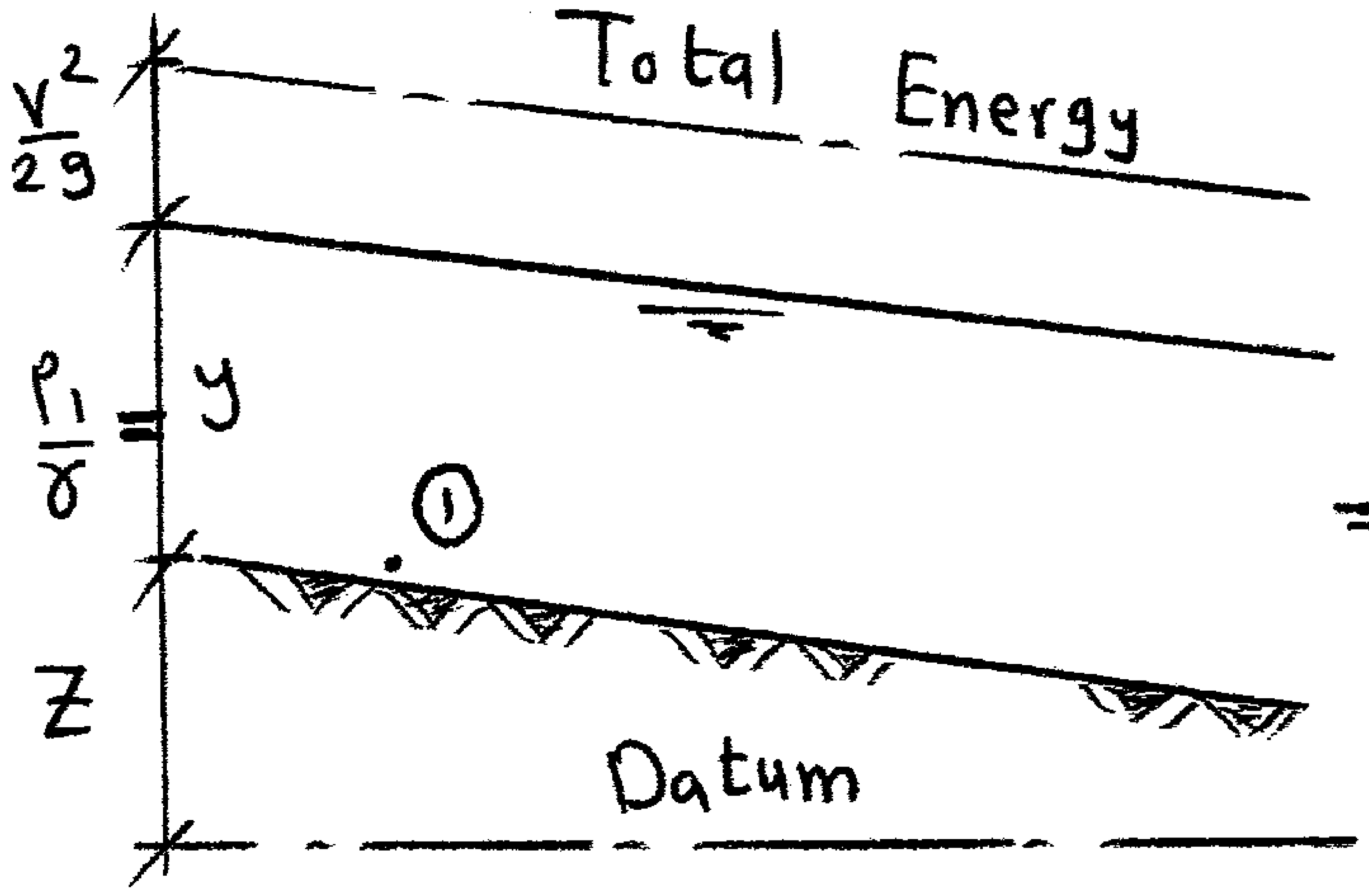
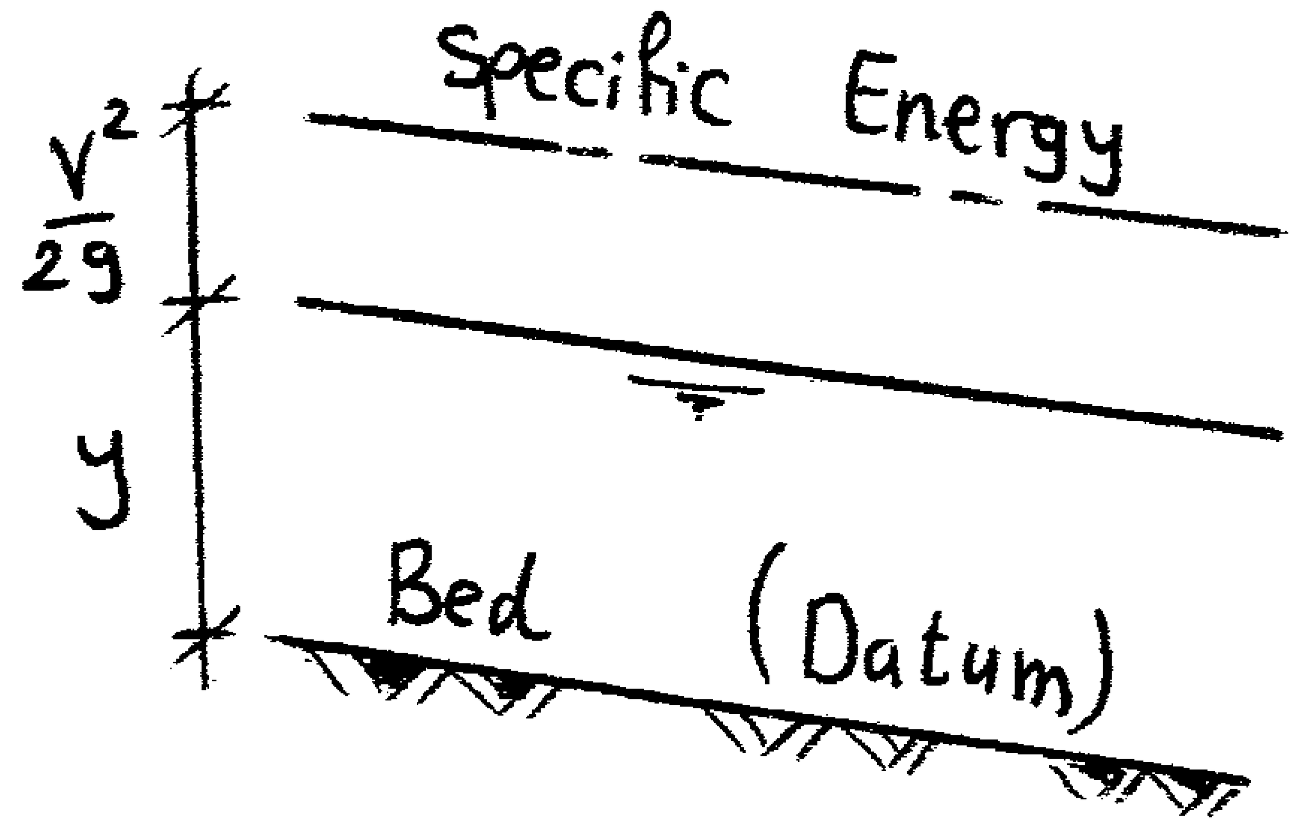
# Specific Energy




