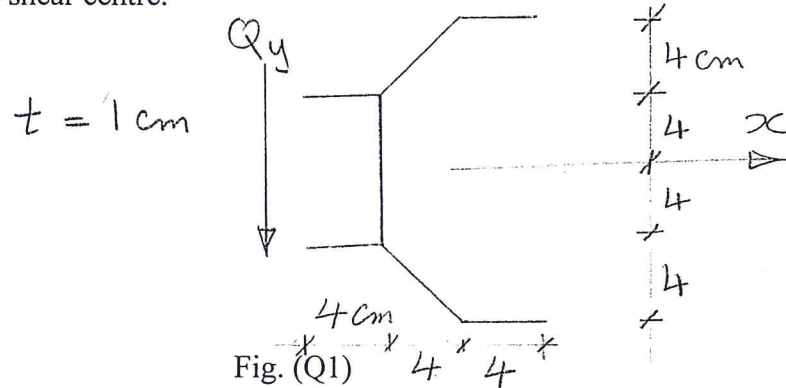


**Q1**

For the given cross-section:

1. Find  $I_x$ .
2. Draw the distribution of the shear flow.
3. Find the shear forces resisted by the elements of the cross-section.
4. Find the shear centre.



ILO's	
[d3]	[4 marks]
[d1,c3]	[6 marks]
[a3]	[4 marks]
[a3]	[2 marks]
<b>[Total 16]</b>	

**Q2**

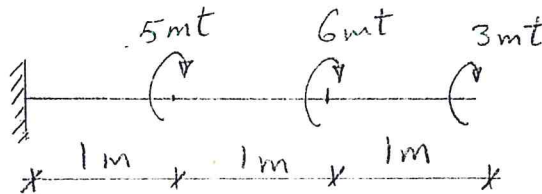
For the shown structure:

1. Draw the T.M.D.
2. Draw the T.A.D.
3. Compute the maximum shear stress.

Given :

$G=800 \text{ t/cm}^2$

Shaft:  
 $D = 0.4 \text{ m}$

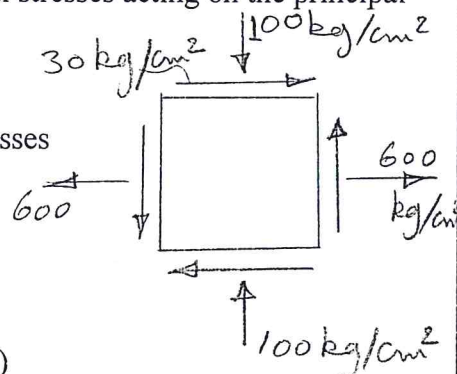


[a3]	[4 marks]
[a3]	[4 marks]
[d1]	[4 marks]
<b>[Total 12]</b>	

**Q3**

For the shown stress element:

1. Compute analytically the principal normal stress plane.
2. the maximum and minimum normal stresses acting on the principal plane.
3. Check graphically.
4. The principal shear plane.
5. The direction and values of the stresses acting on the principal shear plane.



$\sigma_x = + 600 \text{ kg/cm}^2$

$\sigma_y = - 100$

$\tau = - 30$

Fig. (Q3)

[a3,b3]	[3 marks]
[a3,b3]	[3 marks]
[d1]	[3 marks]
[a3,b3]	[3 marks]
[a3,b3]	[2 marks]
<b>[Total 12]</b>	

F. Jan 2014

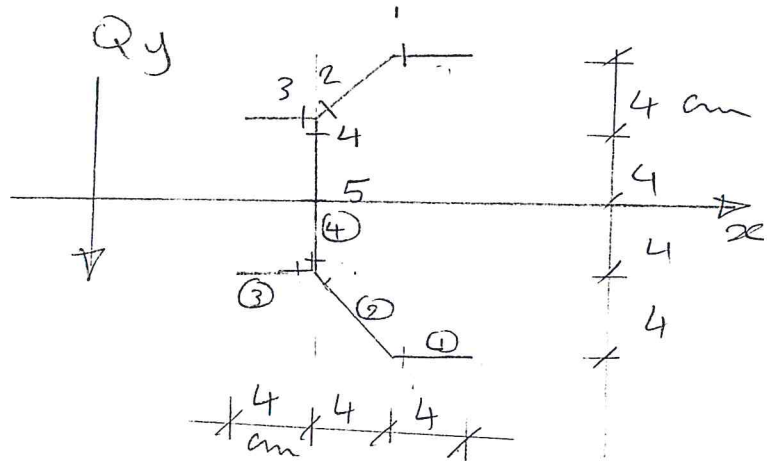
Q1

$t = 1 \text{ cm}$

$$F = \frac{Q_y S_{xx}}{I_{xx}}$$

Assume  $Q_y = I_{xx}$

$$\therefore \boxed{F \equiv S_{xx}}$$



$$\begin{aligned} * I_{xx} = 2 & \left[ (4 \times 1)(8)^2 + \frac{(4\sqrt{2} \times 1)(4)^2}{12} + (4\sqrt{2} \times 1)(6)^2 \right. \\ & \left. + (4 \times 1)(4)^2 \right] + \frac{(1)(8)^3}{12} = 1105.05 \text{ cm}^4 \end{aligned}$$

$$* S_{xx1} = (4 \times 1)(8) = 32$$

$$S_{xx2} = S_{xx1} + (1)(4\sqrt{2})(6) = 65.94$$

$$S_{xx3} = 4 \times 1 \times 4 = 16$$

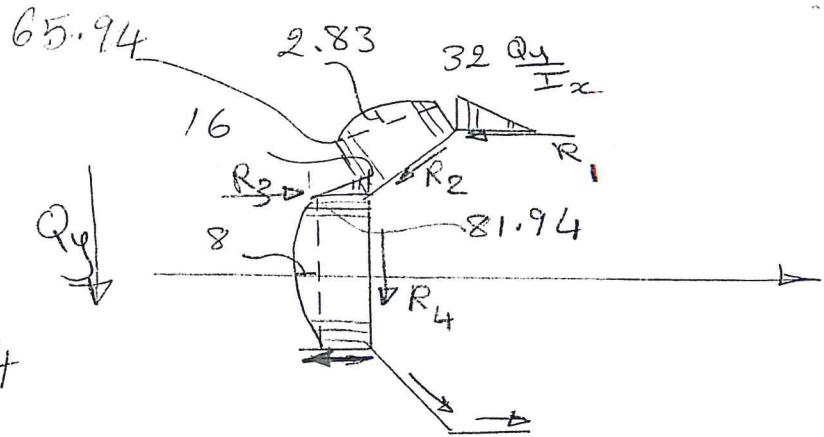
$$S_{xx4} = S_{xx2} + S_{xx3} = 81.94$$

$$S_{xx5} = S_{xx4} + (4 \times 1)(2) = 89.94$$

$$* \Delta_1 = \frac{(1 \times 4\sqrt{2}) \times 4}{8} = 2.83$$

$$\Delta_2 = \frac{(1 \times 8) \times 8}{8} = 8$$

F. Jan 2014



$$* R_1 = \frac{1}{2} \times 4 \times 32 = 64$$

$$R_2 = \left( \frac{32 + 65.94}{2} \right) (4\sqrt{2})$$

$$+ \frac{2}{3} * (4\sqrt{2}) (2.83) = 287.7 \text{ Shear flow Diagram}$$

$$R_3 = \frac{1}{2} \times 4 \times 16 = 32$$

$$R_4 = 81.94 * 8 + \frac{2}{3} * 8 * 8 = 698.19$$

\* Shear Center

$$\sum M_0 = 0$$

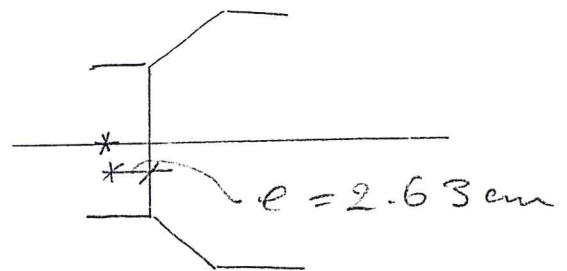
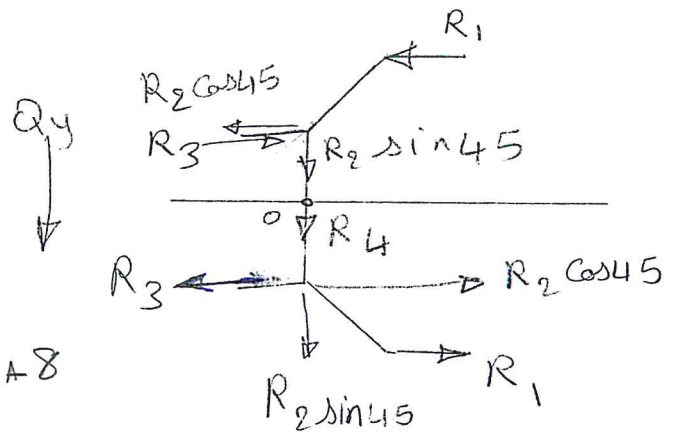
$$Q_y e = R_1 * 16$$

$$+ R_2 \cos 45 * 8 - R_3 * 8$$

$$Q_y e = 2395.4$$

$$L = I_x$$

$$e = 2.47 \text{ cm}$$



F. Jan 2014

Q2

$$D = 0.4 \text{ m}$$

$$\tau_{\text{max}} = \frac{M_t R}{I_p}$$

$$R = 0.2 \text{ m}$$

$$I_p = 2 \left[ \frac{\pi D^4}{64} \right]$$

$$= 2 \left[ \frac{\pi (40)^4}{64} \right]$$

$$= 251327.4 \text{ cm}^4$$

$$\tau_{\text{at sec 1}} = \frac{14 \times 100 \times 20}{I_p}$$

$$= 0.111 \text{ t/cm}^2$$

$$\tau_2 = \frac{9 \times 100 \times 20}{I_p} = 0.072 \text{ t/cm}^2$$

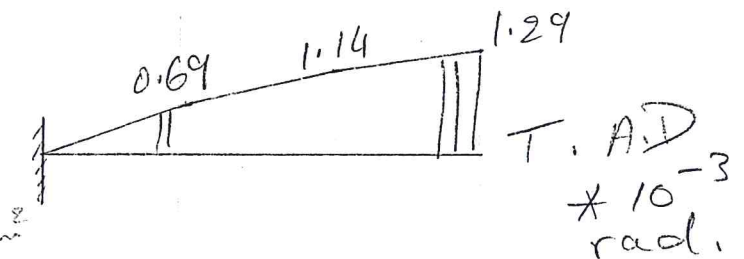
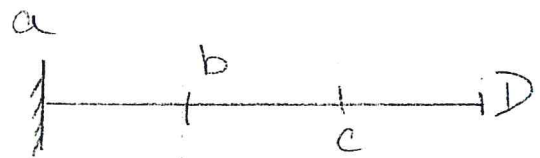
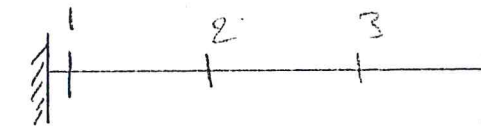
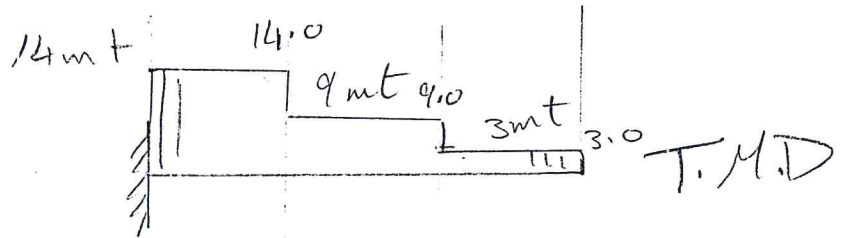
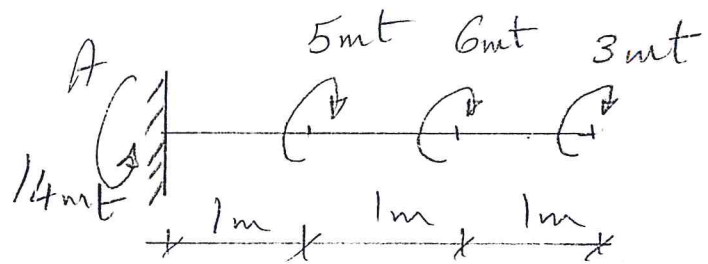
$$\tau_3 = \frac{3 \times 100 \times 20}{I_p} = 0.024 \text{ t/cm}^2$$

$$\theta = \frac{M_t l}{G I_p} = \frac{M_t \times l}{800 \times I_p} \quad , \quad \theta_a = 0$$

$$\theta_b = \theta + \frac{14 \times 100 \times 100}{800 I_p} = 6.96 \times 10^{-4}$$

$$\theta_c = \theta_b + \frac{9 \times 100 \times 100}{800 I_p} = 1.144 \times 10^{-3}$$

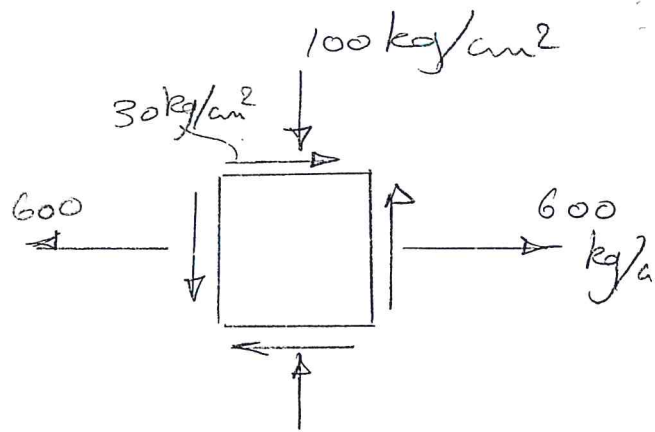
$$\theta_D = \theta_c + \frac{3 \times 100 \times 100}{800 I_p} = 1.293 \times 10^{-3}$$





F. Jan 2014

Q3



$$\therefore \sigma_x = +600 \text{ kg/cm}^2$$

$$\sigma_y = -100$$

$$\tau = -30$$

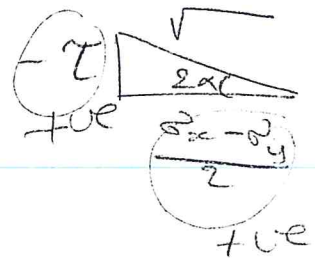
Principal Normal Stresses

$$\begin{aligned} \sigma_{\max/\min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau)^2} \\ &= \frac{600 - 100}{2} \pm \sqrt{\left[\frac{600 - (-100)}{2}\right]^2 + (+30)^2} \\ &= 250 \pm 351.3 \end{aligned}$$

$$\therefore \sigma_{\max} = 601.3$$

$$\sigma_{\min} = -101.3$$

$$\begin{aligned} 2\alpha &= \frac{-\tau}{\frac{\sigma_x - \sigma_y}{2}} = \frac{-(-30)}{\frac{600 - (-100)}{2}} \\ &= \frac{30}{350} = 0.086 \end{aligned}$$



$$\therefore 2\alpha = 4.9^\circ \quad \therefore \alpha = 2.45^\circ$$

$\therefore$  1st quad.

