream

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_L + H_T$$

$$120 + 0 + 0 = 0 + 0 + 0 + 35 + H_T$$

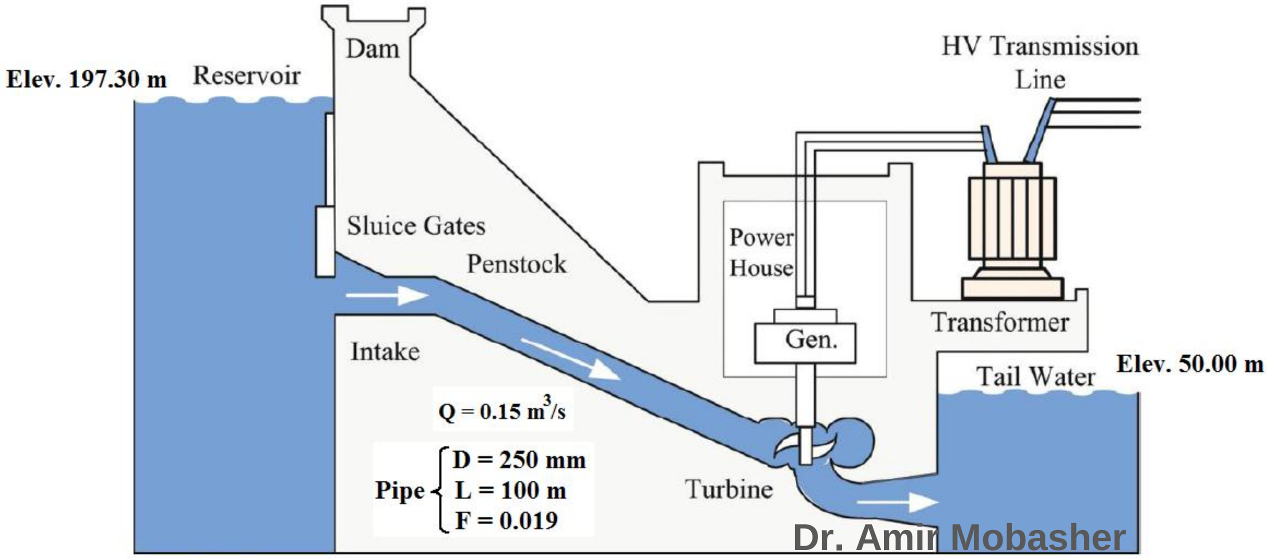
$$H_T = 85 \text{ m}$$

$$P = 9810 x 100 x 85 x 0.80 = 66.70 MW$$

$$HP = \frac{1000 x 100 x 85 x 0.80}{75} = 90666.67 Horsepower$$

Example 3-7

Water flows from an upper reservoir to a lower one while passing through a turbine, as shown in Fig. Find the power generated by the turbine. Neglect minor losses. The efficiency of the turbine–generator is 90 percent



Solution

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_L + H_T$$

$$h_L = h_f = \frac{8FL}{g\pi^2 D^5} Q^2 = \frac{8x0.019x100x0.15^2}{9.81x\pi^2 x0.25^5} = 3.65m$$

$$147.30 + 0 + 0 = 0 + 0 + 0 + 3.65 + H_T$$

$$H_T = 143.65 \text{ m}$$

$$P = 9810x0.15x143.65x0.90 = 190.20kW$$

$$HP = \frac{1000x0.15x143.65x0.90}{75} = 258.57 Horsepower$$

Or HP = 190.20 / 0.746 = 255 Horsepower

Best regards,
Dr. Amir Mobasher

CHAPTER 4

MOMENTUM PRINCIPLE

4.1 Development of the Momentum Equation معادلة كمية الحركة:

It is based on the law of conservation of momentum principle, which states that "***the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction***" .the force acting on fluid mass "m" is given by the Newton`s second law of motion

F = m x a

Where a is the acceleration acting in the same direction as force F

But

$$a = \frac{dv}{dt}$$

$$F = m\frac{dv}{dt}$$

"**m**" is constant and can be taken inside the differential

T $F = \frac{d(mv)}{dt}$ is known as "***the momentum principle***"

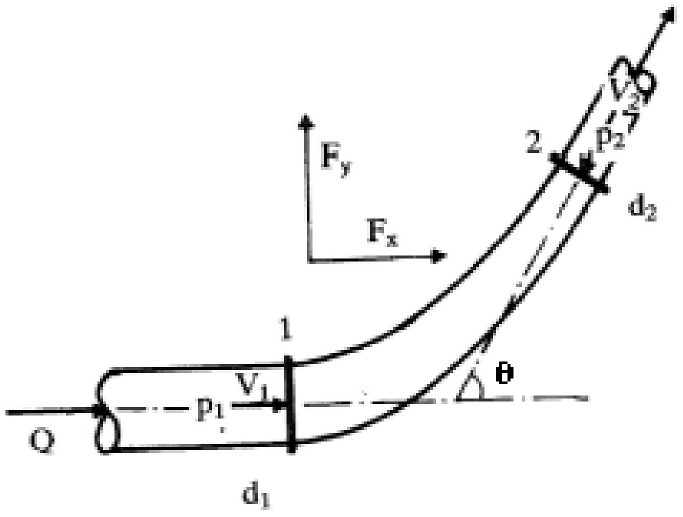
This equation can be written as:

F dt = d(mv)

Which is known as "***the impulse –momentum equation***" and states that:

The impulse of a force acting on a fluid of mass in short interval of time is equal to the change of momentum (**d(mv)**) in the direction of force.

4.2 Force exerted by a flowing fluid on a Pipe –Bend



The "impulse-momentum equation" is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two section (1) and (2), as shown in Fig

Let,

V_1 = Velocity of flow at section (1),

P_1 = Pressure intensity at section (1),

A_1 = Area of cross-section of pipe at section (1) and ,

V_1, P_1, A_1 Corresponding values of velocity, pressure and area at section (2).