$\Rightarrow$




Energy (Bernoulli)

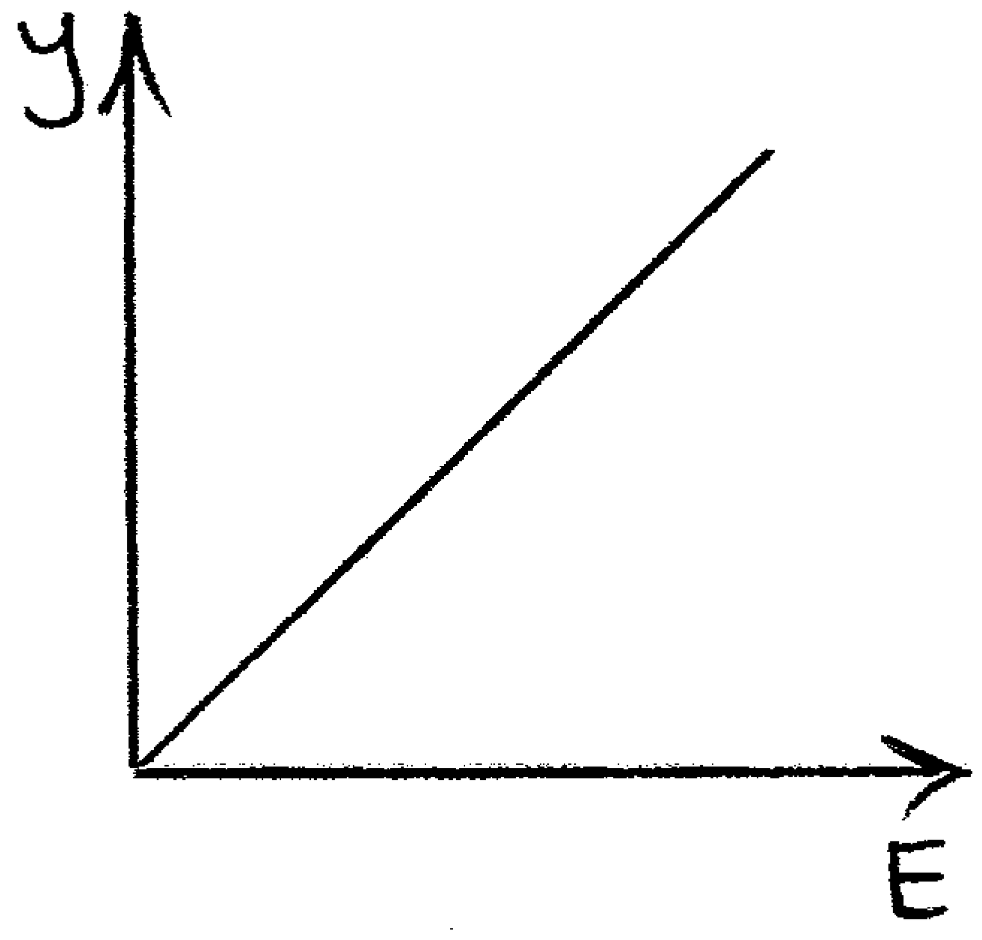
$$E = \frac{P_1}{\gamma} + \frac{V^2}{2g} + Z \Rightarrow E = y + \frac{V^2}{2g}$$