Principal Shear plane

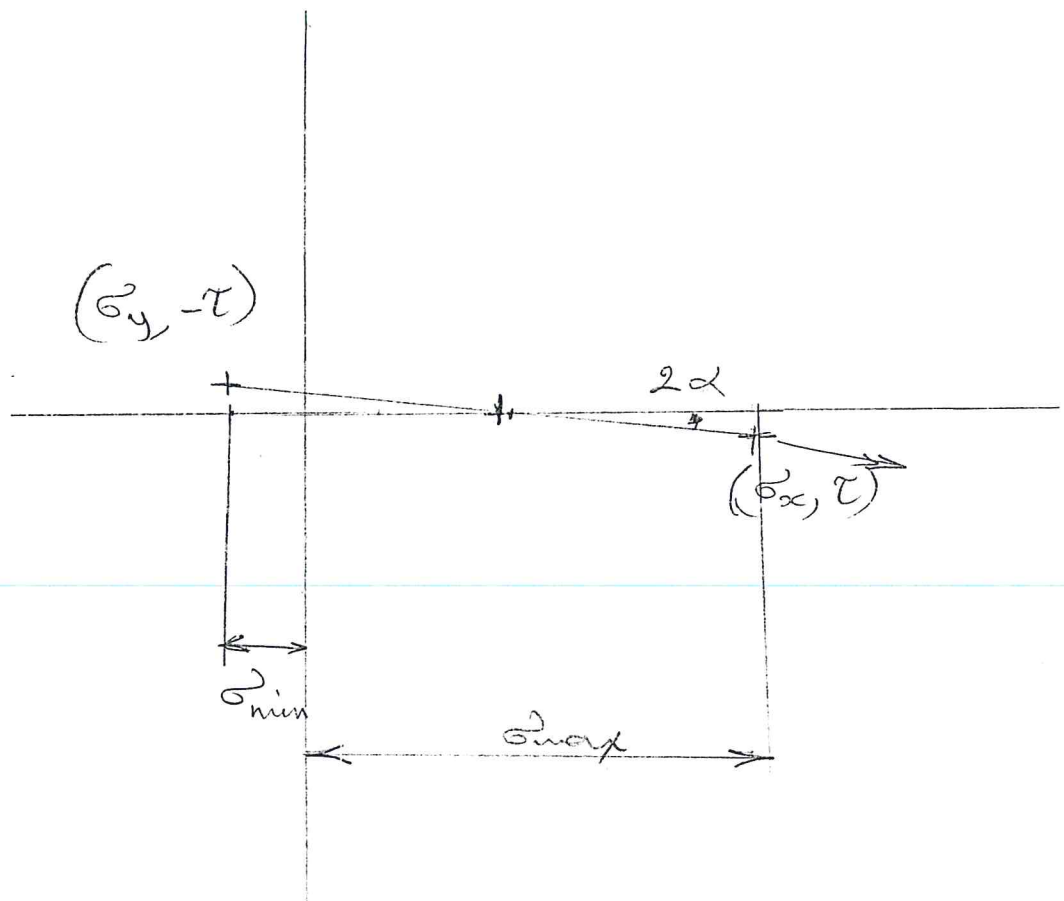
$$\theta = \alpha + 45^\circ = 47.45^\circ$$

F. Jan 2014

$$\begin{aligned}\sigma_{\max/\min} &= \pm \left( \frac{\sigma_{\max} - \sigma_{\min}}{2} \right) \\ &= \pm \left[ \frac{601.3 - (-101.3)}{2} \right] \\ &= \pm 351.3 \text{ kg/cm}^2\end{aligned}$$

---

Mohr's circle for  
principal normal stresses.

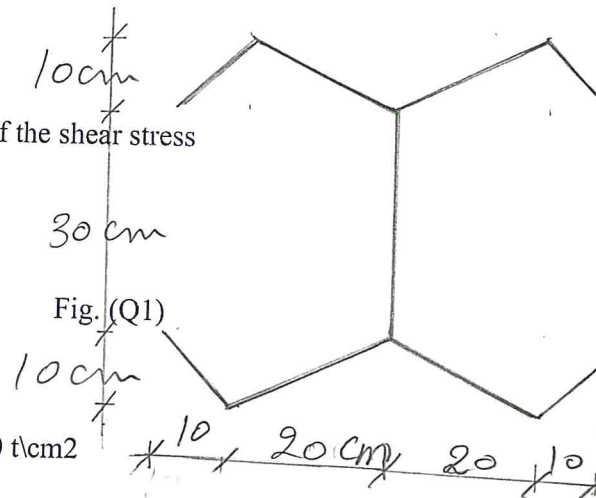


Q1

For the given cross-section:

1. Find  $I_x$ .
2. Draw the distribution of the shear stress due to  $Q_y = 10t$

$t = 1.0 \text{ cm}$



ILO's

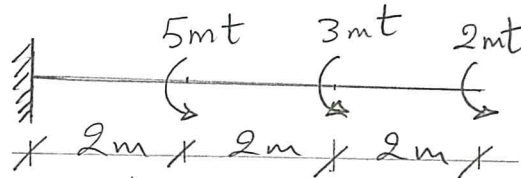
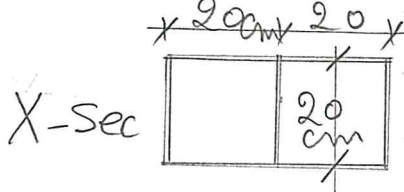
- [d1 1] [2 marks]
- [d1] [4 marks]
- [a3] [4 marks]

[Total 10]

Q2

For the shown structure:  $G = 800 \text{ t/cm}^2$

1. Draw the T.M.D.
2. Draw the T.A.D. for multi-square vent offside length 20cm and  $t = 1 \text{ cm}$ .



$2G\theta A_0/L = \sum (f^*s/t)$

$M_t = M_{t1} + M_{t2} = 2 A_{01} * f_1 + 2 A_{01} * f_1$

Fig. (Q2)

- [a3] [4 marks]
- [a3] [4 marks]

[Total 12]

Q3

For section S-S of the shown column:

1. Find the properties of the cross-section:  $A, I_x, I_y$ .
2. Find the weight of the column:  $\gamma = 2 \text{ t/m}^3$ .
3. Compute the straining actions:  $N, M_x, M_y, Q_x, Q_y, M_t$ .
4. Find the maximum shear stress due to  $Q_x, Q_y$  and  $M_t$ .
5. Find total shear stress at point "n".
6. Find the normal stress at point "n".
7. Draw the stress block.
8. Compute analytically: the principal normal stress plane:
9. the maximum and minimum normal stresses on the principal plane.
10. The principal shear plane.
11. The direction and values of the stresses acting on the principal shear plane.
12. Draw Mohr's circle.

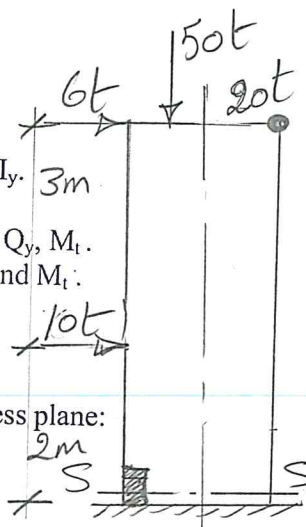
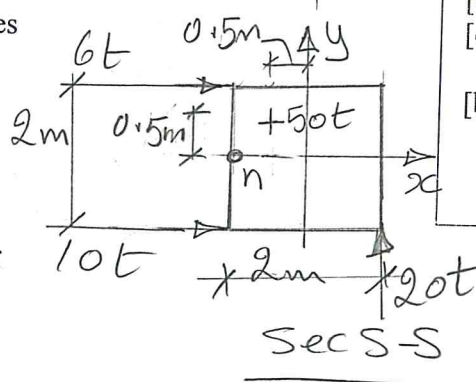


Fig. (Q3)



- [d1] [2 marks]
- [a3,b3] [2 marks]
- [d1] [3 marks]
- [a3,b3] [3 marks]
- [a3,b3] [1 marks]
- [a3,b3] [2 marks]
- [a3,b3] [1 marks]
- [a4] [1 marks]
- [a3,b3] [2 marks]
- [b2,b3] [1 marks]
- [d1,d3] [2 marks]
- [b4,b3] [2 marks]

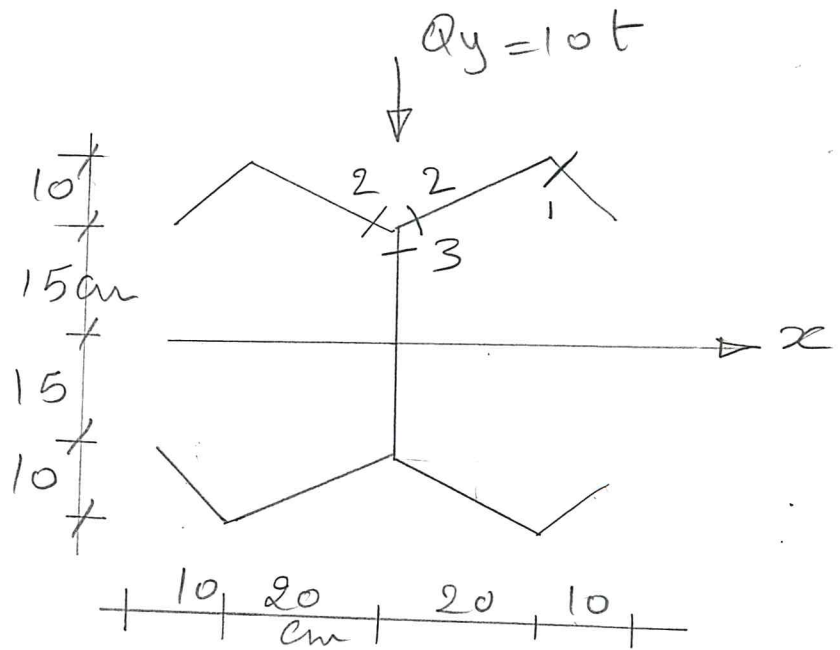
[Total 18]

Mail:

$$\tau_{max} = \frac{M_t}{2ab^2} \text{ t/cm}^2$$

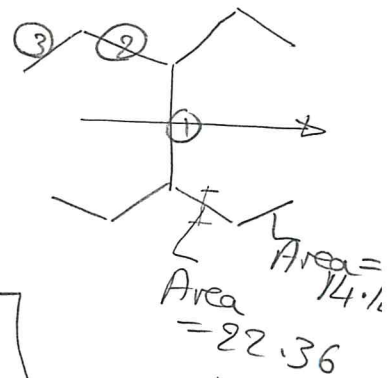
$$\alpha = 0.208$$

Q1



$$I_x = \left[ \frac{(1)(30)^3}{12} \right] + 4 \left[ \frac{22.36(10)^2}{12} + 22.36(5+15)^2 \right] + 4 \left[ \frac{14.14(10)^2}{12} + 14.14(5+15)^2 \right]$$

$$= 2250 + 9130.3 \times 4 + 5773.8 \times 4 = 61866.4$$



$$\tau_y = \frac{Q_y S_x}{I_x b} = S_x \left[ \frac{Q_y}{4 I_x} \right] = 1.6 \times 10^{-4} S_x$$

$$S_1 = (14.14)(20) = 282.8$$

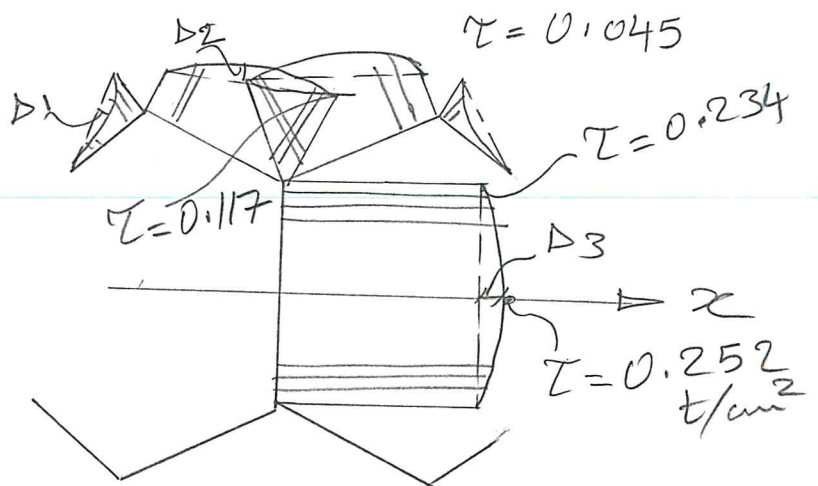
$$S_2 = S_1 + 22.36(20) = 730$$

$$S_3 = 2S_2 = 1460$$

$$\Delta_1 = \frac{14.14 \times 10}{8} = 17.68$$

$$\Delta_2 = \frac{22.36 \times 10}{8} = 28.0$$

$$\Delta_3 = \frac{30 \times 30}{8} = 112.5$$



Sym.





Q3

$$\gamma = 2t/m^3$$

1]  $A = 2 \times 2 = 4 m^2$

$$I_{xc} = I_{yc} = \frac{2(2)^3}{12} = \frac{4}{3} = 1.33 m^4$$

2]  $w = 4 \times 5 \times 2 = 40 t$

3]  $N = -40 - 50 = -90 t$

$$M_{xc} = -50 \times 0.5 - 20 \times 5 = -125 mt$$

$$M_{yc} = -6 \times 5 - 10 \times 2 + 50 \times 0.5 = -25 mt$$

$$Q_x = 6 + 10 = 16 t$$

$$Q_y = +20 t$$

$$M_t = 20 \times 1 + 10 \times 1 - 6 \times 1 = +24 mt$$

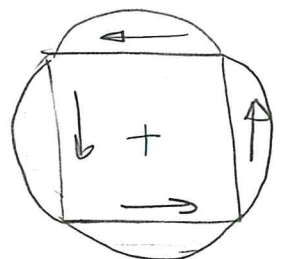
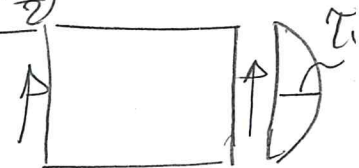
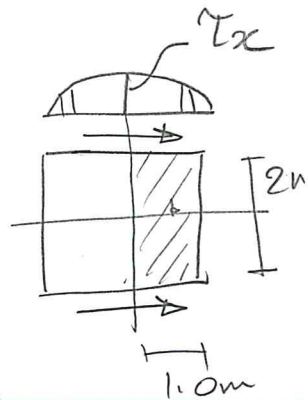
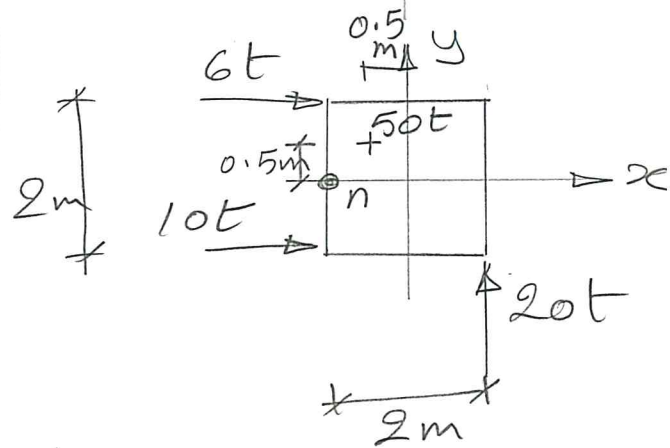
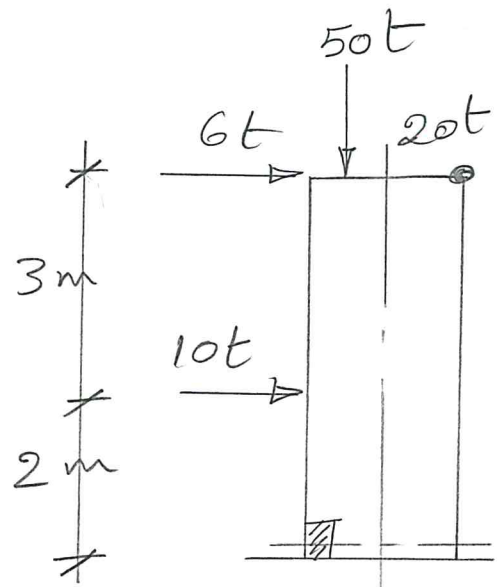
4] Max Shear stress

$$Q_x \Rightarrow \tau_{xc} = \frac{Q_x S_y}{I_{yc} b} = \frac{16 \times (2 \times 1 \times \frac{1}{2})}{1.33 \times 2} = 6.0 t/m^2$$

$$Q_y \Rightarrow \tau_{yc} = \frac{Q_y S_x}{I_{xc} b} = \frac{20 \times (2 \times 1 \times \frac{1}{2})}{1.33 \times 2} = 7.5 t/m^2$$

$$M_t \Rightarrow \tau = \frac{24 \times 100}{0.208 (200)(200)^2} = 7.44 \times 10^3 t/cm^2 = 14.4 t/m^2$$

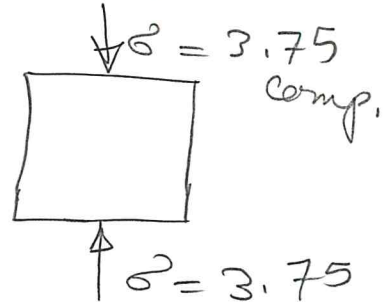
5]  $\tau_{at n} = 7.5 - 14.4 = -6.9 t/m^2$



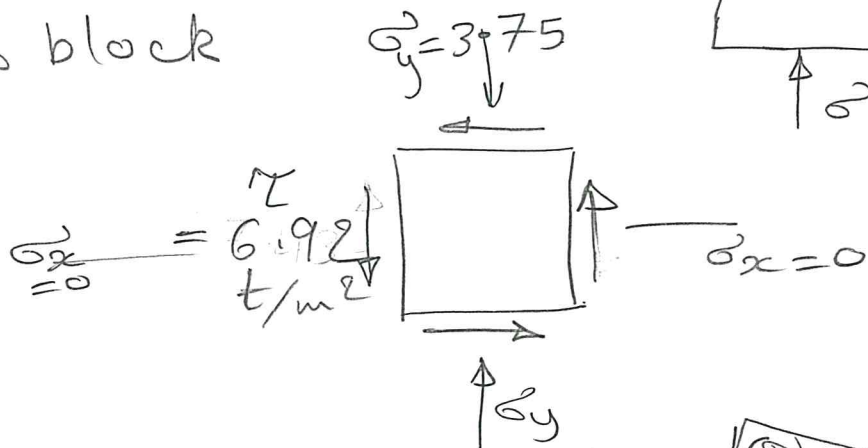
$$\sigma = \frac{N}{A} + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$$

$$\sigma = \frac{-90}{4} - \frac{125}{1.33} * y - \frac{25}{1.33} x$$

$$\begin{aligned} \sigma_n &= -22.5 - 93.75(0) - 18.75(-1) \\ &= -3.75 \text{ t/m}^2 \end{aligned}$$