- Let "F_x" and "F_y" be the components of the forces exerted by the flowing fluid on the bend in "x" and "y" direction respectively.
- Then the force exerted by the bend on the fluid in the "x" and "y" direction will be equal to "F_x" and "F_y" but in the opposite direction.
- The momentum equation in "x" direction is given by:

Total force exerted on the fluid in a control volume in a given direction	=	Rate of change of momentum in the given direction of the fluid passing through the control volume

$$\sum F_{ext} = (\sum \rho QV)_{out} - (\sum \rho QV)_{in}$$

$P_1A_1 - P_2A_2 \cos\theta - F_x$ = (Mass per sec) (change of velocity)

$$P_1A_1 - P_2A_2 \cos\theta - F_x = \rho Q\,(V_2 \cos\theta - V_1)$$

$$F_x = \rho Q\,(V_1 - V_2 \cos\theta) + P_1A_1 - P_2A_2 \cos\theta$$

Similarly the momentum equation in "y" direction gives

$$0 - P_2A_2 \sin\theta - F_y = \rho Q\,(V_2 \sin\theta - 0)$$

$$F_y = \rho Q\,(-V_2 \sin\theta) - P_2A_2 \sin\theta$$

Now the resultant force F_R acting on the bend

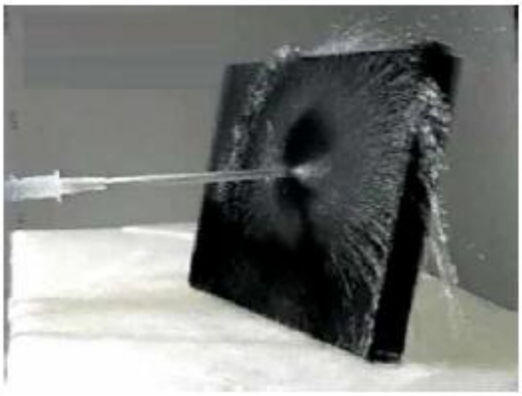
$$F_R = \sqrt{F_x^2 + F_y^2}$$

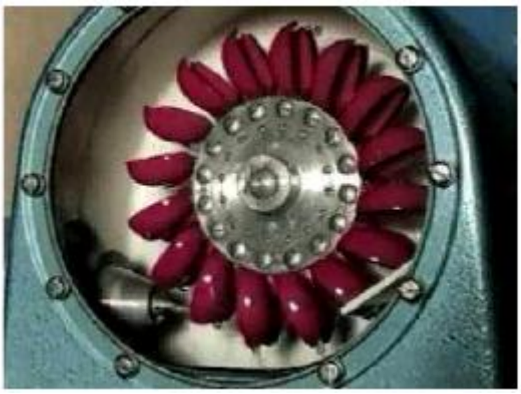
And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x}$$

4.3 Applications of the Linear Momentum Equation:

- A jet of fluid deflected by an object puts a force on the object. This force is the result of the change of momentum of the fluid and can happen even though the speed (magnitude of velocity) remains constant. If a jet of water has sufficient momentum, it can tip over the block that deflects it.
- The same thing can happen when a garden hose is used to fill sprinkles.
- Similarly, a jet of water against the blades of a Pelton wheel turbine causes the turbine wheel to rotate.





Problem 4-1:

A nozzle 5 cm diameter delivers a jet of water that strikes a flat plate normally. If the jet velocity is 60 m/sec, calculate he force acting on the plate if:

- The plate is stationary.
- The plate is moving at 20 m/sec. in the same direction of the jet.
- If the plate is replaced by a series of plates moving at the same speed, determine the rate of doing work and the efficiency of the system.

Solution

a) $F_x = \rho\ Q\ v = 1000 * \pi(0.05)^2/4 * 60 * 60 = 7068.58$ N

b) $F_x = \rho\ Q_r\ v_r = 1000 * \pi(0.05)^2/4 * (60 - 20) * (60 - 20) = 3141.6$ N

c) $F_x = \rho\ Q\ v_r = 1000 * \pi(0.05)^2/4 * 60 * (60 - 20) = 4712.4$ N

Output power $= F_x * u = 4712.4 * 20 = 94247.78$ watt

Input power $= \gamma\ Q\ H = \gamma\ Q\ v^2/2g =$

$= 9810 * \pi(0.05)^2/4 * 60 * (60)^2/2g = 212057.5$ watt

η = Output power / input power = 44.44 %

Problem 4-2:

Determine the components of divided discharge in the plate direction, and the force exerted by a jet of water that has an area of 6.5 cm^2 and moves at 30 m/sec on an inclined fixed flat plate as a function of its angle of inclination to the jet direction.

Solution

$F_s = 0$

$Q = Q_1 + Q_2$ ------------------------------------(1)

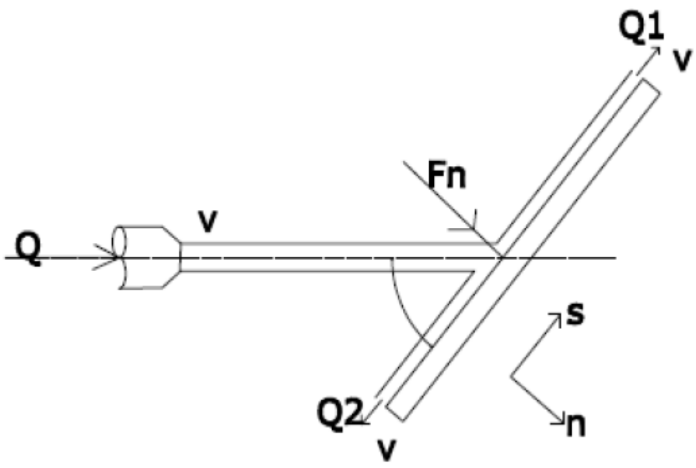
$F_s = \rho\, Q\, v \cos\theta - (\rho\, Q_1\, v - \rho\, Q_2\, v) = 0$

$Q \cos\theta = Q_1 - Q_2$ --------------------------(2)

By solving 1 & 2

$Q_1 = Q\,(1 + \cos\theta)/2$

$Q_2 = Q\,(1 - \cos\theta)/2$



$F_n = \rho\, Q\, v \sin\theta$

$F_x = F_n \sin\theta = \rho\, Q\, v \sin^2\theta$

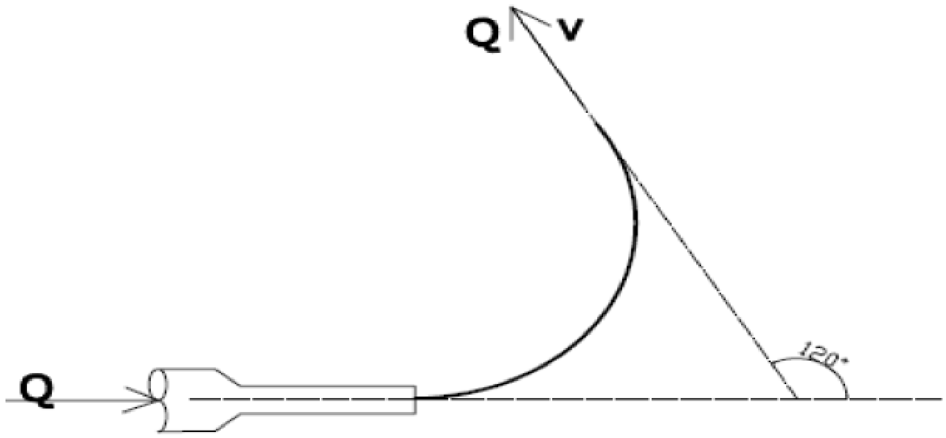
$F_y = - F_n \cos\theta = - \rho\, Q\, v \sin\theta \cos\theta$

Problem 4-3: A jet of water having a velocity of 30 m/sec and area 10 cm^2 impinges on a curved vane. The jet approaches the vane horizontally and deviates an angle of 120^o. Calculate the force acting on the vane if it is:

- Stationary.
- Moving at 15 m/sec in the direction of the jet.