Specific Energy

Definition

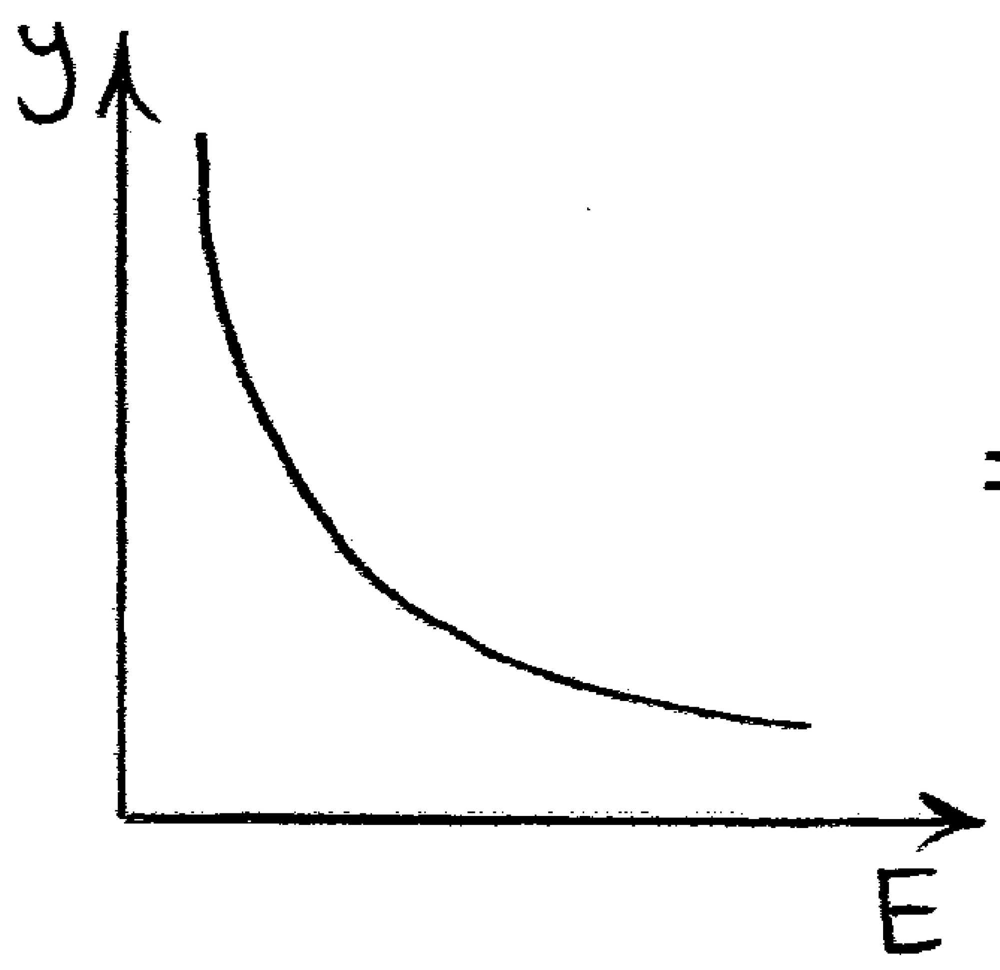
It is the energy referred to channel bed as a Datum.

E = Constant at any point along the channel




$E = y$

+




$E = \frac{V^2}{2g}$

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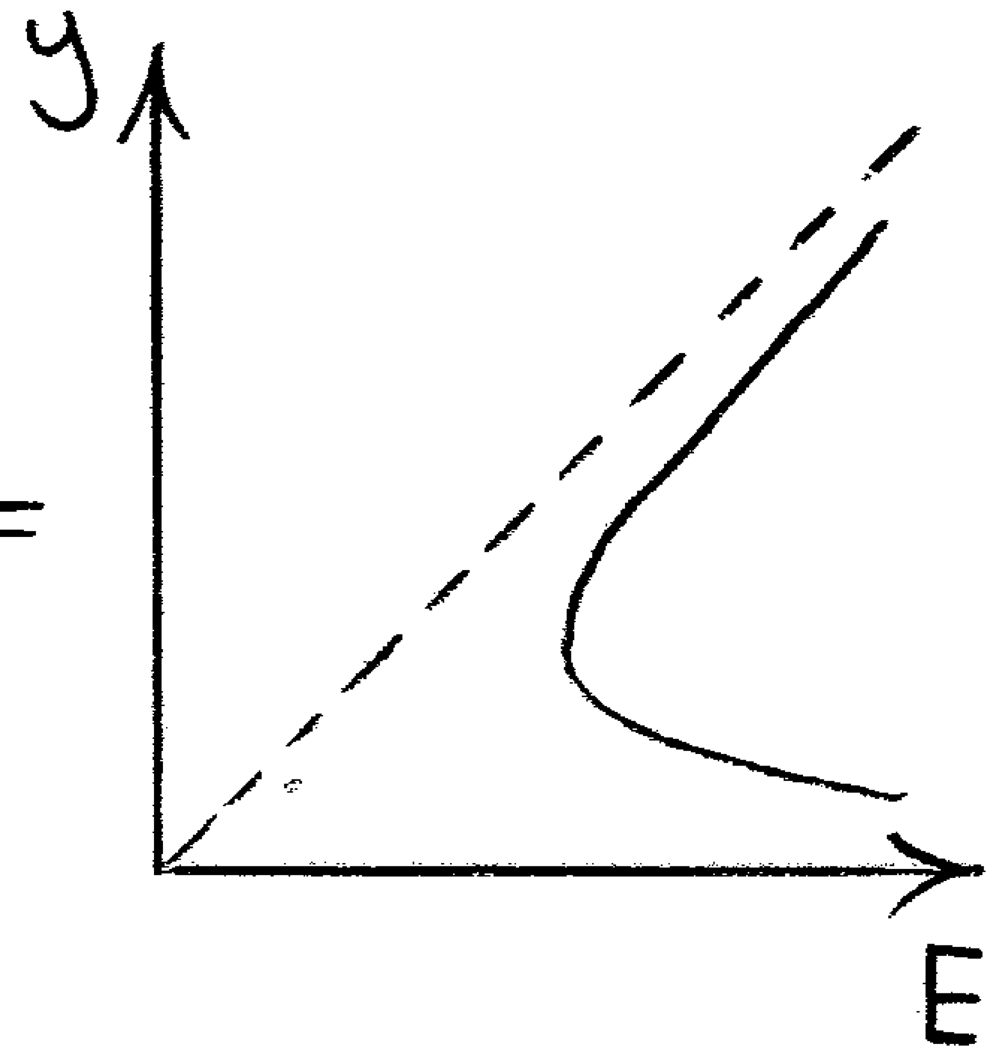
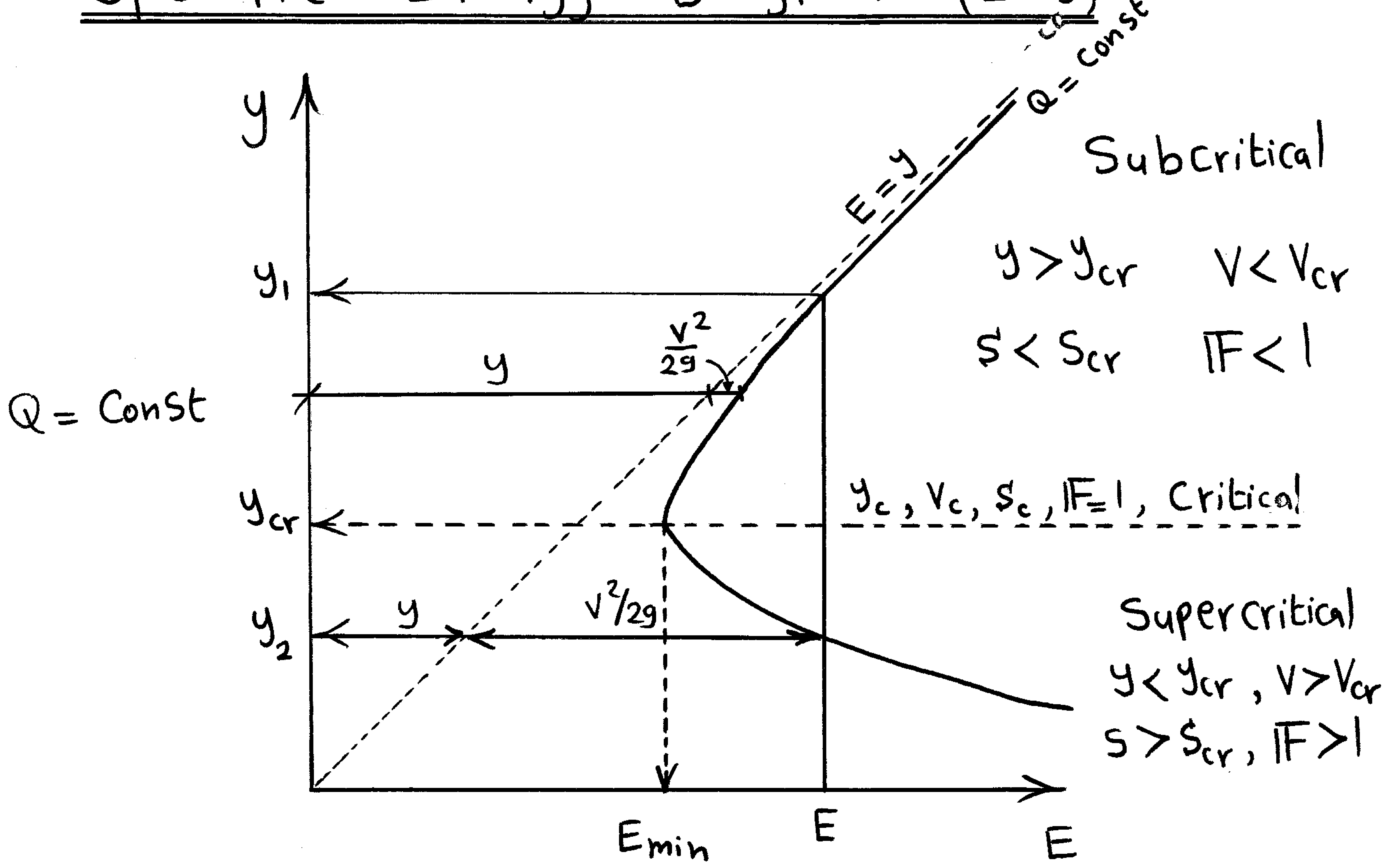