[7] stress block

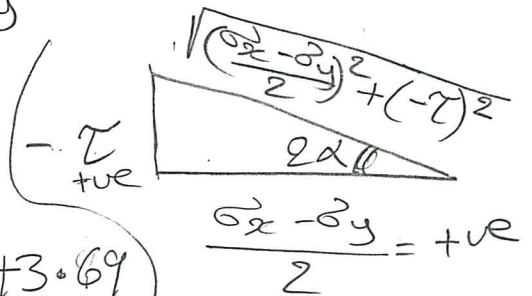


$$\tan 2\alpha = -\frac{\tau}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\therefore \tan 2\alpha = + \frac{6.92}{0 - (-3.75)} = +3.69$$

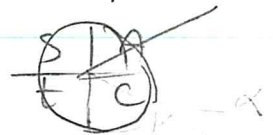
$$2\alpha = 74.84^\circ$$

$$\alpha = 37.42^\circ$$



$$\frac{\sigma_x - \sigma_y}{2} = +ve$$

sin +ve  
cos +ve  
tan +ve



[9]

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (-\tau)^2}$$

$$= \frac{0 - 3.75}{2} \pm \sqrt{\left(\frac{3.75}{2}\right)^2 + (6.92)^2}$$

$$= -1.875 \pm 7.17 = \boxed{+5.29, -9.04 \text{ t/m}^2}$$

$$\tau = 0$$

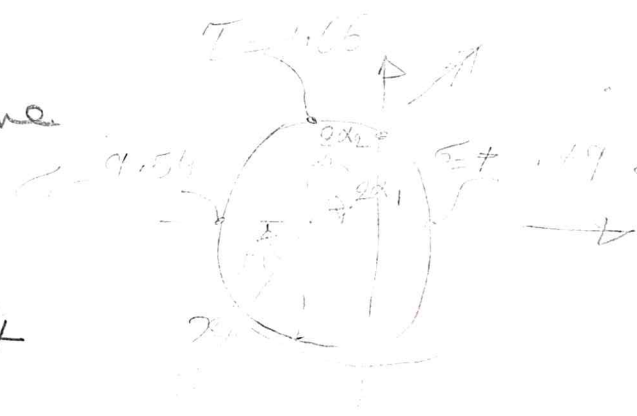
10] Principal shear plane

$$2\alpha_2 = 90 - 2\alpha_1$$

$$= 90 - 74.84$$

$$\therefore \alpha_2 = 7.58^\circ$$

OR  $\alpha_2 = 45 - \frac{74.84}{2} = 7.58^\circ$



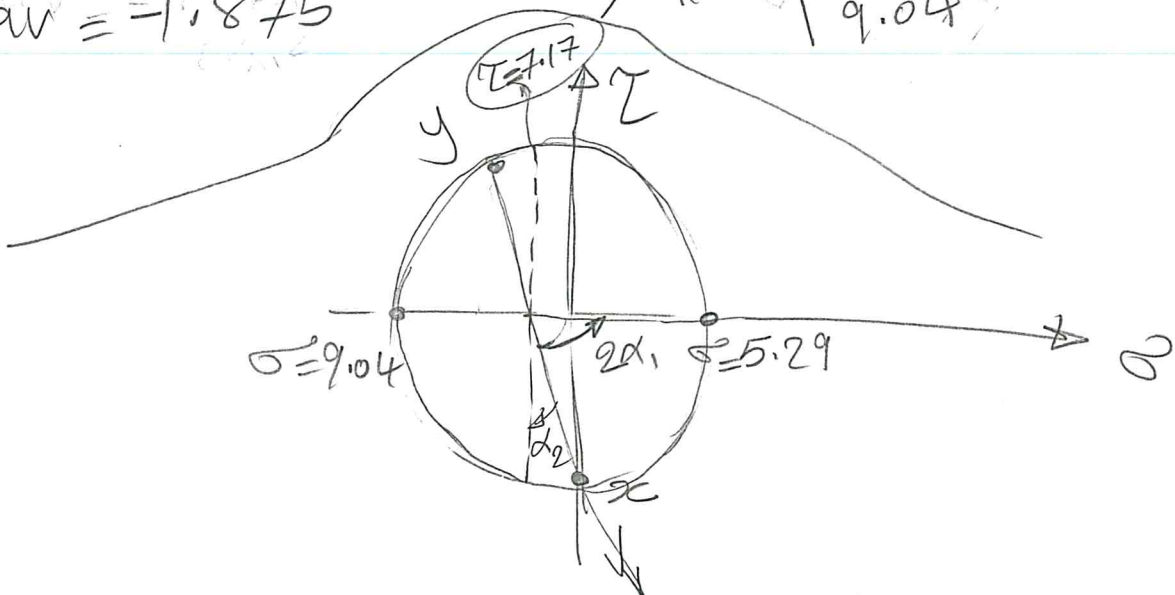
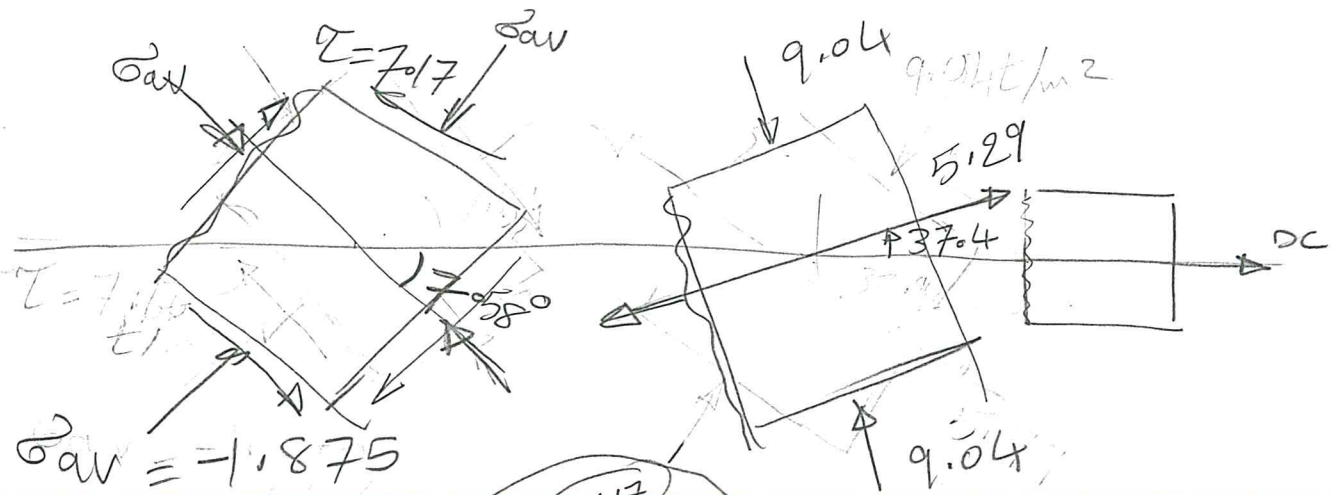
11]

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau)^2} = 7.17 \text{ t/m}^2$$

OR  $\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.29 - (-9.04)}{2}$

$$= 7.17 \text{ t/m}^2$$

$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 3.75}{2} = -1.875 \text{ t/m}^2$$



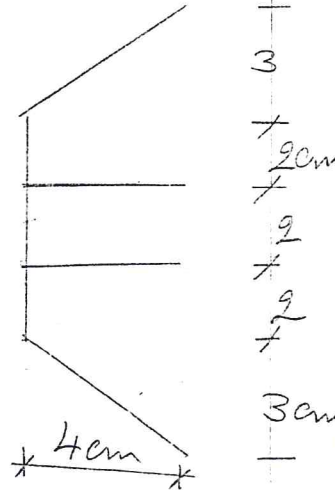
Q1

For the given cross-section:

1. Find  $I_x$ .
2. Draw the distribution of the shear flow due to  $Q_y$ .
3. Find the resultants of the drawn flow.
4. Find the shear centre.

$t = 1\text{ cm}$

Fig. (Q1)



ILO's

[d11]

[4 marks]

[d1]

[6 marks]

[a3]

[3 marks]

[a3]

[2 marks]

[Total 15]

Q2

For the shown structure:  $G = 800\text{ t/cm}^2$

1. Draw the T.M.D.
2. Draw the T.A.D.
3. Find the maximum torsional stress.

$r_{out} = 7\text{ cm}$   
 $r_{in} = 3\text{ cm}$

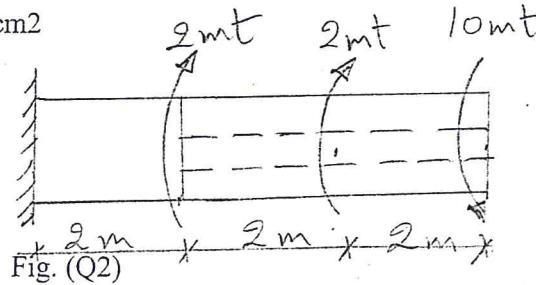


Fig. (Q2)

[a3]

[2 marks]

[a3]

[4 marks]

[d1]

[4 marks]

[Total 10]

Q3

For the shown stress block:

1. Compute analytically: the principal normal stress plane.
2. Compute analytically: the maximum and minimum normal stresses acting on the principal plane.
3. Draw Mohr's circle and check items (1 and 2) graphically.

From Mohr's circle, determine:

1. The principal shear plane.
2. The direction and values of the stresses acting on the principal shear plane.
3. The state of stresses of a plane that makes a clockwise angle  $= 60^\circ$  with the horizontal.

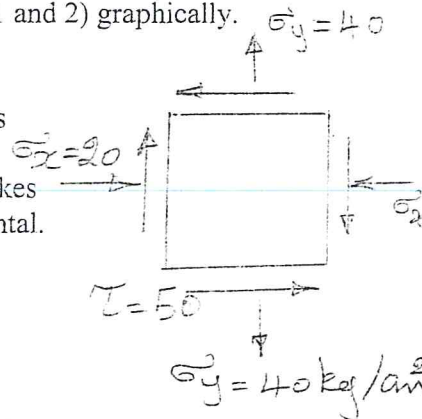


Fig. (Q3)

[d1]

[3 marks]

[a3,b3]

[3 marks]

[a3,b3]

[2 marks]

[a3,b3]

[2 marks]

[a3,a4]

[3 marks]

[b2,b3,

b4,d3]

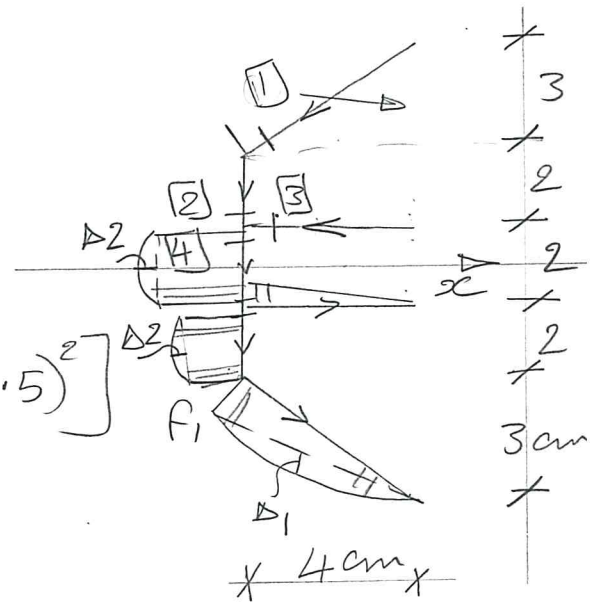
[2 marks]

[Total 15]



Q1

$$\begin{aligned} \textcircled{1} I_x &= \frac{1(6)^3}{12} + 2[4 \times 1(1)^2] \\ &+ 2\left[\frac{(5 \times 1)(3)^2}{12} + (5 \times 1)(4.5)^2\right] \\ &= 236 \end{aligned}$$



② Shear flow Dist.

$$f_y = \frac{Q_y S_x}{I_x} \quad \text{let } Q_y = I_x$$

$$\therefore \boxed{f_y = S_x}$$

$$S_{\text{I}} = 5 \times 1 \times 4.5 = 22.5$$

$$\Delta_1 = \frac{1 \times 5 \times (3)}{8} = 1.88$$

$$S_{\text{II}} = S_1 + (2 \times 1) \times 2 = 26.5$$

$$\Delta_2 = \frac{1 \times 2 \times (2)}{8} = 0.5$$

$$S_{\text{III}} = (4 \times 1) \times 1 = 4$$

$$S_{\text{IV}} = S_2 + S_3 = 30.5$$

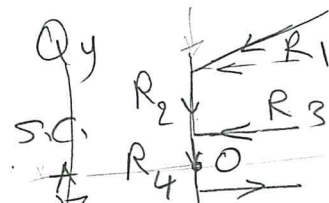
③ Resultants

$$R_1 = \frac{1}{2} \times 5 \times 22.5 + \frac{2}{3} \times 5 \times 1.88 = 62.52 \text{ e}$$

$$R_2 = \frac{1}{2} [22.5 + 26.5] \times 2 + \frac{2}{3} \times 2 \times 0.5 = 49.67$$

$$R_3 = \frac{1}{2} \times 4 \times 4 = 8$$

$$R_4 = \frac{26.5}{30} \times 2 + \frac{2}{3} \times 2 \times 0.5 = 53.67 \text{ } 60.67$$



④ S.C.  $\Sigma M @ 0 = 0 \Rightarrow 2[R_3 \times 1 + R_1 \cos \theta \times 3] = e Q$

$$2 \times [8 \times 1 + 62.52 \times 0.8 \times 3] = e \times 236$$

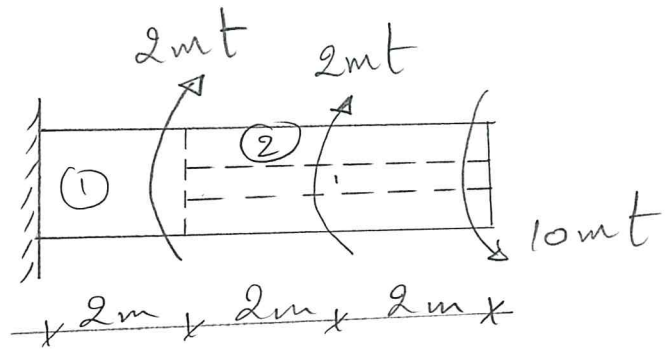
$$\therefore \boxed{e = 0.67 \text{ cm}}$$