- If the vane is replaced by a series of vanes moving at the same speed, calculate the output power and the efficiency of the system.

Solutio



a) $Fx = \rho\, Q\, v - (-\rho\, Q\, v \cos 60) = \rho\, Q\, v\, (1 + \cos 60)$

$= 1000 * 10 * 10^{-4} * 30 * 30\, (1 + 0.5) = 1350$ N

$Fy = \rho\, Q\, v \sin 60 = 1000 * 10 * 10^{-4} * 30 * 30 \sin 60 = 779.42$ N

$R = \sqrt{Fx^2 + Fy^2} = 1558.85$ N

$\theta = \tan^{-1} (Fy / Fx) = 30°$

b) $Fx = \rho\, Q_r\, v_r\, (1 + \cos 60) = 1000 * 10 * 10^{-4} * (30 - 15) * (30 - 15) * 1.5 = 337.5$ N

$Fy = \rho\, Q_r\, v_r \sin 60 = 1000 * 10 * 10^{-4} * (30 - 15) * (30 - 15) \sin 60 = 194.856$ N

$R = \sqrt{Fx^2 + Fy^2} = 389.71$ N

$\theta = \tan^{-1} (Fy / Fx) = 30°$

c) $Fx = \rho\, Q\, v_r\, (1 + \cos 60) = 1000 * 10 * 10^{-4} * 30 * (30 - 15) (\cos 60 + 1) = 675$ N

Output power = $Fx * u = 675 * 15 = 10125$ watt

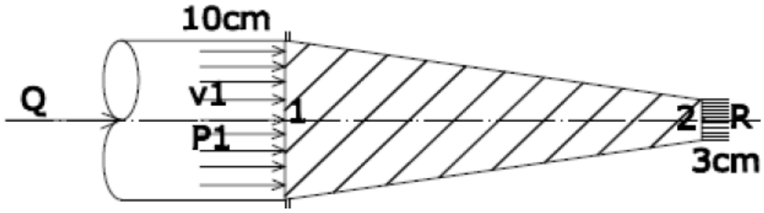
Input power = $\gamma\, Q\, H = \gamma\, Q\, v^2/2g = 9810 * 10 * 10^{-4} * 30 * (30)^2/2g = 13500$ watt

η = Output power / input power = 75 %

Problem 4-4:

A nozzle 3 cm diameter that has a coefficient of velocity of 0.97 is fitted at the end of a pipeline 10 cm diameter. Find the force exerted by the nozzle on the pipeline if the pressure at the end of the pipeline is 0.5 kg/ cm^2

Solution



$P1 = 0.5 \text{ kg/cm}^2 = 0.5 * 10^4 \text{ kg/m}^2$

Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1 / \gamma = Z_2 + v_2^2 / 2g + P_2 / \gamma$

$0 + Q^2 / 2 g A_1^2 + 0.5 * 10^4/1000 = 0 + 0 + Q^2 / 2 g A_2^2$

$Q_{th} = 0.007 \text{ m}^3/\text{sec}$

$Q_{act} = Cd\ Q_{th} = Cv * Cc * Q_{th} =$

$= 0.97 * 1 * 0.007 = 0.00682 \text{ m/sec}$ (Cc = 1 for nozzle)

$v_{1act} = Q_{act} / A_1 = 0.8682 \text{ m/sec}$

$v_2 = Q_{act} / A_2 = 9.646 \text{ m/sec}$

Apply the momentum Eqn. In X-direction

$P_1 A_1 - R = \rho Q v_2 - \rho Q v_1 = \rho Q(v_2 - v_1)$

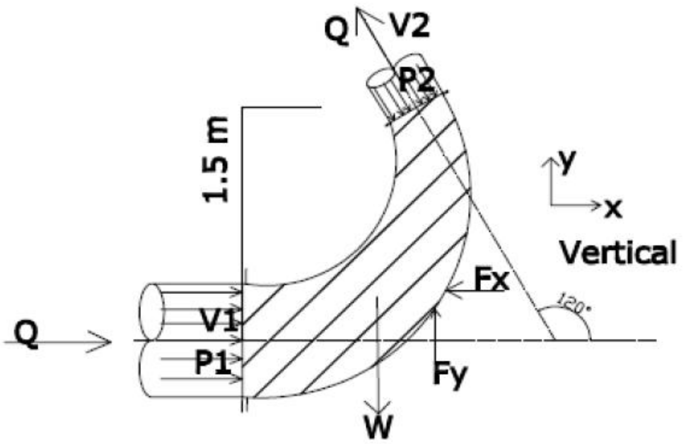
$0.5 * 10^4 * 9.81 * \pi/4 (0.1)^2 - R = 1000 * 0.00682 * (9.646 - 0.8682)$

$R = 325.37$ N Force exerted on water

Problem 4-5:

Water flows at a rate of 100 lit/sec in a 30 cm diameter horizontal pipe connected to a 20 cm diameter pipe through a vertical reducer bend where the change in water direction is 120^o. The vertical distance between the entrance and outlet of the bend is 1.5 m and the pressure at its entrance is 0.7 kg/ cm^2. The total weight of the bend material and water contained in it is 100 kg. Calculate the horizontal and vertical components of the force required to hold the bend in place.

Solution



$$Q = A_1\ v_1 = A_2\ v_2$$

$$V_1 = Q / A_1 = 1.415\ m/sec$$

$$v_2 = Q / A_2 = 3.183\ m/sec$$

Apply B.E bet. 1 & 2

$$Z_1 + v_1^2 / 2\ g + P_1 / \gamma = Z_2 + v_2^2 / 2\ g + P_2 / \gamma$$

$$0 + (1.415)^2 / 2\ g + 0.7 * 10^4 / 1000 = 1.5 + (3.183)^2 / 2g + P_2 / \gamma$$

$$P_2 = 49890.37\ N/m^2$$

Apply the momentum Eqn. In X-direction

$$P_1\ A_1 + P_2\ A_2 \cos 60 - Fx = - \rho\ Q\ v_2 \cos 60 - \rho\ Q\ v_1 = - \rho\ Q(v_2 \cos 60 + v_1)$$

$$0.7 * 10^4 * 9.81 * \pi/4(0.3)^2 + 49890.37 * \pi/4(0.2)^2 * 0.5 - Fx$$
$$= - 1000*0.1(3.183 * 0.5 + 1.415)$$

Fx = 5938.32 N Force exerted on water

Apply the momentum Eqn. In Y-direction

$$Fy – W - P_2\ A_2 \sin 60 = \rho\ Q\ v_2 \sin 60 - 0$$

$$Fy – 100 * 9.81 * 94890.37 * \pi/4(0.2)^2 \sin 60 = 1000 * 0.1\ (3.183 \sin 60)$$