$E = y + \frac{V^2}{2g}$

# Specific Energy Diagram (E - y)




$y > y_{cr} \Rightarrow$ Subcritical flow

$y < y_{cr} \Rightarrow$ Supercritical flow

$y = y_{cr} \Rightarrow$ Critical flow at $E_{min}$

For any E except $E_{min}$ there are 2 depths

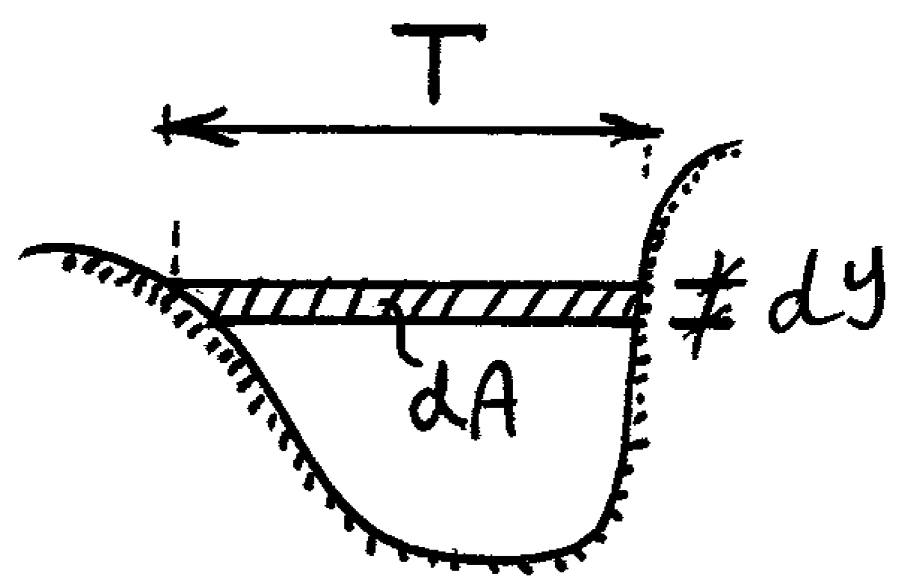
one is <u>subcritical</u> ($y_1$) and the other is <u>Supercritical</u> ($y_2$).