Q2

$$r_{out} = 7 \text{ cm}$$

$$r_{in} = 3 \text{ cm}$$



$$\theta = \frac{M_t L}{I_p G}$$

$$I_{p1} = \frac{\pi (7)^4}{2} = 3771.5$$

$$I_{p2} = \frac{\pi ((7)^4 - (3)^4)}{2}$$

$$= 3644.2$$

$$\theta_A = 0$$

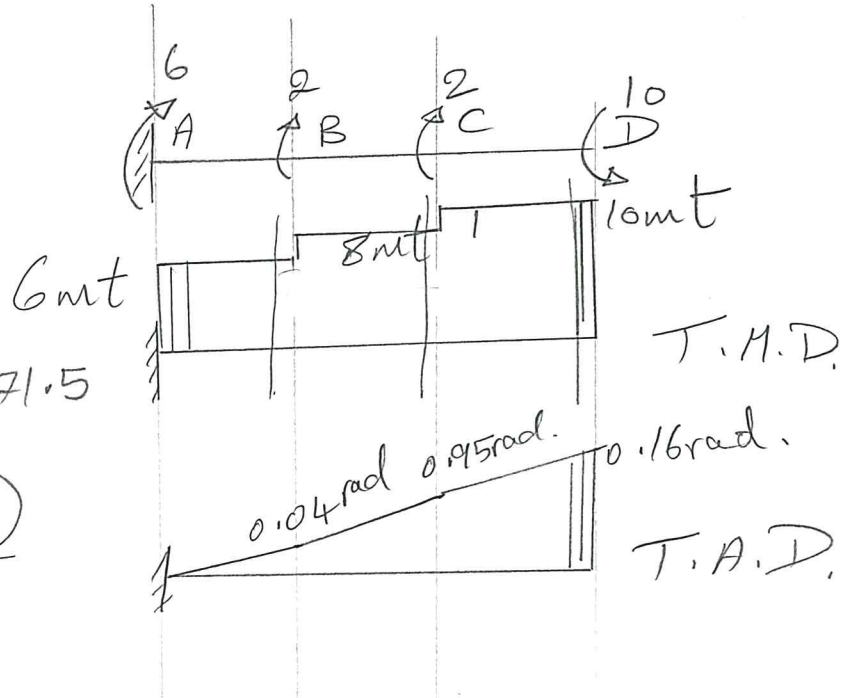
$$\theta_B = 0 + \frac{600 \times 200}{I_{p1} \times 800} = 0.04 \text{ rad.}$$

$$\theta_C = \theta_B + \frac{800 \times 200}{I_{p2} \times 800} = 0.095 \text{ rad}$$

$$\theta_D = \theta_C + \frac{1000 \times 200}{I_{p2} \times 800} = 0.16 \text{ rad.}$$

$$\tau_{max} = \frac{M_t r}{I_p}$$

$$\therefore \tau_D = \frac{1000 \times 7}{3644.2} = 1.92 \text{ t/cm}^2$$

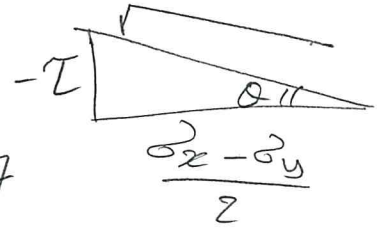
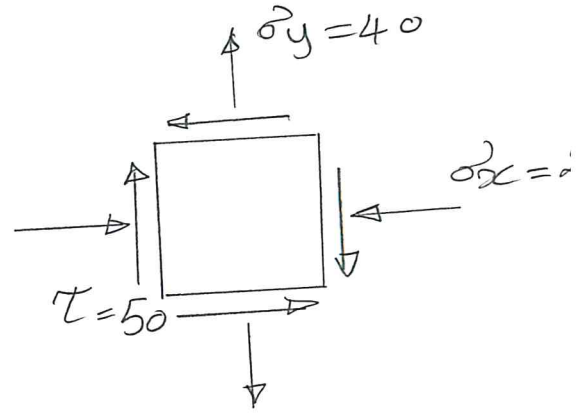


Q3

$$\sigma_x = -20$$

$$\sigma_y = +40$$

$$\tau = +50$$



cos -ve  
sin -ve  
tan +ve

①

$$\tan 2\theta = \frac{-\tau}{\frac{\sigma_x - \sigma_y}{2}}$$

$$= \frac{-50}{\frac{-20 - 40}{2}} = +1.67$$

$$2\theta = 180 + 59^\circ = 239.1^\circ$$

$$\therefore \theta = 119.5^\circ$$

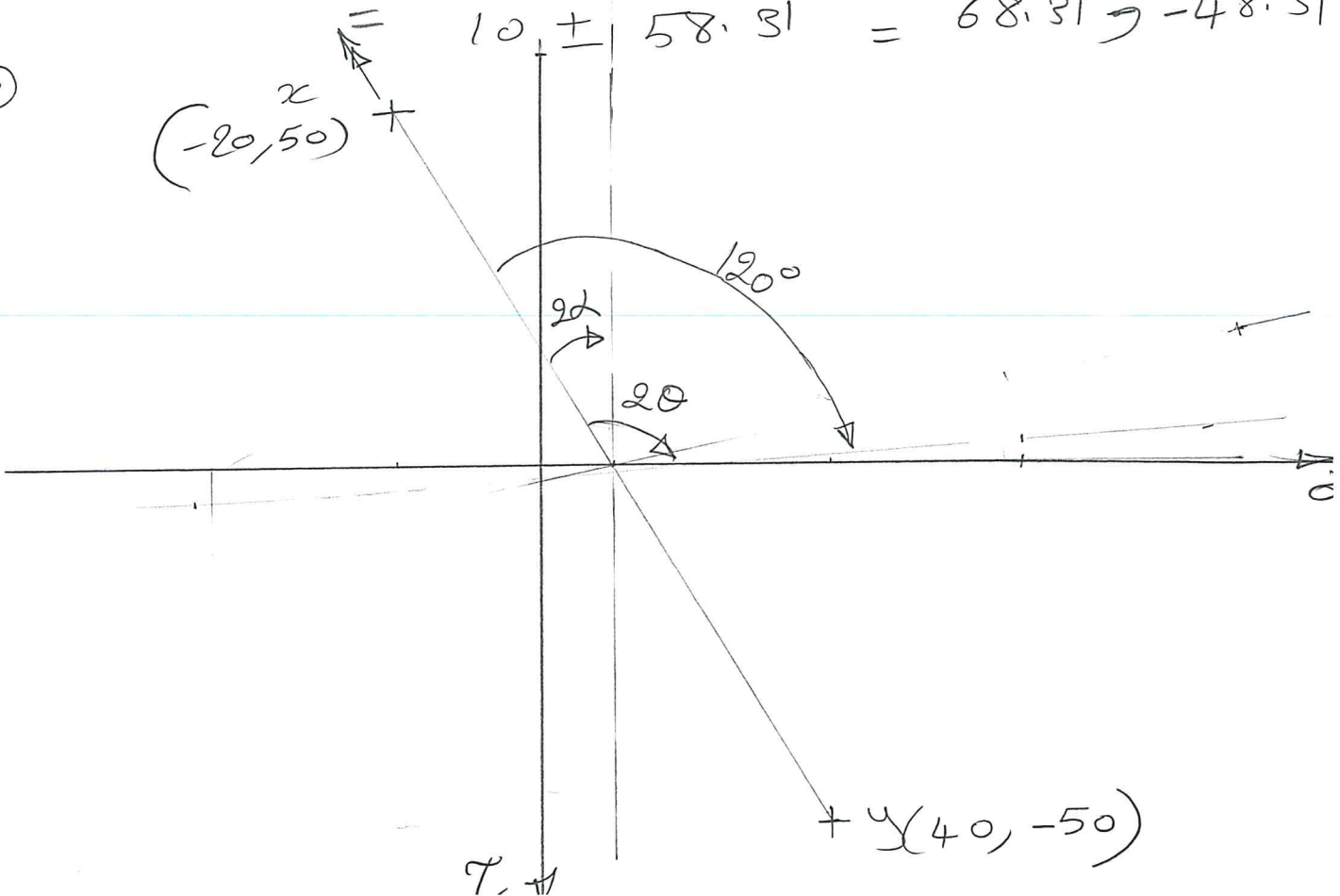
②

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (-\tau)^2}$$

$$= \frac{-20 + 40}{2} \pm \sqrt{\left(\frac{-20 - 40}{2}\right)^2 + (-50)^2}$$

$$= 10 \pm 58.31 = 68.31, -48.31$$

③



1- Principal Shear Plane

$$2\alpha_1 = 29^\circ$$

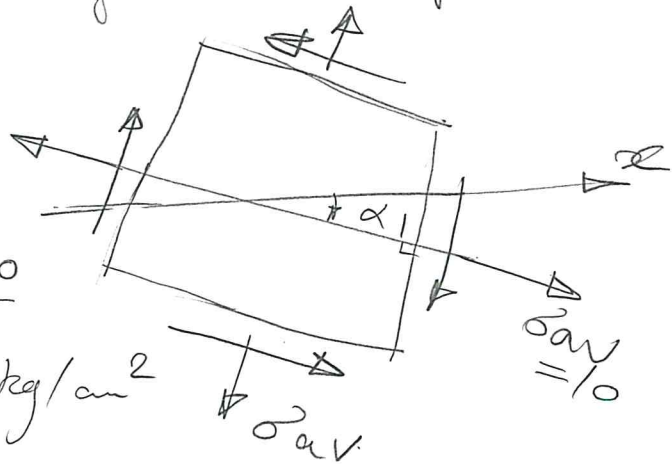
$$\therefore \alpha_1 = 14.5^\circ$$

check  $2\theta - 2\alpha = 90$

$$119 - 29 = 90$$

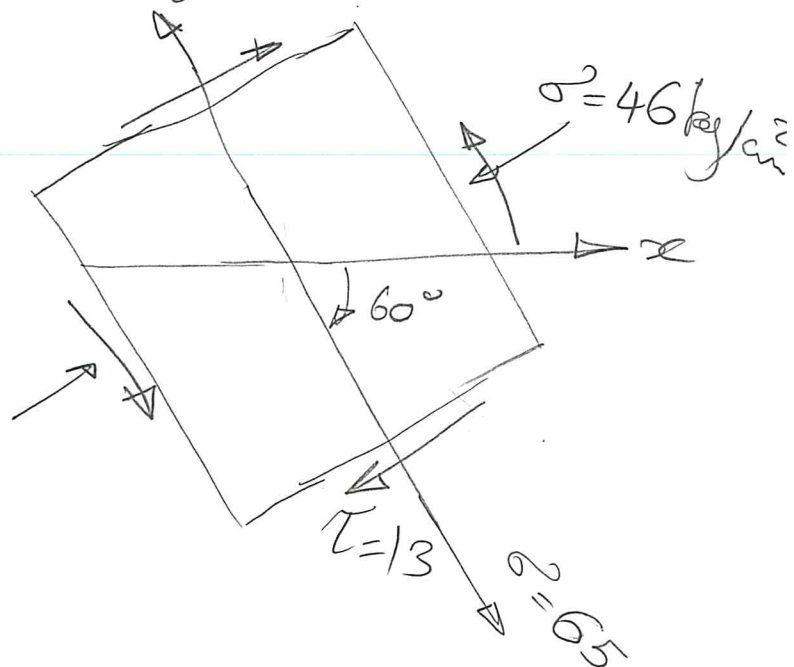
2 - stresses on Principal Shear plane

check  $\sigma_{av} = \frac{-20 + 40}{2}$   
 $= +10 \text{ kg/cm}^2$



check  $T_{\max \min} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (-\tau)^2}$   
 $= 58 \text{ kg/cm}^2$

3 -

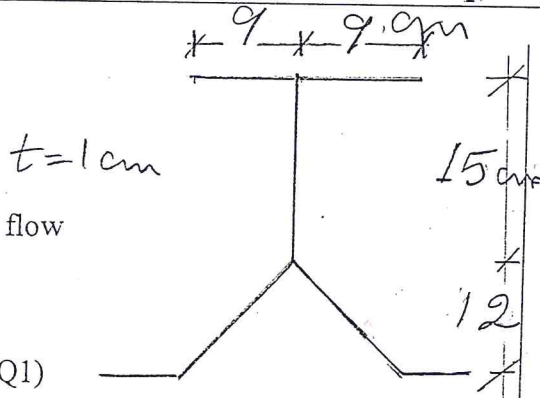


Q1

For the given cross-section:

1. Find the centroid.
2. Find  $I_x$ .
3. Draw the distribution of the shear flow due to  $Q_y=10t$

Fig. (Q1)



ILO's

[d11]

[2 marks]

[d1]

[4 marks]

[a3]

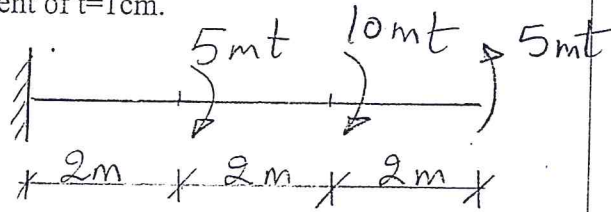
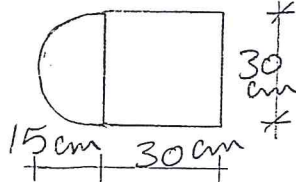
[4 marks]

[Total 10]

Q2

For the shown structure:  $G=800 \text{ t/cm}^2$

1. Draw the T.M.D.
2. Draw the T.A.D. for multi vent of  $t=1 \text{ cm}$ .



$$2G\theta A_0/L = \sum (f^*s/t)$$

$$M_t = M_{t1} + M_{t2} = 2 A_{01} * f_1 + 2 A_{02} * f_2$$

Fig. (Q2)

[a3]

[4 marks]

[a3]

[4 marks]

[d1]

[4 marks]

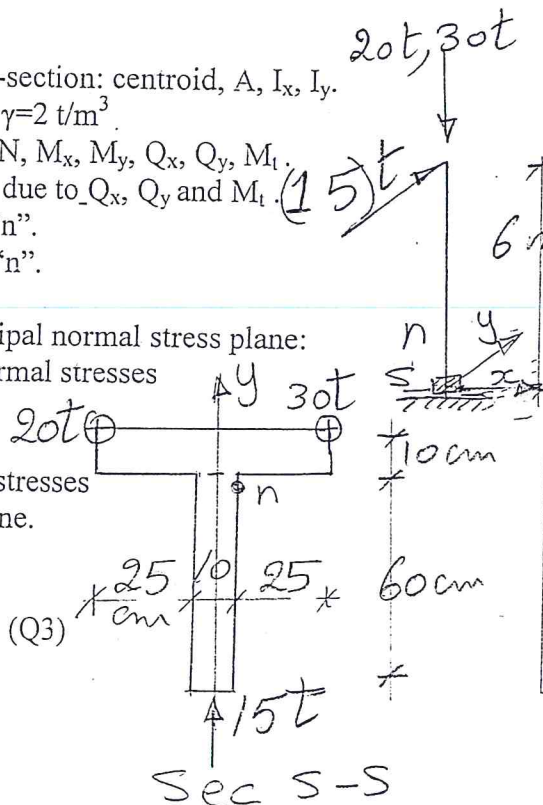
[Total 12]

Q3

For section S-S of the shown column:

1. Find the properties of the cross-section: centroid,  $A$ ,  $I_x$ ,  $I_y$ .
2. Find the weight of the column:  $\gamma=2 \text{ t/m}^3$ .
3. Compute the straining actions:  $N$ ,  $M_x$ ,  $M_y$ ,  $Q_x$ ,  $Q_y$ ,  $M_t$ .
4. Find the maximum shear stress due to  $Q_x$ ,  $Q_y$  and  $M_t$ .
5. Find total shear stress at point "n".
6. Find the normal stress at point "n".
7. Draw the stress block.
8. Compute analytically: the principal normal stress plane:
9. the maximum and minimum normal stresses on the principal plane.
10. The principal shear plane.
11. The direction and values of the stresses acting on the principal shear plane.
12. Draw Mohr's circle.