Fy = 2614.02 N Force exerted on water

Force exerted on the bend

$Fx = 5938.32$ N

$Fy = 2614.02$ N

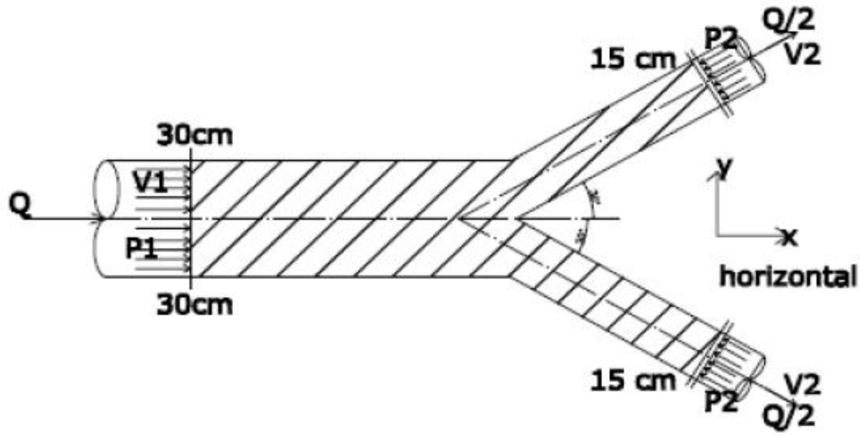
Force required to hold the bend in position.

$Fx = 5938.32$ N

$Fy = 2614.02$ N

Problem 4-6: Water flows through a Y-shaped horizontal pipe connection. The velocity in the stem is 3 m/sec and the diameter is 30 cm. Each branch is of 15 cm diameter and inclined at an angle 30° to the stem. If the pressure in the stem is 2 kg/cm^2, calculate the magnitude and direction of the force of water on the Y-shaped pipe connection if the flow rates in both branches are the same.

Solution



$P_1 = 2 \text{ kg/cm}^2$

$v_1 = 3 \text{ m/sec}$

$Q = A_1 v_1 = \pi/4 (0.3)^2 * 3 = 0.212 \text{ m}^3/\text{sec}$

$v_2 = Q / A_2 = 6 \text{ m/sec}$

Apply B.E bet. 1 & 2

$$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_2 + v_2^2/2g + P_2/\gamma$$

$$0 + (3)^2/2g + 2 * 10^4/1000 = 0 + (6)^2/2g + P_2/\gamma$$

$$P_2 = 18623.853 \text{ kg/m}^2$$

Apply the momentum Eqn. In X-direction

$P_1 A_1 - 2 * P_2 A_2 \cos 30 - Fx = 2 * \rho Q/2\ v_2 \cos 30 - \rho Q v_1$

$2 * 10^4 * 9.81 * \pi/4(0.3)^2 - 2 * 18623.853 * 9.81 * \pi/4(0.15)^2 * \cos30 - Fx$
$= 1000 * 0.212\ (6 \cos60 - 3)$

$Fx = 7810.92$ N Force exerted on water

Apply the momentum Eqn. In Y-direction

$Fy - P_2 A_2 \sin 30 + P_2 A_2 \sin 30 = \rho Q/2\ (v_2 \sin 30 - v_2 \sin 30)$

$Fy = 0$

Force exerted on the Y – shaped connection = 7810.92 N

Problem 4-7:

A 45° horizontal reducer elbow has 15 cm diameter at the upstream end and 10 cm at the other end and is connected at the end of a pipeline. Neglecting any losses in the elbow, determine the magnitude and direction of the force affecting the pipeline when a discharge of 75 lit/sec of water is flowing through the elbow into the atmosphere.

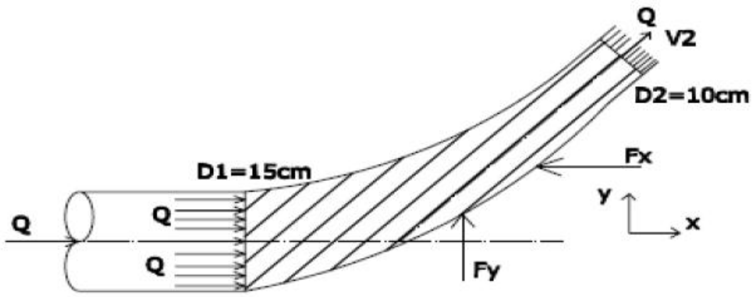
Solution

$Q = 75$ lit/sec $= 0.075\ m^3/sec$

$Q = A_1 v_1 = A_2 v_2$

$V_1 = Q / A_1 = 4.244$ m/sec

$v_2 = Q / A_2 = 9.55$ m/sec



Apply B.E bet. 1 & 2

$Z_1 + v_1^2 / 2g + P_1/\gamma = Z_2 + v_2^2 / 2g + P_2/\gamma$

$0 + (4.244)^2 / 2g + P_1/\gamma = 0 + (9.55)^2 / 2g + 0$

$P_1 = 36588.21\ N/m^2$

Apply the momentum Eqn. In X-direction

$P_1 A_1 - F_x = -\rho Q v_2 \cos 45 - \rho Q v_1 = -\rho Q(v_2 \cos 45 - v_1)$

$36588.21 * \pi/4(0.15)^2 - F_x = -1000*0.06 (9.55 \cos 45 - 4.244)$

$F_x = 496.073$ N Force exerted on water

Apply the momentum Eqn. In Y-direction

$F_y = \rho Q v_2 \sin 45$

$F_y = 1000 * 0.075* (9.55 \sin 45)$

$F_y = 506.43$ N Force exerted on water

Force exerted on the elbow

$F_x = 496.073$ N

$F_y = 506.43$ N

$R = \sqrt{F_x^2 + F_y^2} = 708.913$ N

$\theta = \tan^{-1} (F_y/F_x) = 45.6°$

R = the force exerted by elbow on the pipeline.