$y_1, y_2$ are called <u>alternate depths</u>

They have the same Energy (E) and Discharge (Q)

## Minimum Specific Energy ($E_{min}$)

$$E = y + \frac{Q^2}{2gA^2}$$




$dA = T\,dy$

$\frac{dA}{dy} = T$

for $E_{min}$ $\quad \frac{dE}{dy} = 0$

$$\frac{dE}{dy} = 1 + \frac{(-2)\,Q^2}{2g\,A_c^3}\left(\frac{dA}{dy}\right) = 0$$

$$= 1 - \frac{Q^2 T}{g\,A_c^3} = 0$$

$$\frac{Q^2 T}{g\,A_c^3} = 1 \quad \Rightarrow \quad \boxed{\frac{Q^2}{g} = \frac{A_c^3}{T}}$$

any Section

## Rectangular Section

$$Q = qb \quad , \quad A_c = b\,y_{cr}$$

$$\frac{q^2 b^2}{g} = \frac{b^3 y_{cr}^3}{b}$$

$$y_{cr}^3 = \frac{q^2}{g}$$

$$\Rightarrow \quad \boxed{y_{cr} = \sqrt[3]{\frac{q^2}{g}}}$$

Rec - Section only

$$E = y + \frac{Q^2}{2gA^2} = y + \boxed{\frac{Q^2}{g}} \times \frac{1}{2A^2}$$

$$E_{min} = y_{cr} + \frac{A_c^3}{T} \times \frac{1}{2A_c^2} = y_{cr} + \frac{A_c}{2T}$$

$$\boxed{E_{min} = y_{cr} + \frac{1}{2} D_{cr}}$$ , $D =$ Hyd mean Depth $= \frac{A}{T}$

<u>For Rectangular section</u> $D = y_{cr}$

$$E_{min} = y_{cr} + \frac{1}{2} y_{cr}$$

$$\boxed{E_{min} = \frac{3}{2} y_{cr}}$$ Rec - Section only

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$$\frac{1}{2} D_{cr} = \frac{V_{cr}^2}{2g} \Rightarrow V_{cr}^2 = g D_{cr}$$

$$\boxed{V_{cr} = \sqrt{g D_{cr}}}$$

$$\boxed{V_{cr} = \sqrt{g\, y_{cr}}}$$ Rec - Section only

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## Froude number

$$F_N = \frac{V}{\sqrt{gD}}$$

$$F_N = \frac{V}{\sqrt{gy}}$$ Rec - Section only

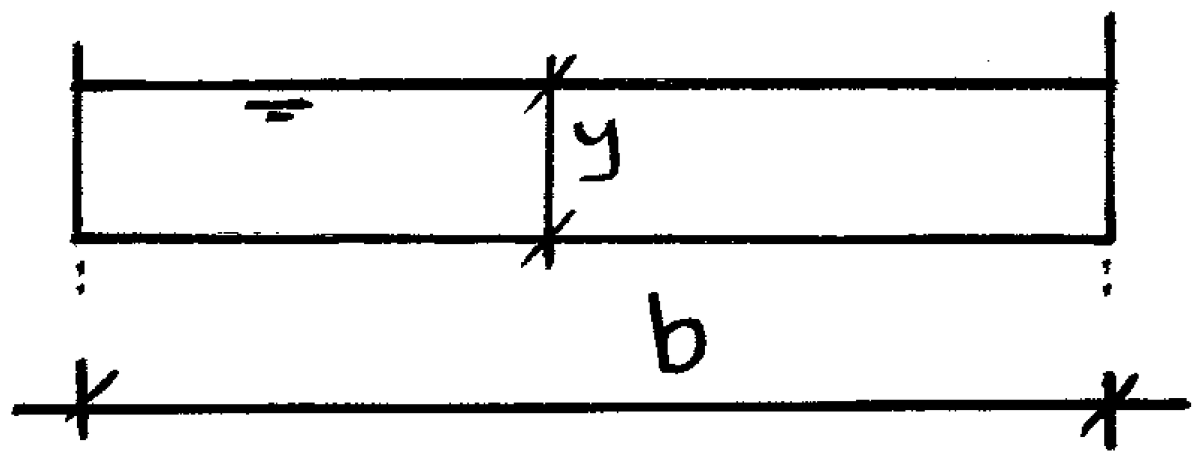
$F > 1$ supercritical , $F < 1$ Subcritical , $F = 1$ critical