Fig. (Q3)



[d1]

[2 marks]

[a3,b3]

[2 marks]

[d1]

[3 marks]

[a3,b3]

[3 marks]

[a3,b3]

[1 marks]

[a3,b3]

[2 marks]

[a3,b3]

[1 marks]

[a4]

[1 marks]

[a3,b3]

[2 marks]

[b2,b3]

[1 marks]

[d1,d3]

[2 marks]

[b4,b3]

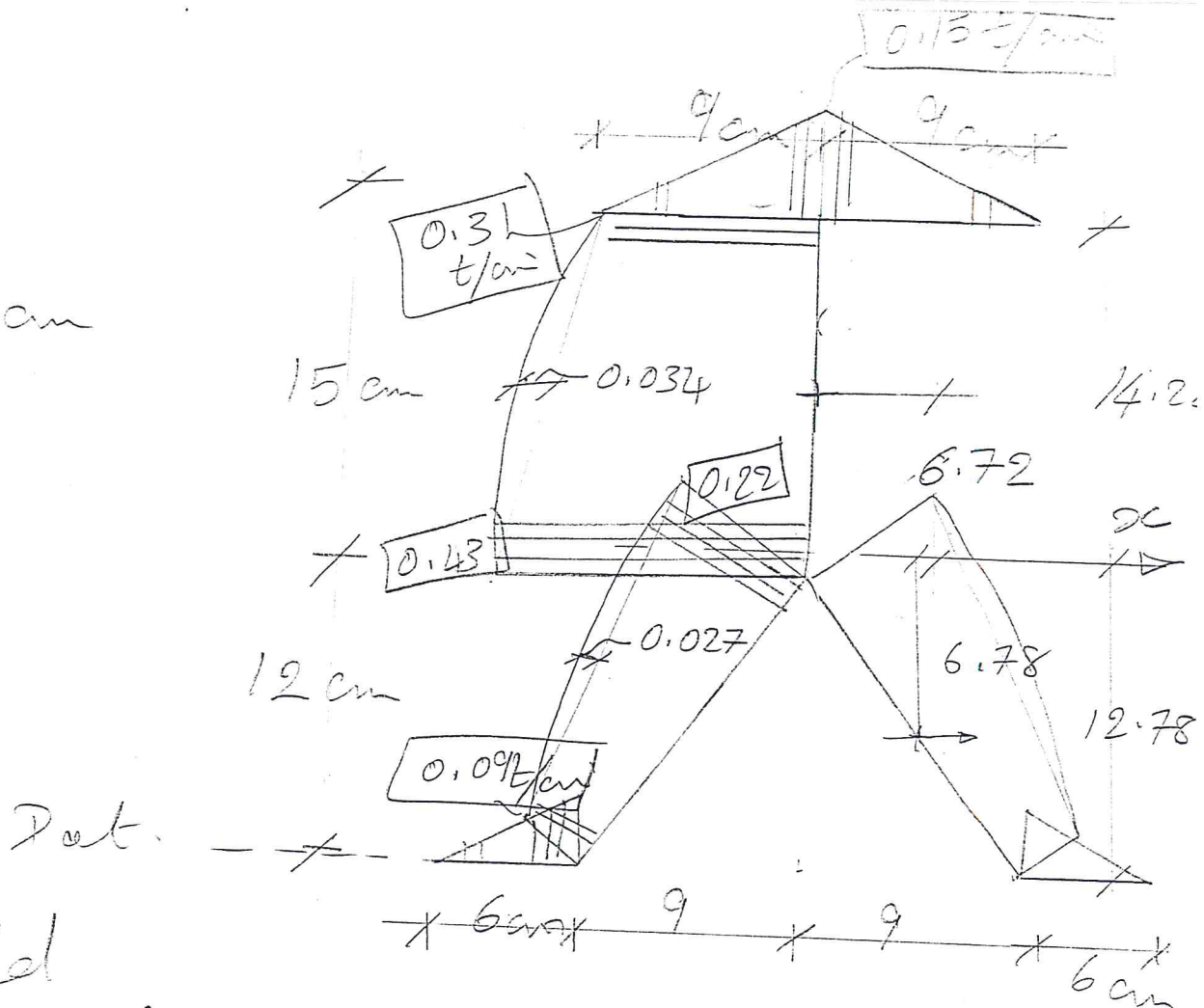
[2 marks]

[Total 18]

Mail:



$$t = 1 \text{ cm}$$



1) Centroid

$$\text{Area} = 1(15) + 1 * 2(9 + 6 + 15) = 75 \text{ cm}^2$$

$$\bar{y} = \frac{2[15 * 1 * 6] + [15 * 1 * 19.5] + 2[9 * 1 * 27]}{75}$$

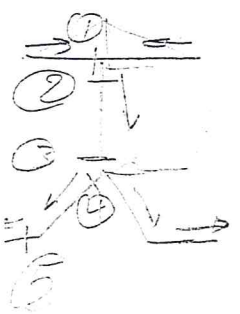
$$\bar{y} = 12.78 \text{ cm}$$

$$2) I_{xc} = 18 * 1 (14.2)^2 + \left[ \frac{1(15)^3}{12} + 15(6.72)^2 \right] + 2 \left[ \frac{6 * 1 (12.78)^2}{12} + 2 \left[ \frac{15 * 1 (12)^2}{12} + 15(6.78)^2 \right] \right]$$

$$= 3639.75 + 958.63 + 1959.94 + 1739.05$$

$$= 8297.4 \text{ cm}^4$$

$$3) f = \frac{Q_y S_{xc}}{I_{xc}} = \frac{10}{8297.4} * S_{xc} = 1.2 * 10^{-3} S_{xc}$$





$$F_1 = 1.2 \times 10^{-3} \times 9 \times 1 (4.22) = 1.2 \times 10^{-3} \times 1279$$

$$= 0.15 \text{ t/cm}$$

$$F_2 = 2 F_1 = 0.31 \text{ t/cm}$$

$$F_3 = F_2 + 1.2 \times 10^{-3} \times (15 \times 1 \times 6.72)$$

$$= F_2 + 1.2 \times 10^{-3} \times 100.8 = 0.43$$

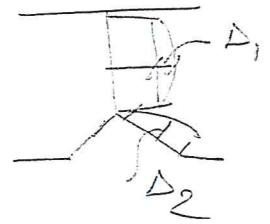
$$F_4 = \frac{F_3}{2} = 0.22 \text{ t/cm}$$

$$F_5 = (1 \times 6 \times 12.78) \times 1.2 \times 10^{-3} = 1.2 \times 10^{-3} \times 76.68$$

$$= 0.09 \text{ t/cm}$$

$$\Delta_1 = \frac{15 \times 1 \times 15}{8} \times 1.2 \times 10^{-3}$$

$$= 28.13 \times 1.2 \times 10^{-3} = 0.034$$



$$\Delta_2 = \frac{15 \times 1 \times 12}{8} \times 1.2 \times 10^{-3}$$

$$= 22.5 \times 1.2 \times 10^{-3} = 0.027$$



$$A_{o1} = 30 \times 30 = 900 \text{ cm}^2$$

$$A_{o2} = \frac{\pi (15)^2}{2} = 353.4 \text{ cm}^2$$

Part AB  $M_t = 10 \text{ mt}$

$$M_t = 2A_{o1} f_1 + 2A_{o2} f_2$$

$$= 2(900) f_1 + 2(353.4) f_2$$

$$10 \times 100 = 1800 f_1 + 706.8 f_2 \rightarrow \textcircled{1}$$

Vent 1

$$\frac{2G\theta A_0}{L} = \sum f + \frac{S}{t}$$

$$\frac{2G\theta}{L} \times 900 = 4 \left( \frac{30}{1} \right) f_1 - \frac{30}{1} f_2 \rightarrow \textcircled{2}$$

Vent 2

$$\frac{2G\theta \times 353.4}{L} = \frac{30}{1} f_2 - \frac{30}{1} f_1 + \pi (15) f_2 \rightarrow \textcircled{3}$$

Divide ② by ③

$$\frac{900}{353.4} = \frac{120 f_1 - 30 f_2}{-30 f_1 + 77.1 f_2}$$

$$\div -27000 f_1 + 6941.5 f_2 = 42408 f_1 - 10602 f_2$$

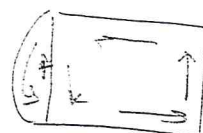
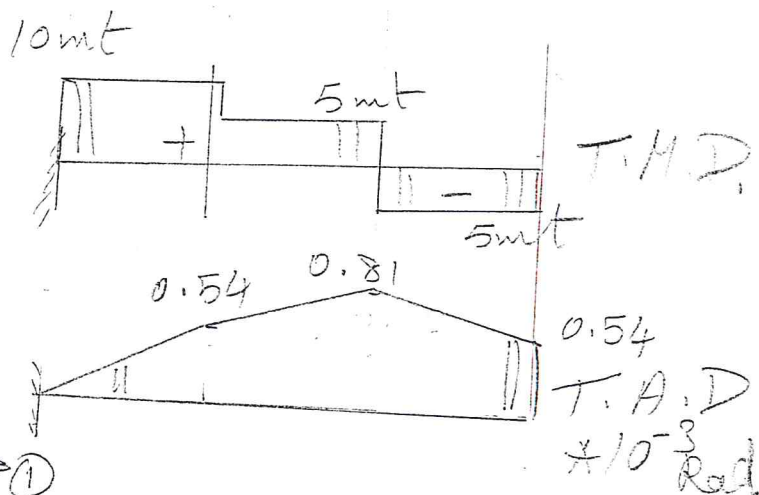
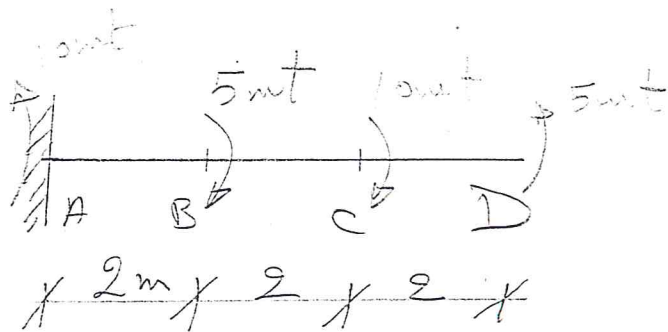
$$\div 80013.5 f_2 = 69408 f_1 \Rightarrow f_1 = 1.153 f_2$$

Subs in ①

$$10 \times 100 = 1800 \times 1.153 f_2 + 706.8 f_2$$

$$\Rightarrow f_2 = 0.36 \text{ t/cm}$$

$$f_1 = 0.41 \text{ t/cm}$$



Subs in (2)

$$-\frac{2 \times 800 \theta_A}{200} \times 900 = \frac{4(30)}{1} (0.41) - \frac{30}{1} (0.35)$$

$$\theta_1 = 5.41 \times 10^{-3} \text{ rad}$$

Part BC

$$\theta_2 = \frac{\theta_1}{2} = 2.7 \times 10^{-3}$$

Part CD

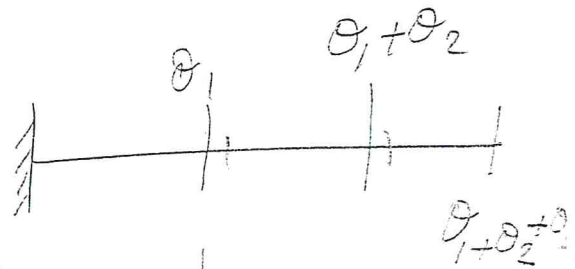
$$\theta_3 = -\theta_2 = -2.7 \times 10^{-3}$$

$$\theta_A = 0$$

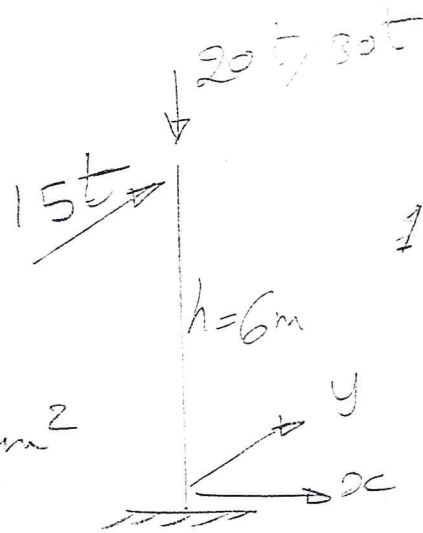
$$\theta_B = 0 + \theta_{AB} = 5.41 \times 10^{-3} \text{ rad}$$

$$\theta_C = \theta_B + \theta_{BC} = 5.41 \times 10^{-3} + 2.7 \times 10^{-3} = 8.1 \times 10^{-3}$$

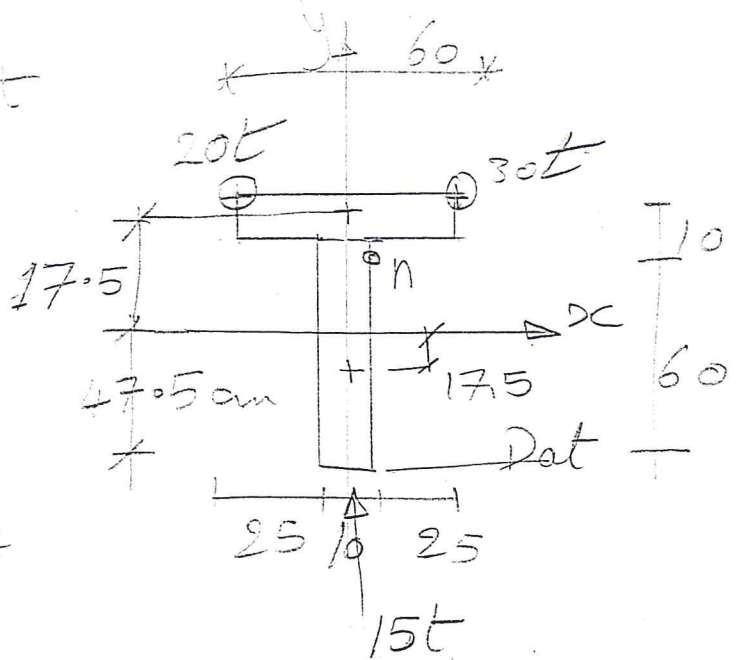
$$\theta_D = \theta_C + \theta_{CD} = 8.1 \times 10^{-3} - 2.7 \times 10^{-3} = 5.4 \times 10^{-3}$$



Q3



$$A = 60 \times 10 \times 2 = 1200 \text{ cm}^2$$



1) Centroid

$$\bar{y} = \frac{10 \times 60 \times 30 + 60 \times 10 \times 65}{1200} = 47.5$$

$$I_x = \left[ \frac{10(60)^3}{12} + 600(17.5)^2 \right] + \left[ \frac{60(10)^3}{12} + 600(17.5)^2 \right]$$

$$= 552500 \text{ cm}^4$$

$$I_y = \frac{60(10)^3}{12} + \frac{(10)(60)^3}{12} = 185000$$

$$2) \omega = \frac{1200}{(100)^2} \times 6 \times 2 = 1.44 \text{ t}$$

$$3) N = -(30 + 20 + 1.44) = -51.44 \text{ t}$$

$$M_x = -20 \times \frac{22.5}{100} - 30 \times \frac{22.5}{100} - 15 \times 6$$

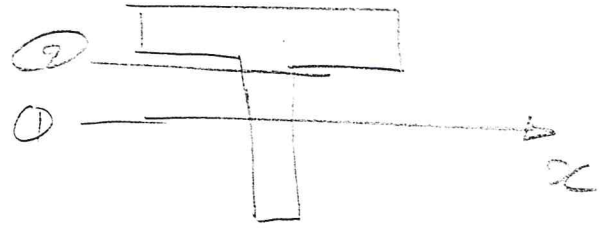
$$= -101.25 \text{ mt}$$

$$M_y = -30 \times 0.3 + 20 \times 0.3 = -3.0 \text{ t}$$

$$Q_x = 0, \quad Q_y = +15, \quad M_t = 0$$

$$4) Q_y = +15t$$

$$\tau_y = \frac{Q_y S_x}{I_x b}$$



$$\tau_1 = \frac{Q}{I_x} * \frac{1}{10} * 47.5 * 10 * \frac{47.5}{2} = \boxed{2256.3 \frac{Q}{I_x}}$$

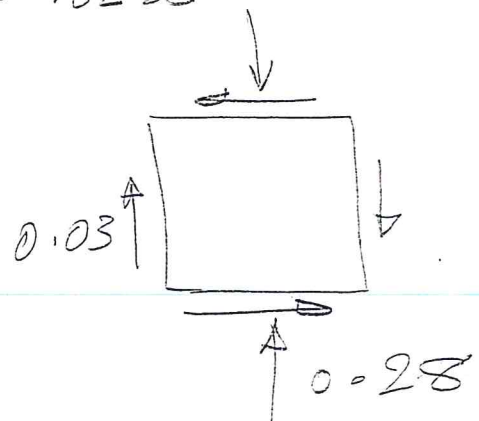
$$\tau_2 = \frac{Q}{I_x} * \frac{1}{10} * 60 * 10 * 17.5 = 1050 \frac{Q}{I_x} = 0.03 \frac{t}{cm^2}$$

$$5) \tau_n = \frac{1050 * 15}{552500} = 0.03 \text{ t/cm}^2$$

$$6) \sigma = \frac{N}{A} + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$$

$$\begin{aligned} \sigma_n &= \frac{-51.44}{1200} - \frac{101.25 * 100}{552500} y - \frac{3 * 100}{185000} x \\ (5, 12.5) & \\ &= -0.0424 - 0.0183y - 0.00162x \\ &= -0.28 \end{aligned}$$