Best regards,
Dr. Amir Mobasher

ASSIGNMENTS

ASSIGNMENT 1

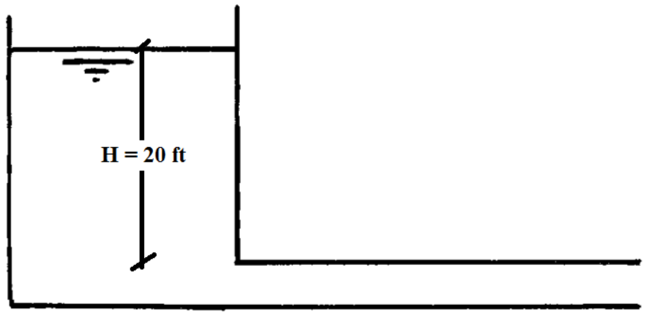
FLOW THROUGH PIPES

Q1: Determine the type of flow in a 400 mm diameter pipe when

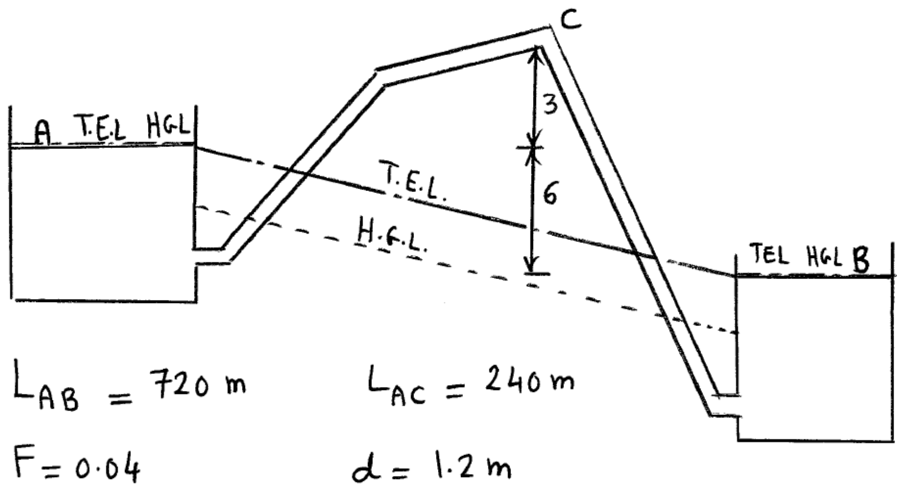
a) Water flows at average velocity of 1.2 m/sec and $\nu = 1.13\text{x}10^{-6}$ m^2/sec.

b) Glycerin flows at a rate of 2.0 lit/min having $\rho = 1260$ kg/m^3 and $\mu = 0.9$ N.s/m^2.

Q2: The losses in the shown figure equals $3(V^2/2g)$ft, when H is 20 ft. What is the discharge passing in the pipe? Draw the TEL and the HGL.

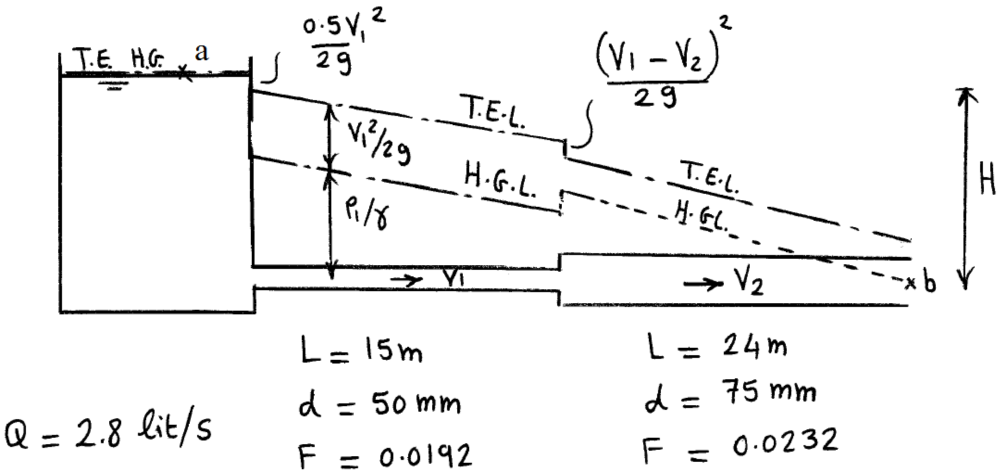


Q3: Find the discharge through pipe and the pressure at point "C".

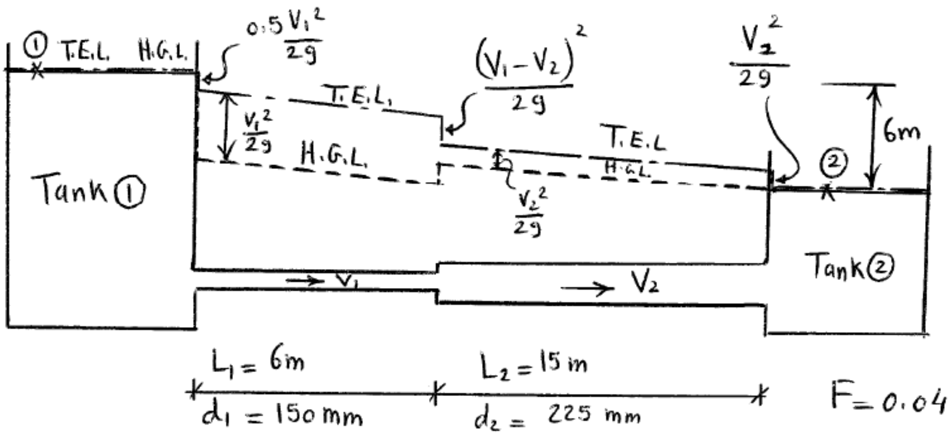


Q4: A syphon filled with oil of specific gravity 0.9 discharges 300 lit/s to the atmosphere at an elevation of 4.0 m below oil level. The syphon is 0.2 m in diameter and its invert is 5.0 m above oil level. Find the losses in the syphon in terms of the velocity head. Find the pressure at the invert if two thirds of the losses occur in the first leg.

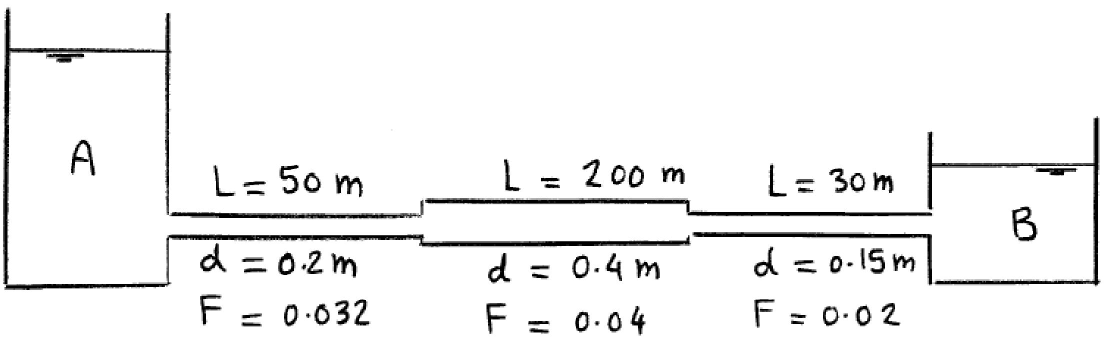
Q5: Find difference in the level "H" between point "a" and "b" for the shown figure.



Q6: Find the discharge through pipe for the shown figure.