## Great Width channel $R = \frac{A}{P} \simeq y$

If $b > 10y$

$$R = \frac{A}{P} = \frac{by}{b+2y}$$

$$R = \frac{y}{1 + \underbrace{2\frac{y}{b}}_{\simeq 0}} = y$$

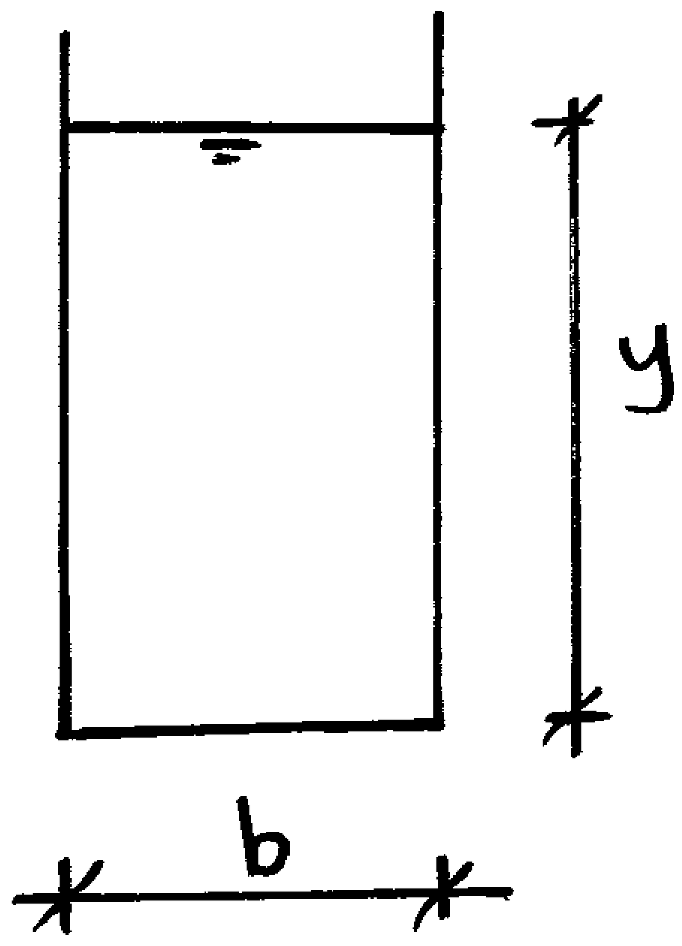



$\frac{y}{b} \simeq 0$

## Deep channel $R = \frac{b}{2}$

$$R = \frac{by}{b+2y} = \frac{b}{\underbrace{\frac{b}{y}}_{=0} + 2}$$

$$R = \frac{b}{2}$$




$\frac{b}{y} \simeq 0$

## Very wide channel $R = y$

$$Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$Q = \frac{1}{n} y^{2/3} S^{1/2} by$$

$$q = \frac{1}{n} S^{1/2} y^{5/3}$$

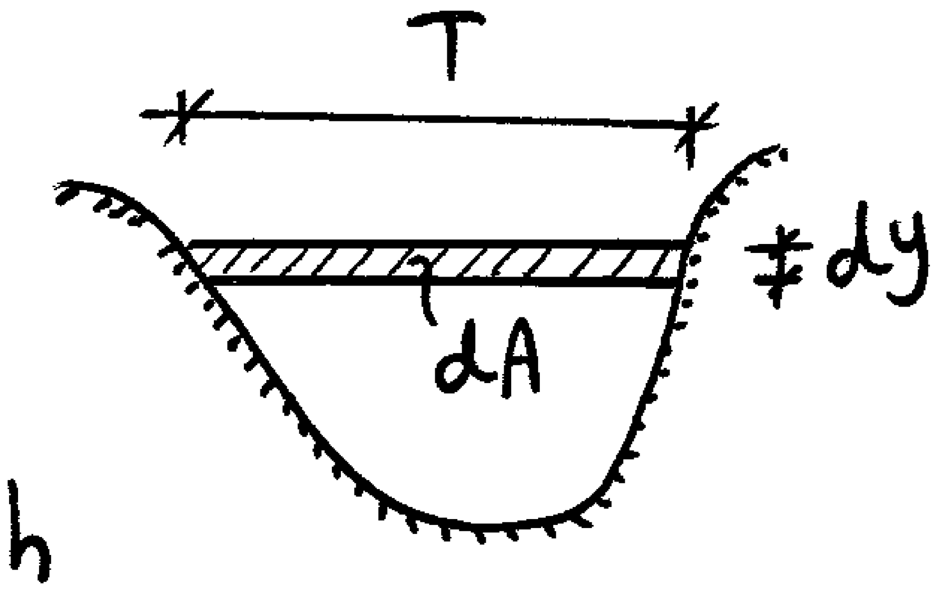
## To get $S_{cr}$ for very wide rect channel

$$q^2 = \frac{1}{n^2} S_{cr} y_c^{10/3}$$

$$g y_c^3 = \frac{S_{cr} y_{cr}^{10/3}}{n^2} \Rightarrow \boxed{S_{cr} = \frac{g n^2}{y_{cr}^{1/3}}}$$

| Any channel | Rectangular channel |
|---|---|
| $E = y + \dfrac{Q^2}{2gA^2}$ | $E = y + \dfrac{q^2}{2gy^2}$ |
| $\dfrac{Q^2}{g} = \dfrac{A_c^3}{T}$ | $y_c = \sqrt[3]{\dfrac{q^2}{g}}$ |
| $E_{min} = y_{cr} + \frac{1}{2} D_{cr}$ | $E_{min} = \frac{3}{2} y_c$ |
| $V_c = \sqrt{g D_c}$ | $V_c = \sqrt{g y_c}$ |
| $\mathbb{F}_N = \dfrac{V}{\sqrt{gD}}$ | $\mathbb{F}_N = \dfrac{V}{\sqrt{gy}}$ |
| $D = \dfrac{A}{T}$ | $y = \dfrac{A}{b}$ |
| | $S_c = \dfrac{gn^2}{y_c^{1/3}}$ wide rect channel |

$T = \dfrac{dA}{dy}$ = Top width




$D = \dfrac{A}{T}$ = Hydraulic mean depth

$M = \sqrt{\dfrac{Q^2}{g}} = \sqrt{\dfrac{A_c^3}{T}}$ = Section factor (M curve)