7)



$$8) \tan 2\alpha = \frac{-\tau}{\frac{\sigma_x - \sigma_y}{2}} = \frac{-0.03}{\frac{0.28}{2}} \left| \begin{array}{l} \sigma_x = 0 \\ \sigma_y = -0.28 \\ \tau = +0.03 \end{array} \right.$$

$$= -0.214$$

$$\Rightarrow 2\alpha = -12.09^\circ$$

$$\Rightarrow \alpha = -6.04^\circ$$

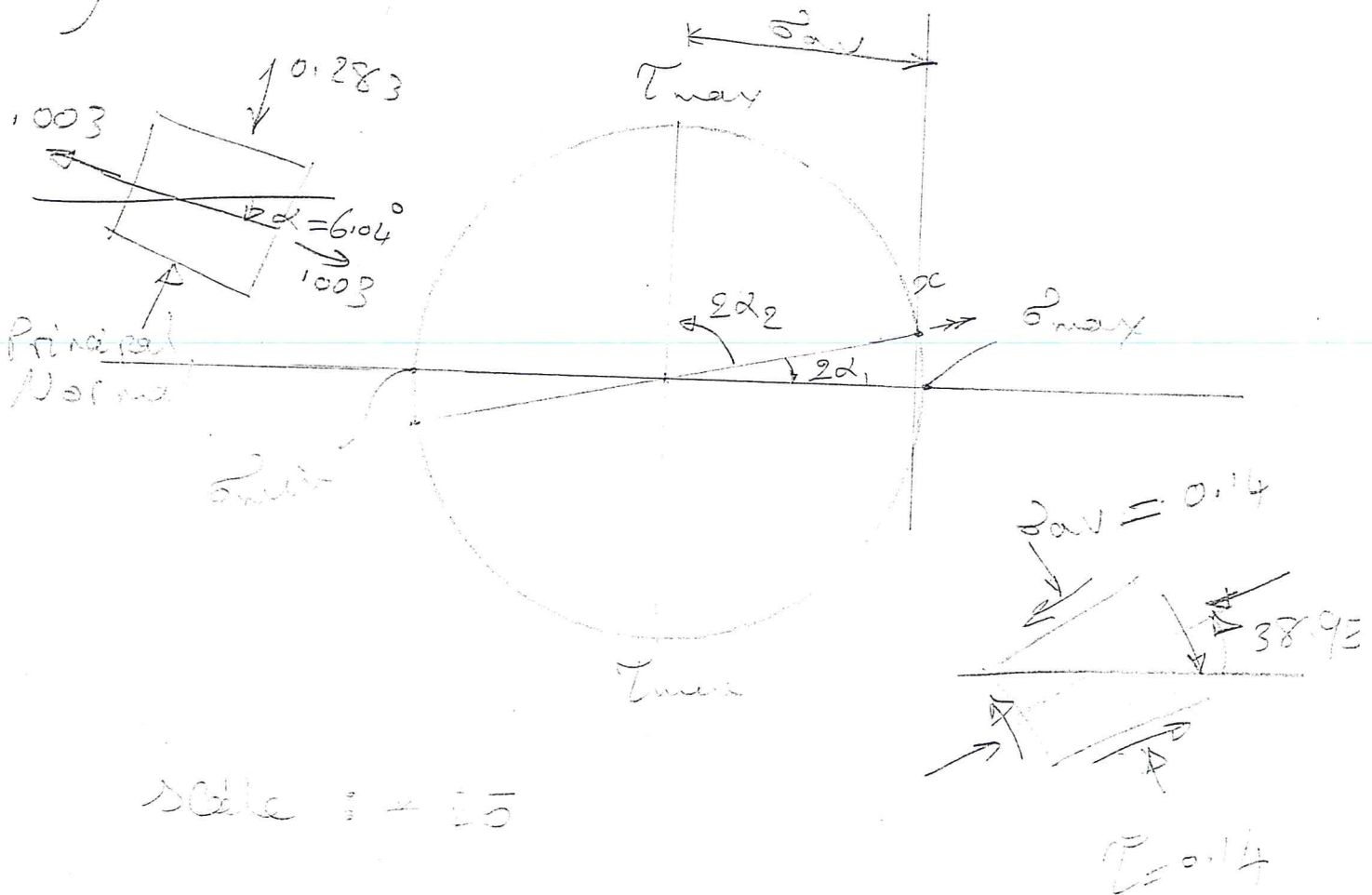


$$\begin{aligned}
 \sigma_{\max/\min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau)^2} \\
 &= \frac{0 - 0.28}{2} \pm \sqrt{\left(\frac{0 + 0.28}{2}\right)^2 + (-0.03)^2} \\
 &= -0.14 \pm 0.143 \pm 0.003, -0.283 \text{ t/cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \tan 2\alpha_2 &= \frac{\sigma_x - \sigma_y}{\tau} = \frac{0.28}{-0.03} = -4.67 \\
 \therefore 2\alpha_2 &= 77.9 \quad \therefore \alpha_2 = 38.95^\circ
 \end{aligned}$$

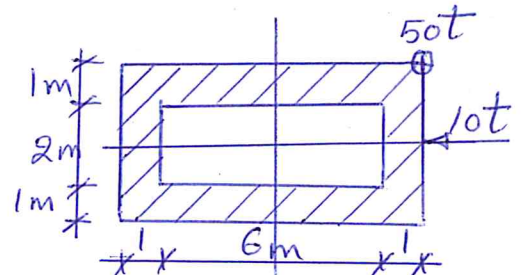
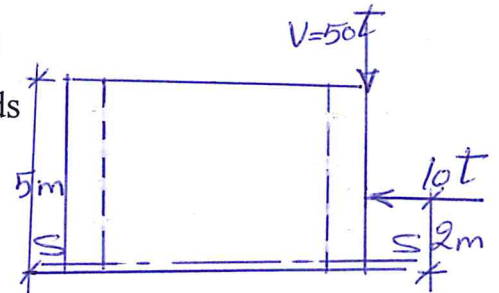
$$\begin{aligned}
 \tau_{\max/\min} &= \frac{\sigma_{\max} - \sigma_{\min}}{2} = \pm 0.143 \text{ t/cm}^2 \\
 \sigma_{av} &= \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 0.28}{2} = -0.14
 \end{aligned}$$

12)



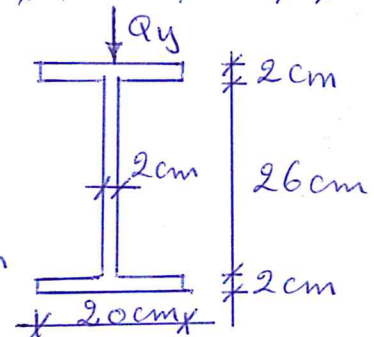
**Question 1** (12 marks)

For section s-s of the shown structure weighing  $2 \text{ t/m}^3$   
 And subjected to the given horizontal and vertical loads  
 -Draw the normal stress distribution  
 -Find the vertical load  $v$  if the maximum allowable compressive normal stress at section s-s is  $20 \text{ t/m}^2$

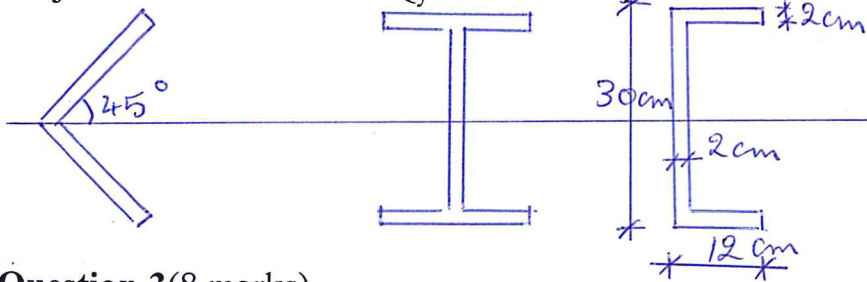


**Question 2** (12marks)

Draw the shear stress distribution for the shown cross-section subjected to a shear force  $Q_y=30 \text{ t}$



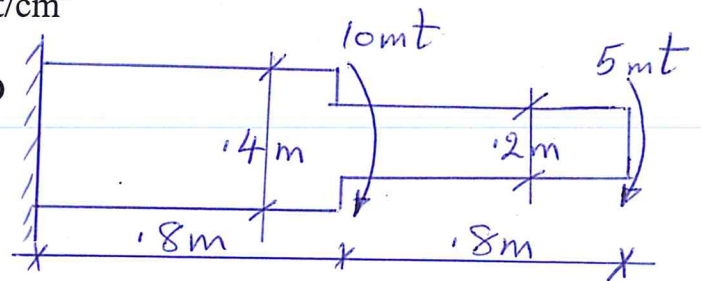
-Determine the shear center for the shown sections subjected to a shear force  $Q_y=20 \text{ t}$



**Question 3** (8 marks)

For the shown circular shaft having  $G=800 \text{ t/cm}^2$

- Draw the torsional moment diagram
- Draw the twisting angle diagram T.A.D
- Determine the maximum shear stress and the maximum angle of twist



**Question 4** (8 marks)

For a section subjected to  $\sigma_x=400 \text{ kg/cm}^2$ ,  $\sigma_y=200 \text{ kg/cm}^2$  and  $\tau=100 \text{ kg/cm}^2$

- Determine analytically the maximum and minimum normal stresses
- Verify your calculations using Mohr's circle

Question 1

1] Straining Actions

$$W = (8 \times 4 - 6 \times 2) \times 5 \times 2$$

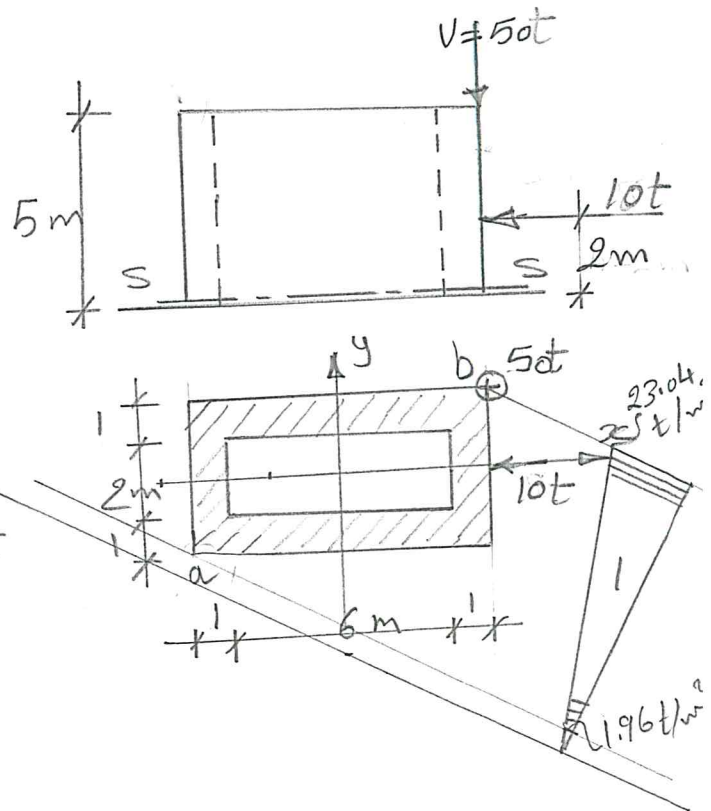
$$= (20) \times 5 \times 2 = 200 \text{ t}$$

$$N = -200 - 50 = -250 \text{ t}$$

$$M_x = -50 \times 2 = -100 \text{ mt}$$

$$M_y = -50 \times 4 + 10 \times 2$$

$$= -180 \text{ mt}$$



2] Prop. of sec.

$$A = 20 \text{ m}^2$$

$$I_x = \frac{8(4)^3}{12} - \frac{6(2)^3}{12} = 38.67 \text{ m}^4$$

$$I_y = \frac{4(8)^3}{12} - \frac{2(6)^3}{12} = 134.67 \text{ m}^4$$

3]

$$\sigma = \frac{N}{A} + \frac{M_{2c}}{I_x} y + \frac{M_y}{I_y} x$$

$$= \frac{-250}{20} - \frac{100}{38.67} y - \frac{180}{134.67} x$$

$$\sigma = -12.5 - 2.59 y - 1.34 x$$

For N.A.  $\sigma = 0$

at  $x = 0 \rightarrow y = -4.83 \text{ m}$

at  $y = 0 \rightarrow x = -9.33 \text{ m}$

$$\sigma_a = -12.5 - 2.59(-2) - 1.34(-4) = -1.96 \text{ t/m}^2$$

(-4, -2)

$$\sigma_b(4, 2) = -12.5 - 2.59(2) - 1.34(4) = -23.04 \text{ t/m}^2$$

$$b) V = ?$$

$$\sigma = 20$$

$$* M_x = -2V, \quad N = -200 - V$$

$$M_y = -4V + 20$$

$$\sigma_b = \frac{-200 - V}{20} - \frac{-2V}{38.67} \sqrt{2} + \frac{(-4V + 20)}{134.67} \sqrt{4} = -20$$

$$\therefore -10 - .05V - .1034V - .1188V + .594 = -20$$

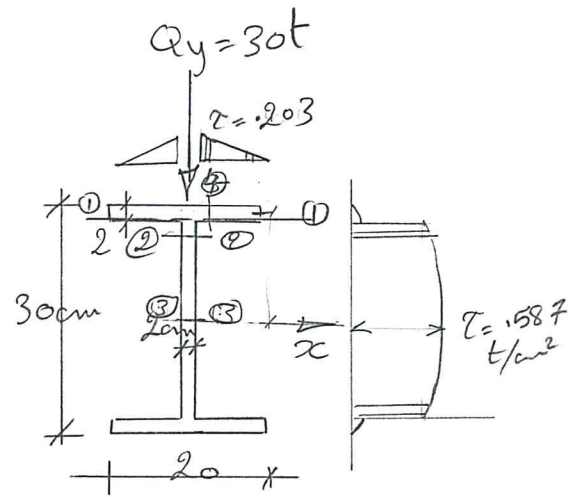
$$\therefore -2.722V = -10.594$$

$$\boxed{V = -38.61 \text{ t}}$$

## Question 2

$$\tau = \frac{Q_y S_x}{I_x b}$$

$$I_x = \left[ \frac{20(2)^3}{12} + 20 \times 2(14)^2 \right] \times 2 + \frac{2(26)^3}{12} = 18636 \text{ cm}^4$$



### Sec 1-1

$$S_x = 2 \times 20 \times 14 = 560, b = 20$$

### Sec 3-3

$$S_x = 2 \times 20 \times 14 + 13 \times 2 \times 6.5 = 729, b = 2$$

### Sec 4-4

$$S_x = 9 \times 2 \times 14 = 252, b = 2$$

### Sec 1-1

$$\tau = \frac{30}{18636} \times \frac{S_x}{b} = 0.045 \text{ t/cm}^2$$

### Sec 2-2

$$b = 2 \rightarrow \tau = 0.451 \text{ t/cm}^2$$

### Sec 3-3

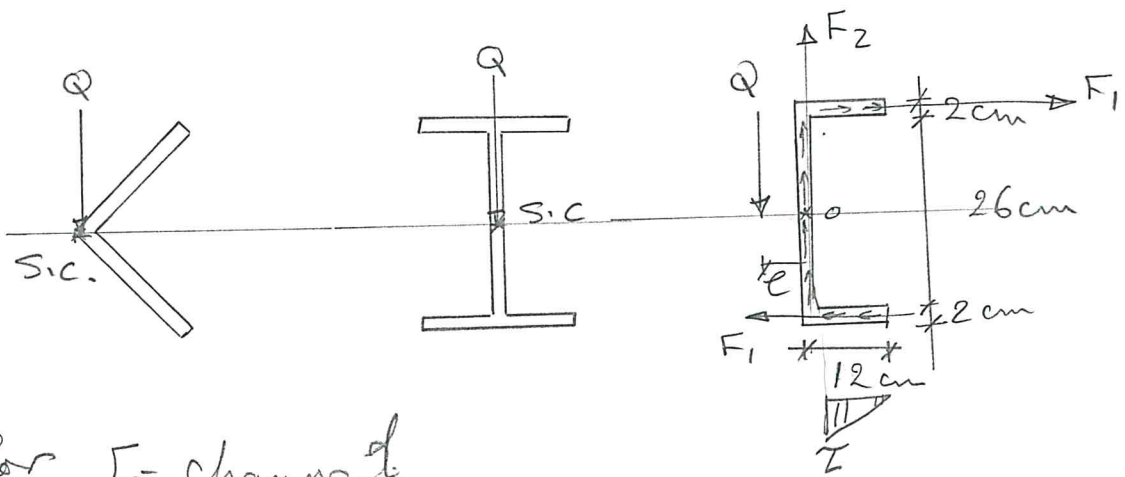
$$\tau = 0.587 \text{ t/cm}^2$$

### Sec 4-4

$$\tau = 0.203 \text{ t/cm}^2$$



2-b)