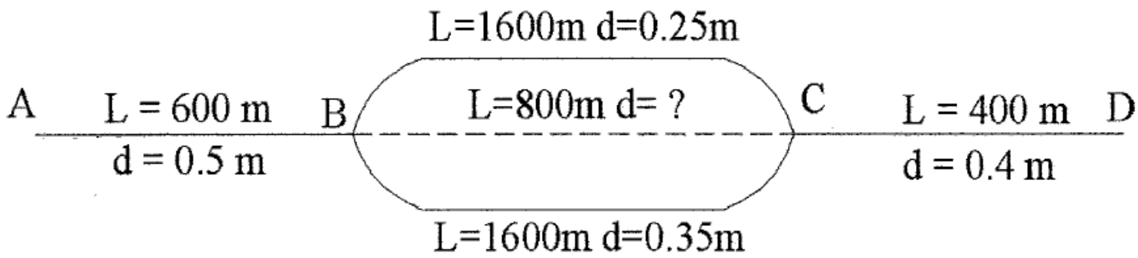
Q7: Two reservoirs are connected by 3 pipes in series as shown in figure. The velocity of water in the first pipe is 5.73 m/s. considering minor losses, calculate the difference in levels of the water in the two reservoirs. Sketch the T.E.L and the H.G.L. showing all vertical dimensions. Kc=0.5.



<u>Q8:</u> Two pipes are connected in parallel to each other between two reservoirs with length L1=2400m, D1=1.20m, F1=0.024; L2=2400m, D2=1.0m, and F2=0.02. Find the total flow, if the difference in elevation is 25m.

<u>Q9:</u> Reservoir A is connected to reservoir B through a pipeline having a length 64 km. The elevation of reservoir A is 180 m higher than that of reservoir B. The pipeline was designed to convey a flow rate of 28,000 m^3/day. It was decided to raise the flow rate from reservoir A to reservoir B by laying an identical pipe connected in parallel to the original pipe for a length of 32 km. Find the new flow rate from reservoir A to reservoir B. (F = 0.016 for all pipes).

<u>Q10:</u> The discharge in the network shown is increased from 250 lit/s to 350 lit/s by adding a new branch pipe BC of length 800 m. calculate the diameter of BC such that the head loss from A to D remains constant.



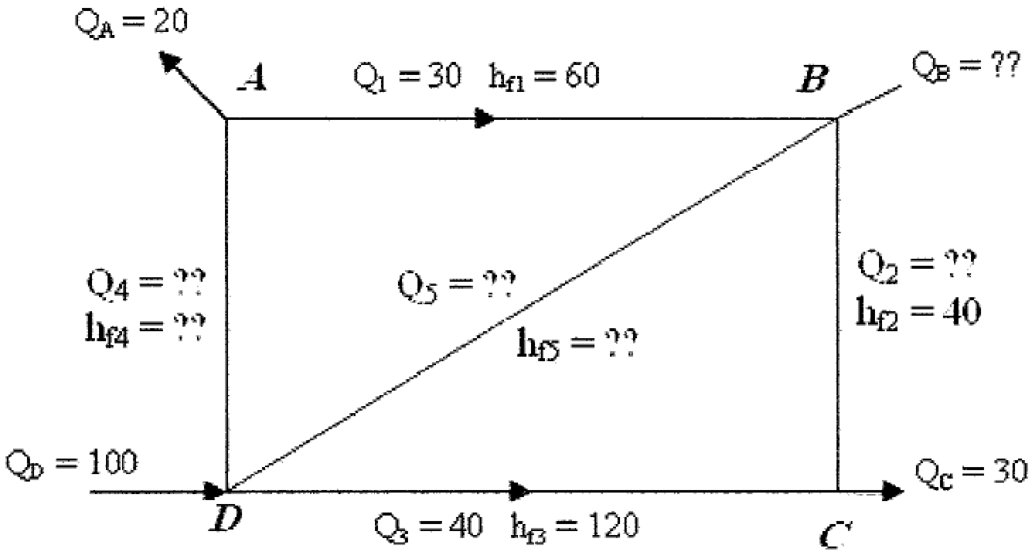
<u>Q11:</u> A reservoir A with its surface at elevation 100 ft above datum supplies water to 2 other reservoirs B and C of surface levels of 25 ft and zero ft respectively above datum. From A to junction J, common pip of 6 inch diameter, 800 ft long is used, the junction being at a level 20 ft above datum. The branch JB is 4 inch diameter, 200 ft long and the branch JC is 5 inch diameter, 300 ft long. **Calculate** the discharge to B and C. Take F for all pipes is taken 0.0075 .

Best regards,
Dr. Amir Mobasher

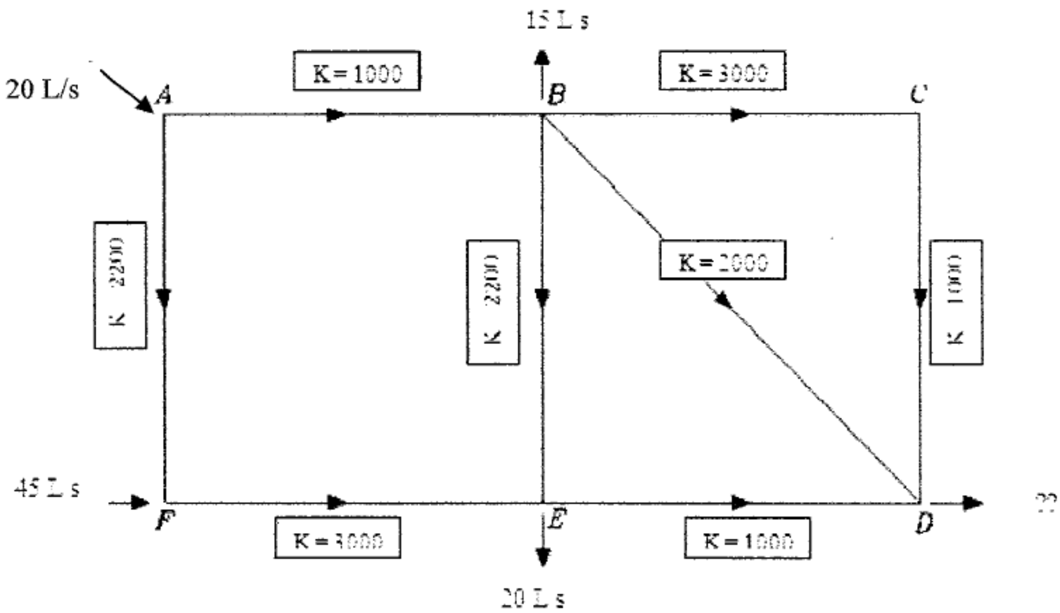
ASSIGNMENT 2

PIPE NETWORKS

Q1: A pipe network is shown in figure in which Q and h_f refers to discharges and pressure head loss respectively. Subscripts 1,2,3,4 and 5 designate respective values in pipe lengths AC, BC, CD, DA, and AC. Subscripts A, B, C, and D designate discharges entering or leaving the junction points A, B, C and D respectively.