for non rect channels $y_c$ لإيجاد (M curve) يستخدم

$S_c = \dfrac{gn^2}{y_c^{1/3}}$ very wide rectangular channel

# Specific Discharge Diagram $(q-y)$ curve

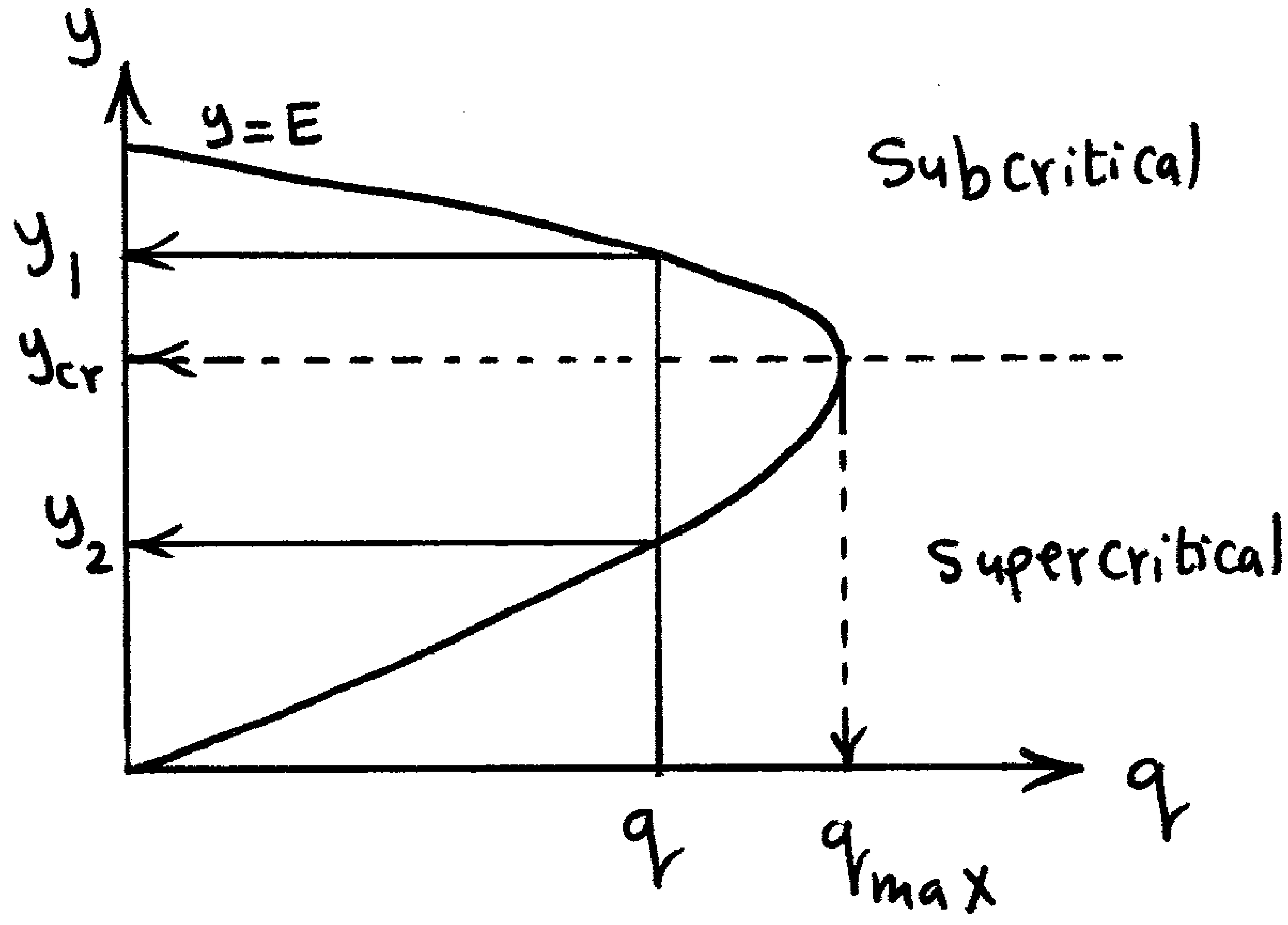
$$E = y + \frac{Q^2}{2gA^2}$$

$$E - y = \frac{Q^2}{2gA^2}$$

$$\boxed{Q = A\sqrt{2g(E-y)}}$$

$$\boxed{q = y\sqrt{2g(E-y)}}$$

Rec - Section only




$q_{max}$ at $y_{cr}$

$$y_{cr} = \frac{2}{3}E$$

$$q_{max} = \sqrt{g\,y_{cr}^3}$$

$$b_{min} = \frac{Q}{q_{max}}$$

E = Const

## Sheet (2) – Specific Energy & Applications

**Q1:** Define each of the following:
a) Specific energy
b) Alternate depths

**Q2:** Develop an expression for each of the following:
a) Critical depth
b) Critical specific energy
c) Critical velocity
For both (I) rectangular section & (II) any other section

**Q3:** Calculate the specific energy when 6.25 $m^3/s$ flow in a rectangular channel of 3m wide at a depth of 1m

**Q4:** Determine the alternate depths for 30 cfs flow in a rectangular channel 6.5 ft wide if the specific energy is 4 ft.

**Q5:** A 7m wide rectangular channel carries a discharge of 160 $m^3/s$ at a uniform depth of 3.40 m. Manning's coefficient equals 0.022. Determine the channel slope, critical depth, and Froude's number.

**Q6:** For a discharge of 40 $m^3/s$, determine the critical depth and critical slope for
(a) Trapezoidal channel with bed width 7.5 m and side slope of 1:1.
(b) Rectangular channel with bed width 7.5 m.

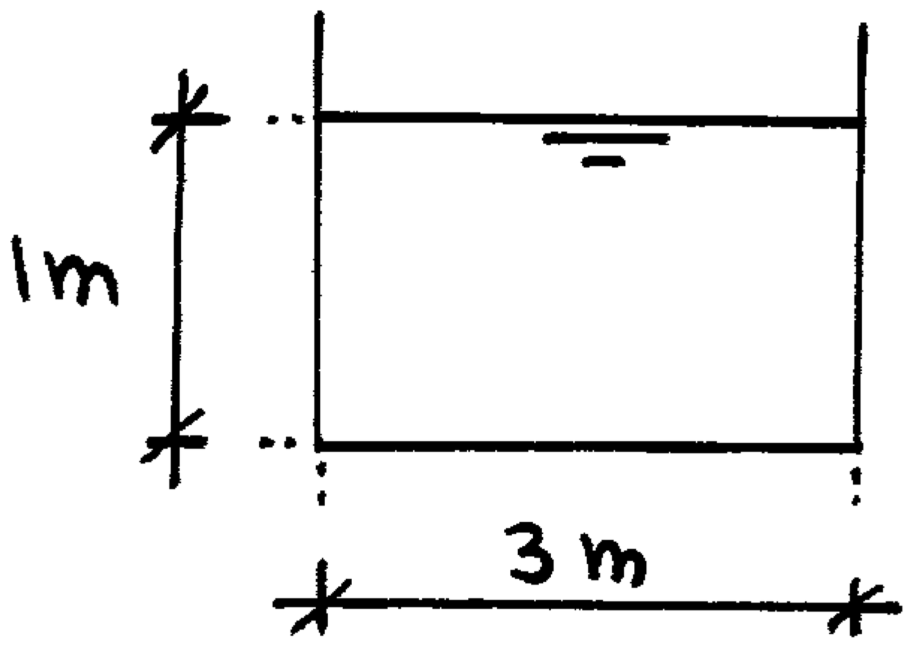
**Q7:** Twenty two cubic meters per second flow in a rectangular channel of 6 m width having n of 0.017. Plot accurately the specific energy diagram for depths from 0 to 3 m using the same scale for y and E. Determine from the diagram:
(a) The critical depth.
(b) The minimum specific Energy.
(c) The specific energy when the depth of flow is 2 m.
(d) The depths when the specific energy is 2.5 m.
(e) The depth which is the alternate depth for 1.5 m depth.
(f) What type of flow exists when the depth is: (i) 0.6 m. (ii) 1.8 m.
(g) What are the channel slopes necessary to maintain these depths?
(h) What types of slopes are these?
(k) What is the critical slope assuming the channel to be of great width?