For I-channel

$$\tau = \frac{Q_y S_x}{I_x b}$$

$$I_x = \left[ \frac{12(2)^3}{12} + 12 \times 2(14)^2 \right] \times 2 + \frac{2(26)^3}{12}$$

$$= 12353.33 \text{ cm}^4$$

$$\tau = \frac{20 \times [2 \times 10 + 14]}{I_x \times 2} = 0.227 \text{ t/cm}^2$$

$$\therefore F_1 = \frac{0.227 \times 10}{2} \times 2 = 2.27 \text{ t}$$

$$\sum M_o = 0$$

$$\therefore F_1 \times 28 = Q e$$

$$\therefore 2.27 \times 28 = 20 e \longrightarrow e = 3.18 \text{ cm}$$

### Question 3

$$\tau = \frac{M_t r}{I_p}$$

$$\theta = \frac{M_t x}{G I_p}$$

$$I_{p1} = \frac{\pi (40)^4}{32}$$

$$= 251327.4 \text{ cm}^4$$

$$\therefore \theta_1 = \frac{15 \times 100 (80)}{800 \times 251327.4}$$

$$= 5.97 \times 10^{-4}$$

$$I_{p2} = \frac{\pi (20)^4}{32}$$

$$= 15707.96 \text{ cm}^4$$

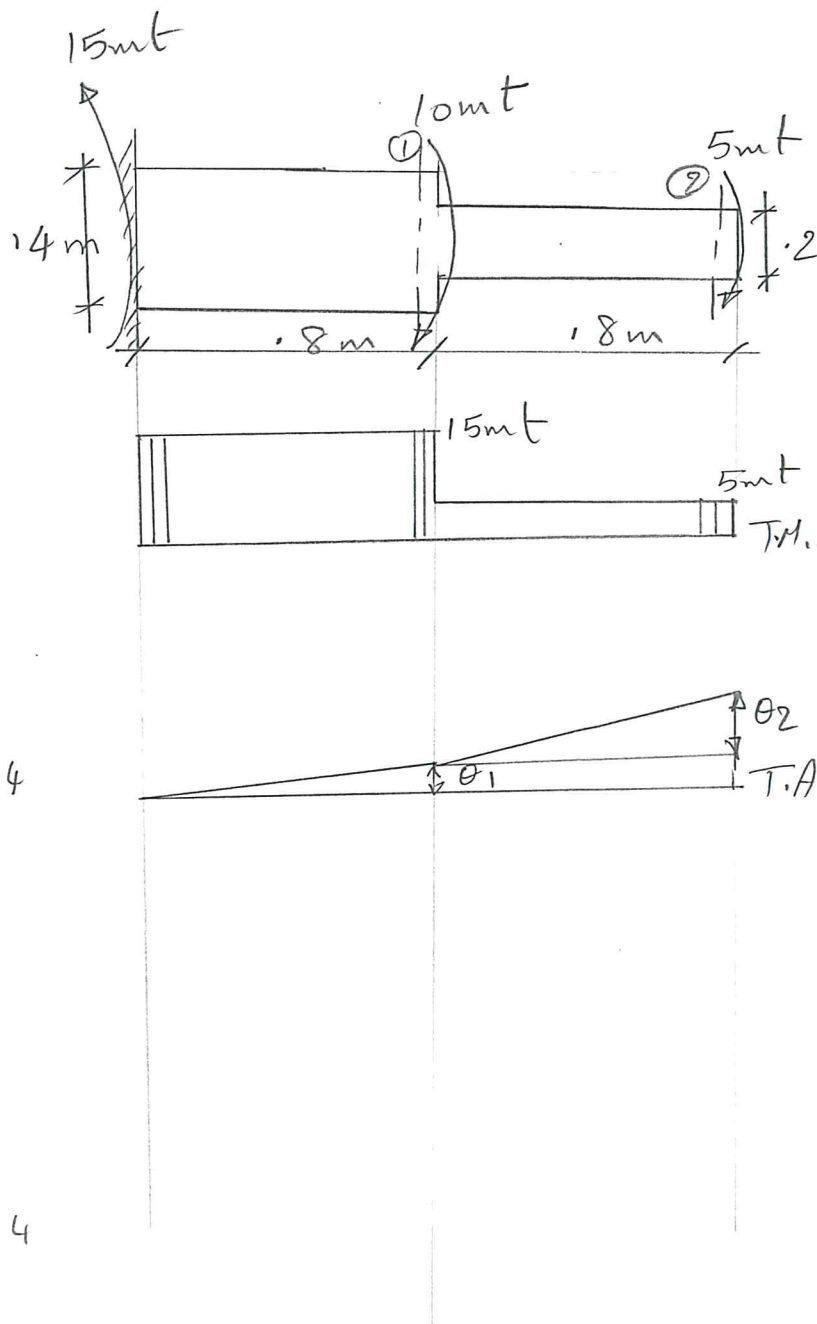
$$\theta_2 = \frac{5 \times 100 (80)}{800 \times 15707.96}$$

$$= 3.18 \times 10^{-3}$$

$$\theta_{\max} = \theta_1 + \theta_2 = 3.78 \times 10^{-3} \text{ rad.}$$

$$\tau_1 = \frac{15 \times 100 \times 20}{251327.4} = 0.1194 \text{ t/cm}^2$$

$$\tau_2 = \frac{5 \times 100 \times 10}{15707.96} = 0.3183 \text{ t/cm}^2 = \tau_{\max}$$



# Question 4

$$\sigma_x = 400 \text{ kg/cm}^2, \quad \sigma_y = 200 \text{ kg/cm}^2$$

$$\tau = 100 \text{ kg/cm}^2$$

$$a) \quad \sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{400 + 200}{2} \pm \sqrt{\left(\frac{400 - 200}{2}\right)^2 + 100^2}$$

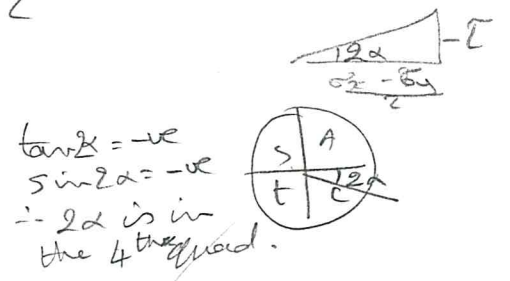
$$\therefore \sigma_{\max} = 441.42 \text{ t/cm}^2$$

$$\sigma_{\min} = 158.58 \text{ t/cm}^2$$

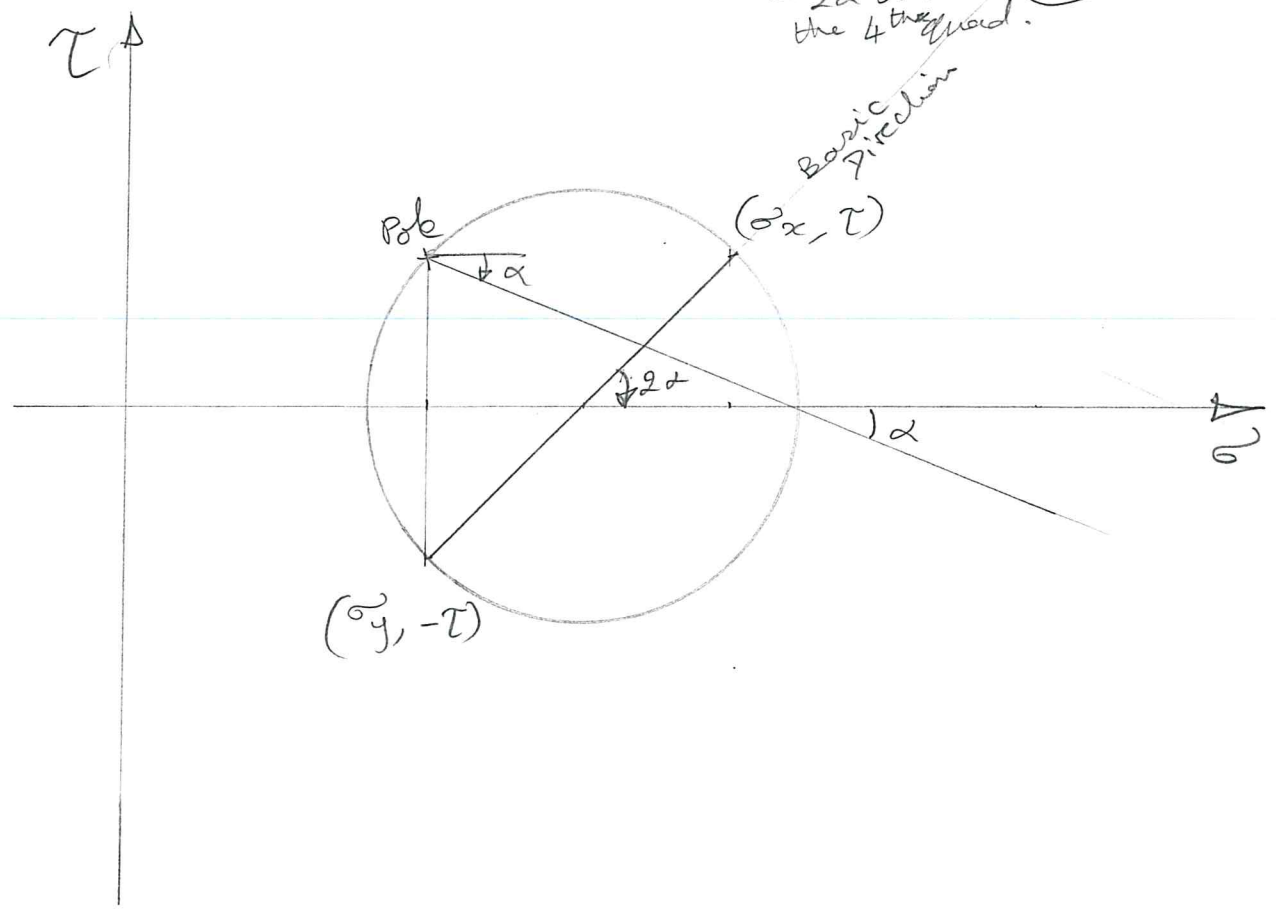
$$\tan 2\alpha = \frac{-\tau}{\frac{\sigma_x - \sigma_y}{2}} = \frac{-100}{\frac{400 - 200}{2}} = -1$$

$$\therefore 2\alpha = -45^\circ$$

$$\therefore \alpha = -22.5^\circ$$



$\tan 2\alpha = -ve$   
 $\sin 2\alpha = -ve$   
 $\therefore 2\alpha$  is in the 4<sup>th</sup> quadrant.

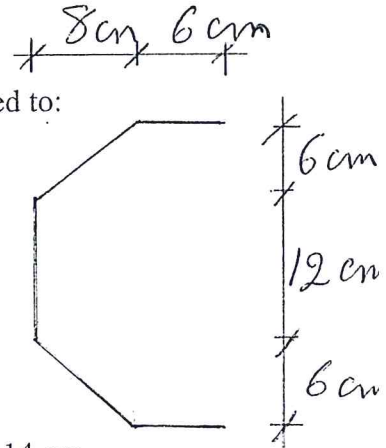


Final Exam

Question 1 [a3,b1,b2,c4] {12 Marks}

The given cross-section has a thickness of 1 cm and is subjected to  $Q_y$ . It is required to:

1. Find the centroidal moment of inertia  $I_x$ . {3 M}
2. Draw the distribution of the shear stress. {5 M}
3. Find the resultant forces due to the drawn shear stresses. {3 M}
4. Find the shear centre if the cross-section. {1 M}

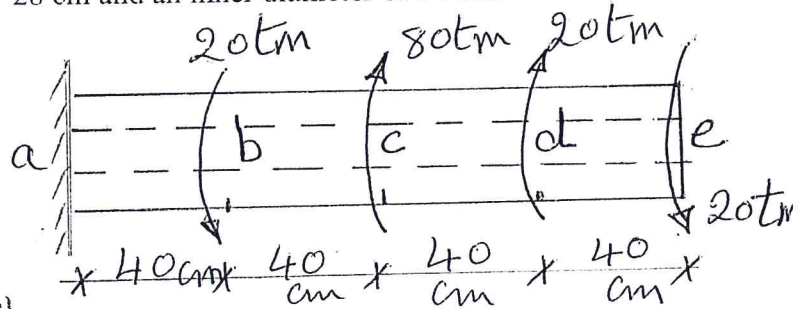


Question 2 [a4,b1,c4] {12 Marks}

The shown hollow shaft has an outer diameter of 28 cm and an inner diameter of 14 cm.

$G=800 \text{ t/cm}^2$

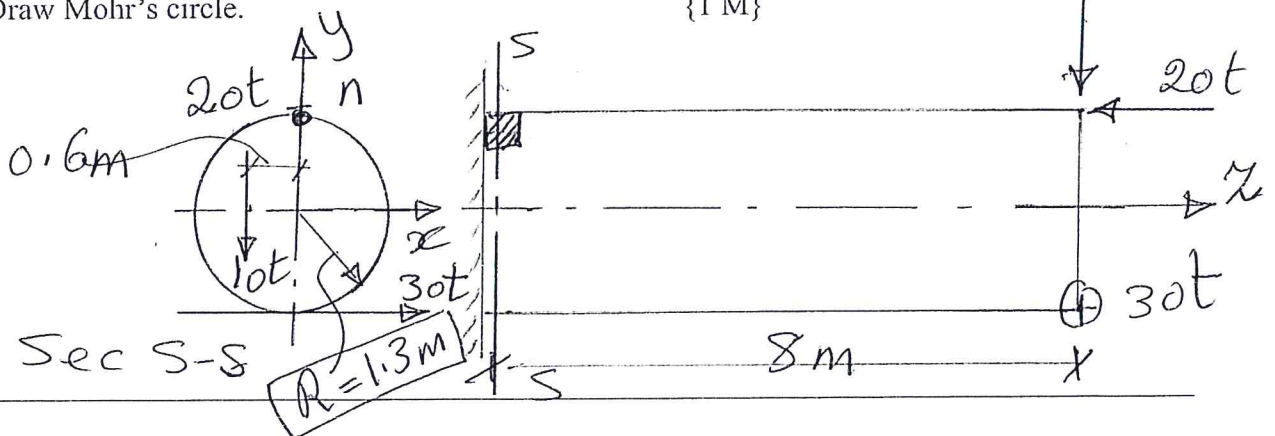
1. Draw the T.M.D. {4 M}
2. Draw the T.A.D. {4 M}
3. Find the maximum shear stress on the shaft at section "a". {4 M}



Question 2 [a5,b1,b3,b4,c2,c3,c4] {16 Marks}

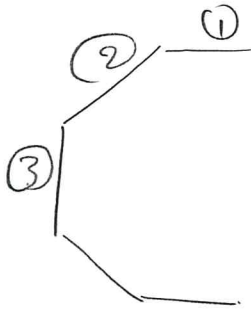
For section S-S of the shown beam:

1. Find the properties of the cross-section:  $A$ ,  $I_x$ ,  $I_y$ . {2 M}
2. Compute the straining actions:  $N$ ,  $M_x$ ,  $M_y$ ,  $Q_x$ ,  $Q_y$ ,  $M_t$ . {3 M}
3. Find the normal stress at point "n". {2 M}
4. Find the maximum shear stress due to  $Q_x$ ,  $Q_y$  and  $M_t$ . {3 M}
5. Find total shear stress at point "n". {1 M}
6. Draw the stress block. {1 M}
7. Compute analytically: {2 M}
  - a. The direction of the principal normal stress plane:
  - b. The maximum and minimum normal stresses acting on the principal plane.
8. Compute analytically: {2 M}
  - a. The principal shear plane.
  - b. The direction and values of the stresses acting on the principal shear plane.
9. Draw Mohr's circle. {1 M}



$8\text{cm} \times 6\text{cm}$

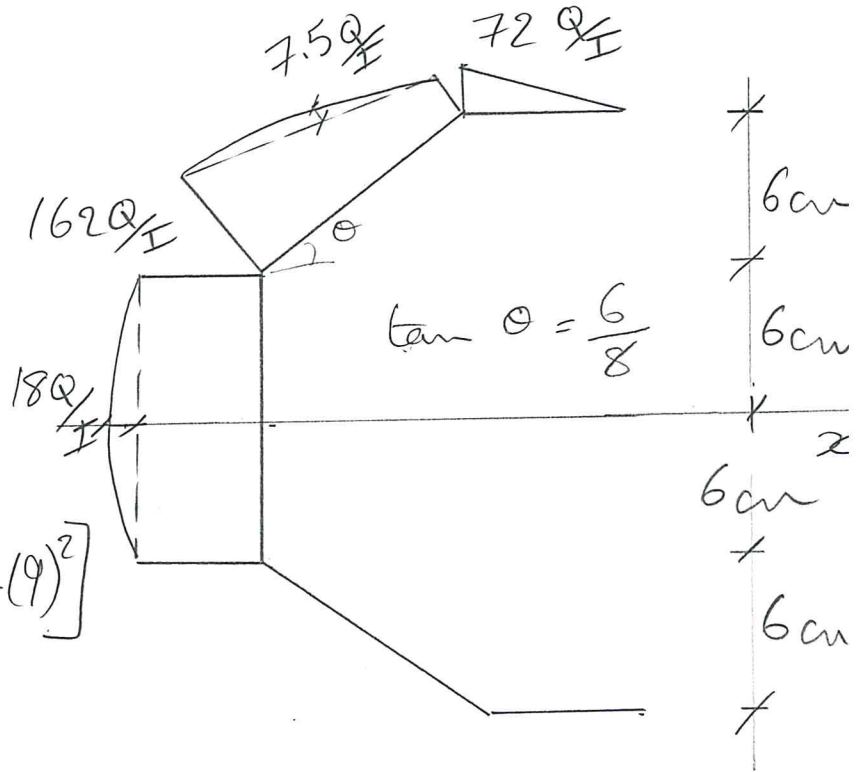
1



$I_x$

$$I_x = 2 \left[ 6 \times 1 \times (12)^2 \right] + 2 \left[ \frac{10 \times 1 (6)^2}{12} + 10 \times (9)^2 \right] + \left[ \frac{1 (12)^3}{12} \right]$$

$$= 1728 + 1680 + 144$$



$I_{xx} = 3552 \text{ cm}^4$

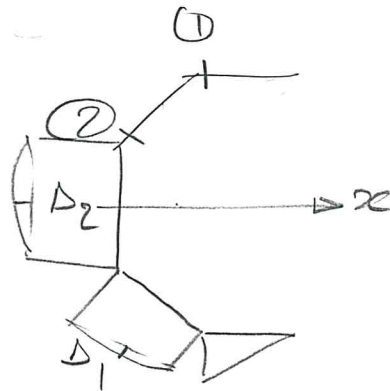
Flow

$$f_1 = 6 \times 1 \times 12 = 72 \text{ Q/I}$$

$$f_2 = f_1 + 10 \times 9 = 162 \text{ Q/I}$$

$$\Delta_1 = \frac{10 \times 6}{8} = 7.5 \text{ Q/I}$$

$$\Delta_2 = \frac{12 \times 12}{8} = 18 \text{ Q/I}$$

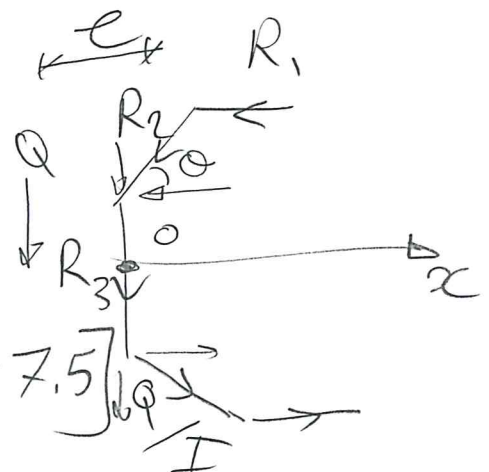


Resultant forces

$$R_1 = \frac{1}{2} \times 6 \times 72 \text{ Q/I} = 216 \text{ Q/I}$$

$$R_2 = \left[ \left( \frac{162 + 72}{2} \right) (10) + \frac{2}{3} \times 10 \times 7.5 \right] \text{ Q/I}$$

$$= 1220 \text{ Q/I}$$





$$R_3 = \left[ 162 \times 12 + \frac{2}{3} \times 12 \times 18 \right] \frac{Q}{I} = 2088 \frac{Q}{I}$$