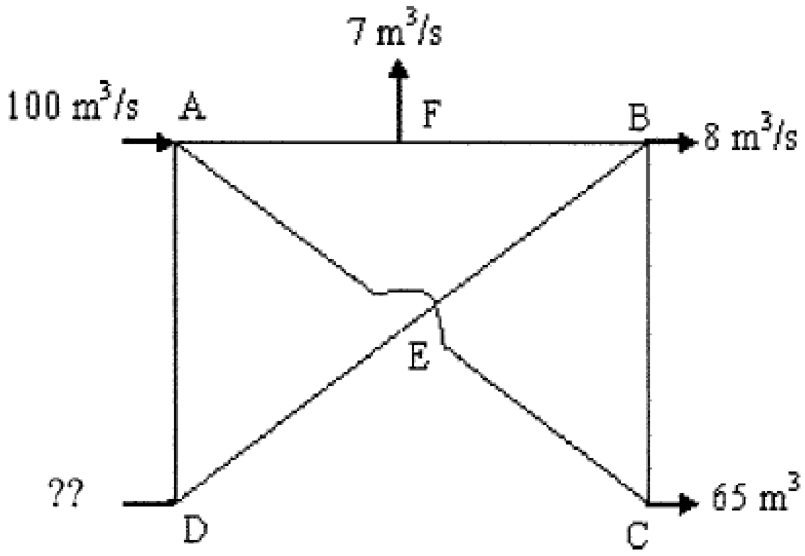


Q2: Determine the flow in each of the cast iron pipes in the shown network using the Hardy Cross method. Take n = 2.0



Q3: Using two steps of hardy-cross method, estimate the flow rate in each of the pipes in the network shown in figure. Assume F=0.02 for all pipes.

Pipe	AF	FB	BC	CE	EA	AD	DE	EB
Diam. (mm)	300	300	300	300	300	300	300	300
Length (m)	400	400	600	500	500	600	500	500



Best regards,
Dr. Amir Mobasher

ASSIGNMENT 3

PUMPS & TURBINES

Q1: A Pump has a cavitation constant = 0.25, this pump was instructed on well using pipe of 20 m length and 300 mm diameter, there are elbow (k_e=1) and valve (k_e= 5) in the system. The flow is 40 m^3 and the total Dynamic Head H_t = 30 m (from pump curve) f = 0.02

Calculate the maximum suction head

Q2: A centrifugal pump running at 1200 rpm gave the following relation between head and discharge:

Discharge (m^3/min)	0	4.5	9.0	13.5	18.0	22.5
Head (m)	22.5	22.2	21.6	19.5	14.1	0

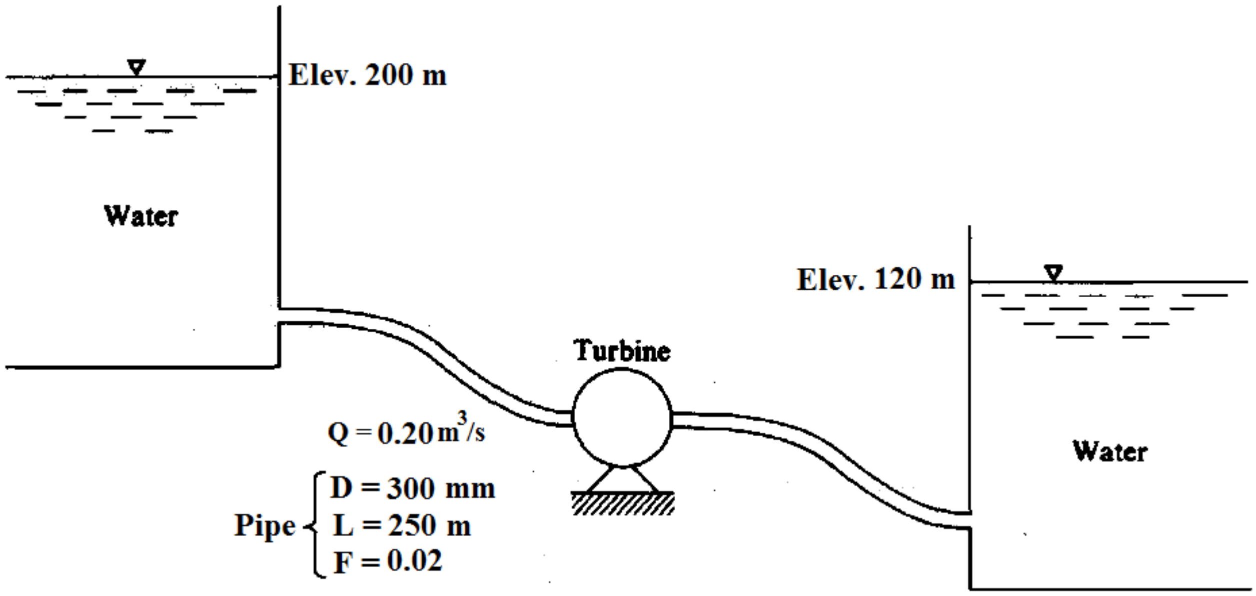
The pump is connected to a 400 mm suction and delivery pipe the total length of which is 80 m and the discharge to atmosphere is 20 m above sump level. The entrance loss is equivalent to an additional 8 m of pipe and f is assumed as 0.02. ***Calculate the discharge in m^3 per minute.***

Q3: A centrifugal pump is used to deliver water against a static lift of 12 m. The head loss due to friction = 150000 Q^2, head loss is in m and the flow is in Q in m^3/s. The pump characteristic is given in the following table. Deduce operating head and flow.

H (m)	30	27	24	18	12	6
Q (L/s)	0	6.9	11.4	15.8	18.9	21.5
η	0	60	70	65	40	20

- Sketch the pump's characteristic curves for the case of two pumps connected in parallel
- Sketch the pump's characteristic curves for the case of two pumps connected in series

Q4: Water flows from an upper reservoir to a lower one while passing through a turbine, as shown in Fig. Find the power generated by the turbine. Neglect minor losses. The efficiency of the turbine–generator is 85 percent



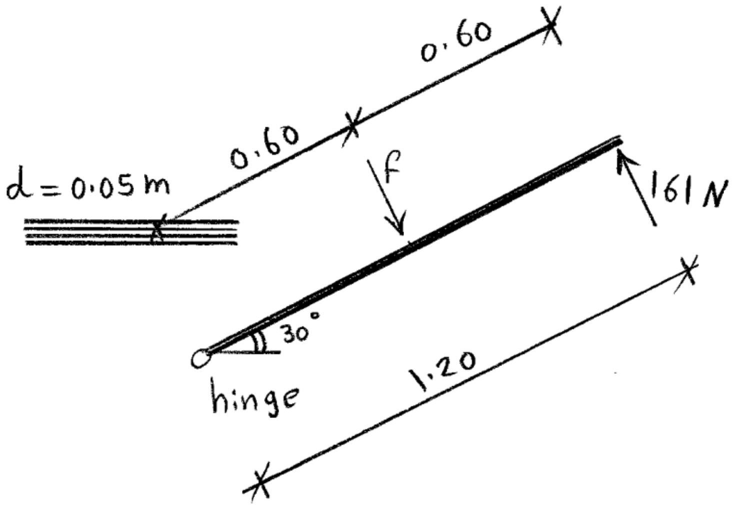
Best regards,
Dr. Amir Mobasher

ASSIGNMENT 4

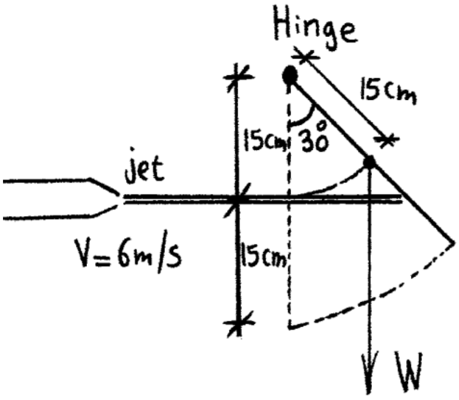
MOMENTUM PRINCIPLE

Q1: The force exerted by a 25mm diameter jet against a flat plate normal to the axis of the jet is 650N. What is the flow in m^3/s?