Shear Centre

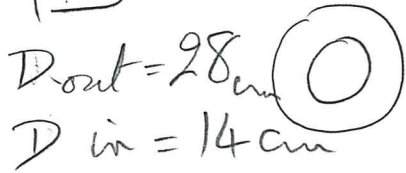
$$R_1 \times 12 \times 2 + R_2 \cos \theta \times 6 \times 2 = Q e$$

$$\therefore 216 \frac{Q}{I} \times 12 \times 2 + 1220 \frac{Q}{I} \times 0.8 \times 6 \times 2 = Q e$$

$$4.75 \text{ cm} = e$$

[2]

$G = 8000 \text{ t/cm}^2$   $60 \text{ tm}$



T.A.D.

$$\theta = \frac{M_t L}{G I_p}$$

$$I_p = \frac{\pi [(14)^4 - (7)^4]}{2}$$

$$I_p = 56595 \text{ cm}^4$$

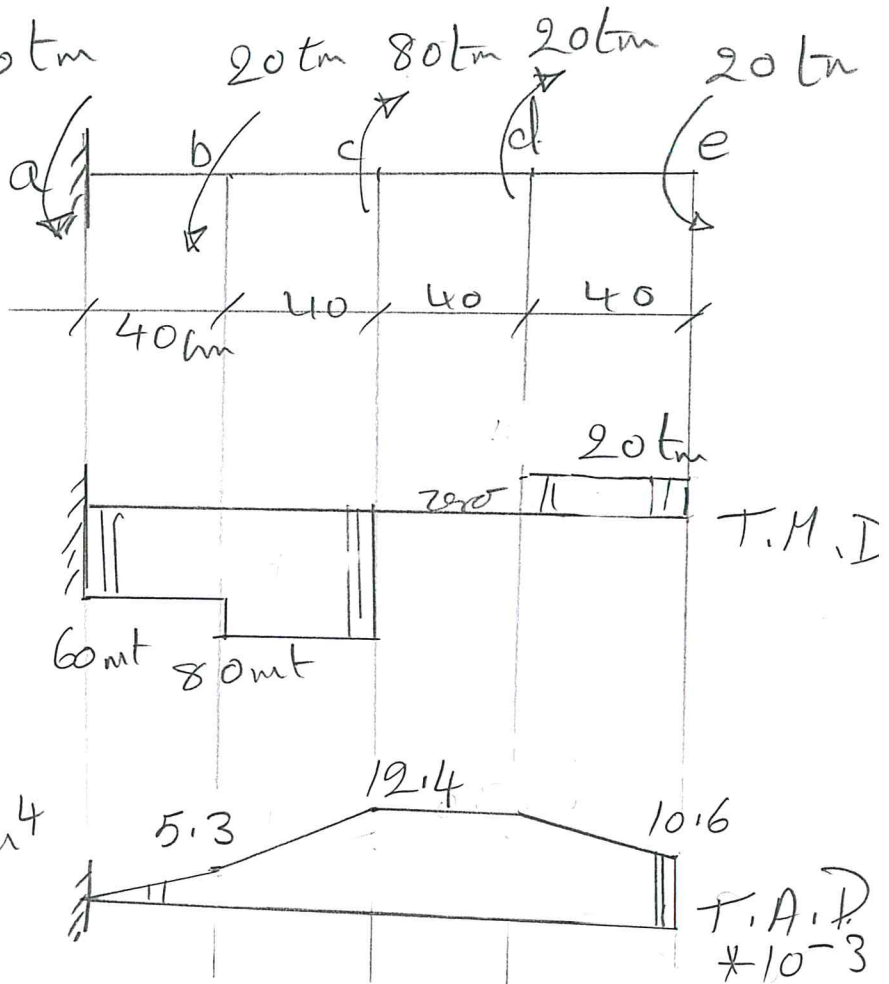
$\theta_a = 0$

$$\theta_b = \theta_a + \frac{60 \times 100 \times 40}{800 I_p} = 5.3 \times 10^{-3} \text{ rad}$$

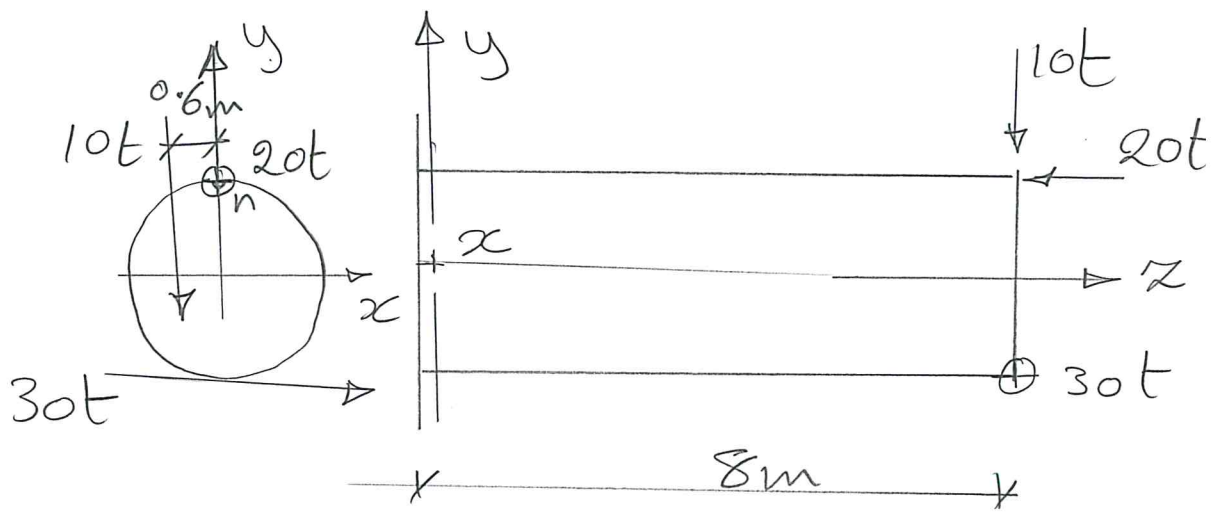
$$\theta_c = \theta_b + \frac{80 \times 100 \times 40}{800 I_p} = 0.0124 \text{ rad} = \theta_d = 12.4 \times 10^{-3}$$

$$\theta_e = \theta_d - \frac{20 \times 100 \times 40}{800 I_p} = 0.0106 \text{ rad} = 10.6 \times 10^{-3}$$

$$\tau_a = \frac{M_t R}{I_p} = \frac{60 \times 100 \times 14}{56595} = 1.48 \text{ t/cm}^2$$



Q3



$$\textcircled{1} A = \pi (1.3)^2 = 5.31 \text{ m}^2 \quad R = 1.3 \text{ m}$$

$$I_x = I_y = \frac{\pi (1.3)^4}{4} = 2.24 \text{ m}^4$$

$$\textcircled{2} N = -20t, \quad M_t = +10 \times 0.6 + 30 \times 1.3 = \boxed{45 \text{ m}t = M_t}$$

$$Q_x = +30t, \quad Q_y = -10t$$

$$M_x = -20 \times 1.3 + 10 \times 8 = \boxed{54 \text{ m}t = M_x}$$

$$M_y = -30 \times 8 = \boxed{-240 \text{ m}t = M_y}$$

$$\textcircled{3} \sigma = \frac{N}{A} + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$$

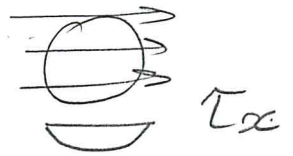
$$\sigma_{(0, 1.3)} = \frac{-20}{5.31} + \frac{54}{2.24} (1.3) - \frac{240}{2.24} (0)$$

$$\therefore \boxed{\sigma_n = 27.57 \text{ t/m}^2}$$

$$\textcircled{4} \quad \tau_x = \frac{Q_x S_y}{I_y b} = \frac{Q_x \times \frac{\pi (1.3)^2}{2} \times 4(1.3)}{2.24 \times 2.6 \times 3\pi}$$

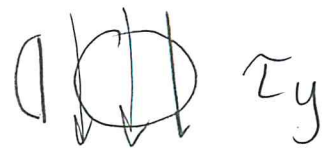
$$= \tau_x = Q_x \times 0.25 = 30 \times 0.25$$

$$= \tau_x = 7.5 \text{ t/m}^2$$



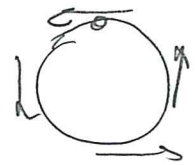
$$\& \quad \tau_y = \frac{Q_y S_x}{I_x b} = Q_y \times 0.25 = 10 \times 0.25$$

$$= \tau_y = 2.5 \text{ t/m}^2$$



$$\tau_t = \frac{M_t R}{I_p} = \frac{45 \times 1.3}{\frac{\pi (1.3)^4}{2}}$$

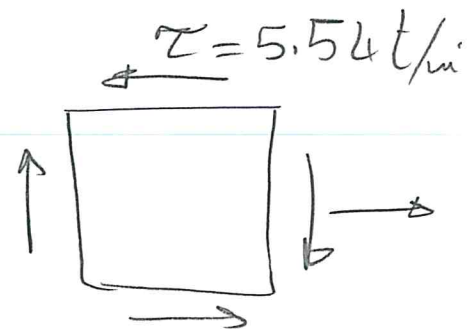
$$\tau_t = 13.04 \text{ t/m}^2$$



$$\textcircled{5} \quad \tau_n = 7.5 - 13.04 = 5.54 = \tau_n$$

\textcircled{6} stress block

$$\sigma = 27.57$$



⑦  $\sigma_x = 27.57$ ,  $\sigma_y = 0$ ,  $\tau = +5.54$

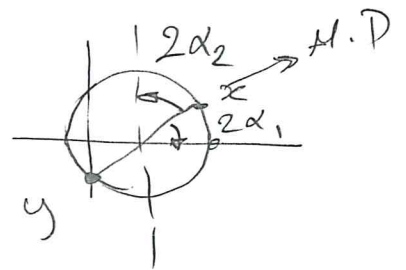
$$\tan 2\alpha_1 = \frac{-\tau}{\frac{\sigma_x - \sigma_y}{2}} = \frac{-5.54}{\frac{27.57 - 0}{2}}$$

$$\therefore \tan 2\alpha_1 = \frac{-5.54}{13.79} = \boxed{-0.40 = \tan 2\alpha_1}$$

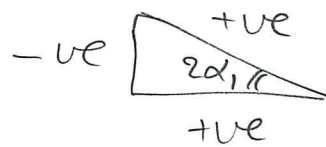
$$\begin{aligned} \sigma_{\max} \\ \sigma_{\min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (-\tau)^2} \\ &= 13.79 \pm 14.86 \end{aligned}$$

$$\therefore \sigma_{\max} = 28.65$$

$$\sigma_{\min} = -1.07$$



$\therefore$  Sin -ve  
Cos +ve  
Tan -ve

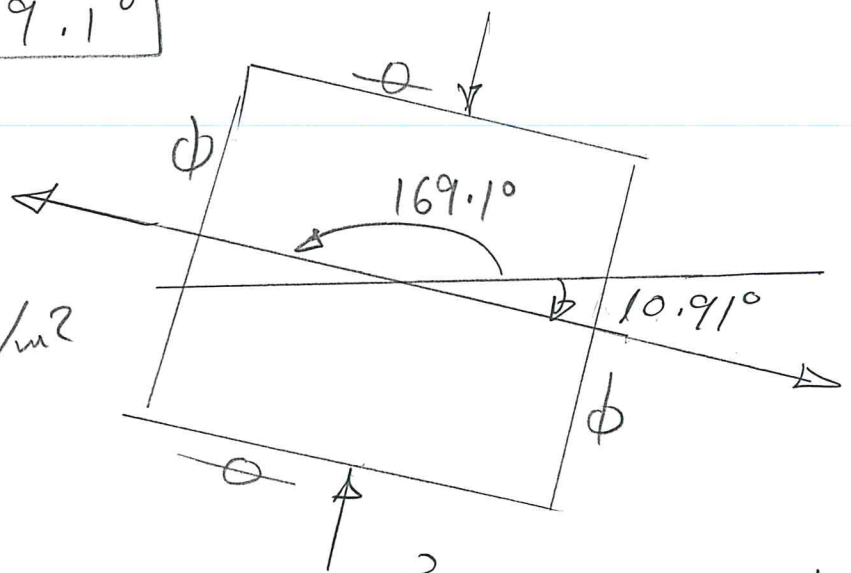


$$\therefore 2\alpha_1 = 360 - 21.8^\circ = 338.2^\circ$$

$$\therefore \boxed{\alpha_1 = 169.1^\circ}$$

$$\begin{aligned} \sigma_{\max} \\ &= 28.65 \text{ t/m}^2 \end{aligned}$$

$$\tau = 0$$



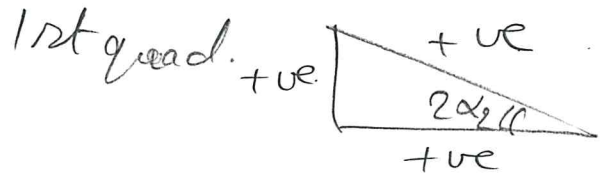
$$\sigma_{\min} = -1.07 \text{ t/m}^2$$



$$\textcircled{8} \quad \tan 2\alpha_2 = + \frac{\frac{\sigma_x - \sigma_y}{2}}{\tau} = 2.5$$

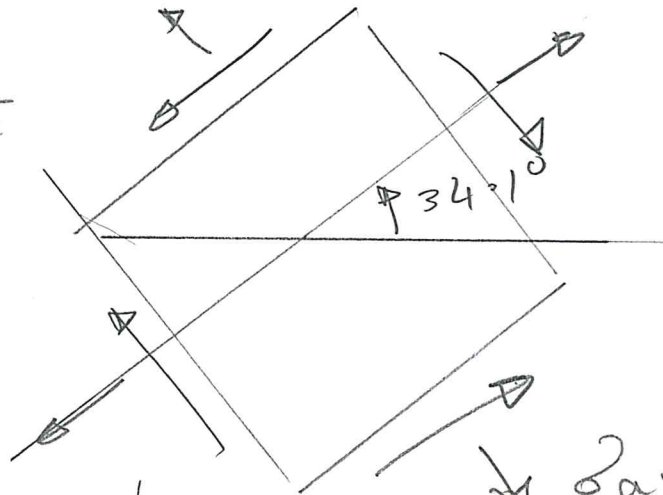
$$\therefore 2\alpha_2 = 68.2^\circ$$

$$\therefore \alpha_2 = 34.1^\circ$$



$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\tau_{\max} = +14.86$$



$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tau_{\max} = 14.86$$

$$\sigma_{av} = 13.79 \text{ t/m}^2$$

$$\therefore \sigma_{av} = 13.79 \text{ t/m}^2$$

$\textcircled{9}$  Mohr's circle