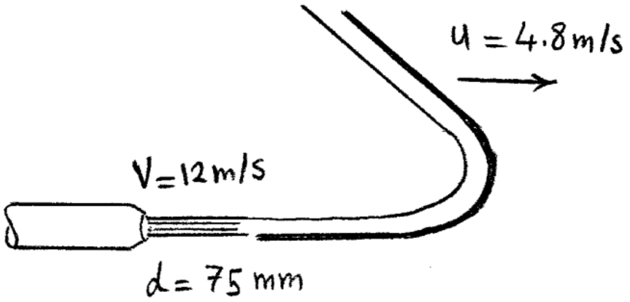
Q2: A water jet 50 mm diameter strikes a 1.20 m plate which is at an angle of 30° with the stream's direction. If the force applied at the edge of the plate, to maintain equilibrium, is 161 N, calculate the rate of flow. Neglect the weight of the plate.



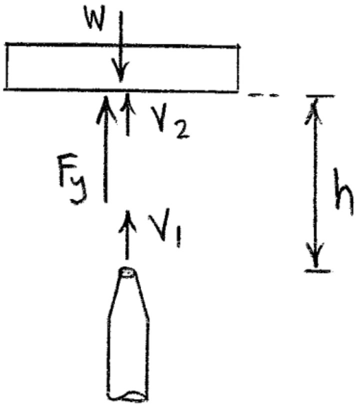
Q3: A square plate of uniform thickness and length of side 30 cm hangs vertically from hinges at its top edge. When a horizontal jet strikes the plate at its center, the plate is deflected and comes to rest at an angle of 30° to the vertical. The jet is 25 mm diameter and has a velocity of 6 m/s. Calculate the mass of the plate and give the distance along the plate, from the hinge, of the point at which the jet strikes the plate in its deflected position.



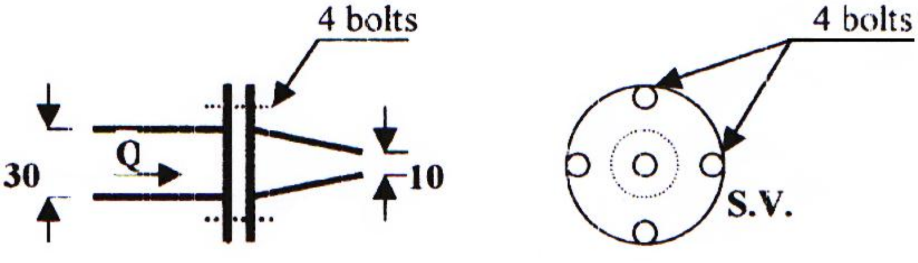
Q4: A jet of water 75 mm in diameter with a velocity of 12 m/sec meets a vane having a velocity of 4.8 m/sec in the direction of the jet. If water meets the vane tangentially and the deflection angle is 120°, find the force of water exerted on the vane.



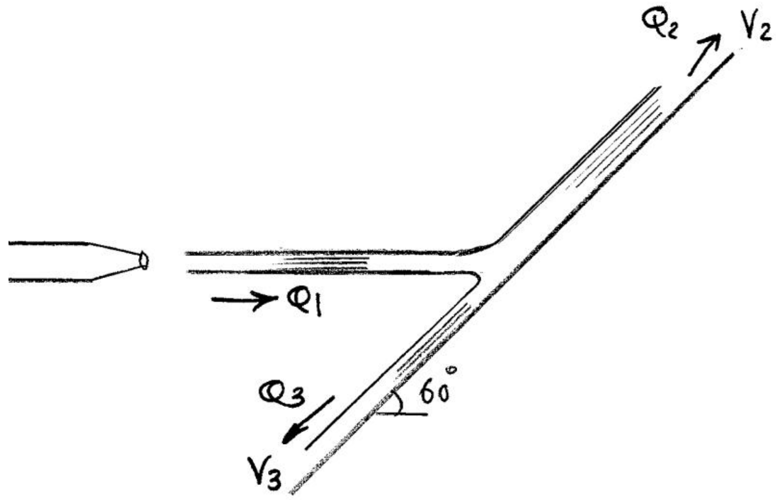
Q5: A liquid jet is issuing upward against a flat board of weight 1 Ib and supporting it as indicated in the figure. Determine the equilibrium height of the board above the nozzle exit, if the discharge is 1.5 lit/s and the nozzle diameter is 2 cm.



Q6: Find the force exerted on each bolt, when the water is flowing at rate of 0.25 m^3/s. The diameters of the pipe and the nozzle are 0.30 m and 0.10m. respectively.



Q7: A horizontal jet of water 2 x 10^3 mm^2 cross-section and flowing at a velocity of 15 m/s hits a flat plate at 60° to the axis (of the jet) and to the horizontal. The jet is such that there is no side spread. If the plate is stationary, calculate a) the force exerted on the plate in the direction of the jet and b) the ratio between the quantity of fluid that is deflected upwards and that downwards. (Assume that there is no friction and therefore no shear force).



Q8: The outlet pipe from a pump is a bend of 45 rising in the vertical plane (i.e. and internal angle of 135). The bend is 150mm diameter at its inlet and 300mm diameter at its outlet. The pipe axis at the inlet is horizontal and at the outlet it is 1m higher. By neglecting friction, calculate the force and its direction if the inlet pressure is 100 KN/m^2 and the flow of water through the pipe is 0.3 m^3/s. The volume of the pipe is 0.075m^3.

Q9: A conveying elbow turns water through an angle of 120° in a vertical plane. The flow cross-sectional diameter is 400 mm at the elbow inlet, section 1, and 200 mm at the elbow outlet, section 2. The elbow flow passage volume is 0.20 m^3 between sections 1&2. The water flow rate is 0.40 m^3/s and the elbow inlet and outlet pressures are 150 KPa and 90 KPa, respectively. Determine the magnitude and direction of the horizontal and vertical components of reaction force exerted by the water on the elbow.

Best regards,
Dr. Amir Mobasher