**Q8:** Flow occurs in rectangular channel of 20 ft width and has a specific energy of 10 ft. Plot accurately the q-curve and determine the following from the curve:
a) The critical depth and maximum flow rate.
b) The flow rate at a depth of 8 ft.
c) The depths at which a flow rate of 1000 cfs may exist
d) The flow condition at these depths

**(3)**

$$Q = 6.25\ m^3/s$$

$$E = ?$$




$$A = 3 \times 1 = 3$$

$$V = \frac{Q}{A} = \frac{6.25}{3} = 2.08\ m/s$$

$$E = y + \frac{V^2}{2g} = 1 + \frac{(2.08)^2}{2 \times 9.81} = 1.22\ m$$

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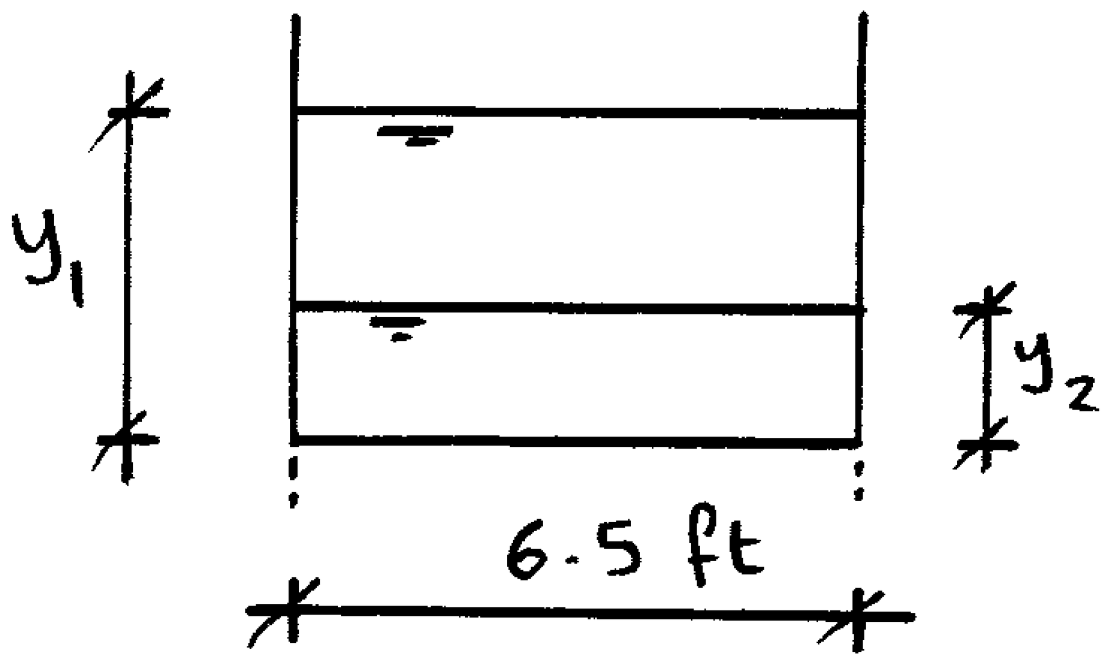
**(4)**

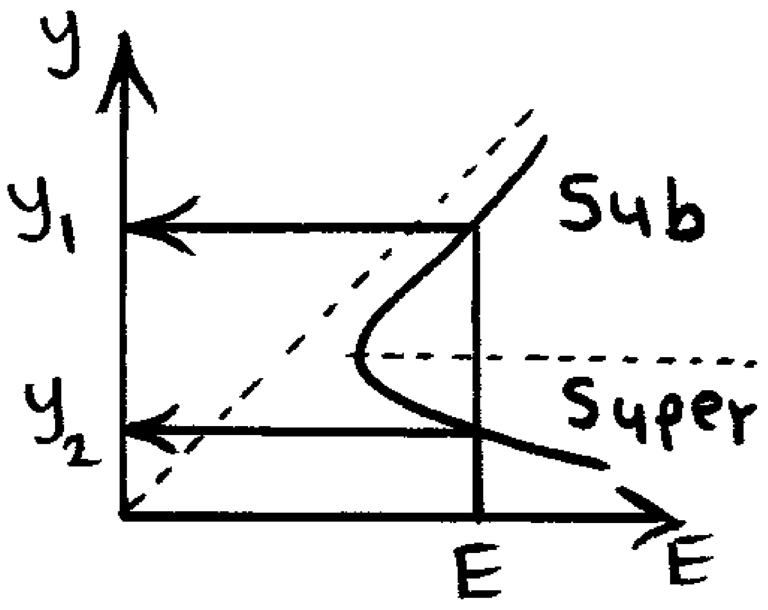
$$Q = 30\ ft^3/s \quad , \quad E = 4\ ft$$

$$y = ?$$




$$E = y + \frac{Q^2}{2gA^2}$$

$$4 = y + \frac{(30)^2}{2 \times 32.2 (6.5)^2 y^2}$$




$$4 = y + \frac{1}{3.02 y^2}$$

$$y_1 = 3.98\ ft$$

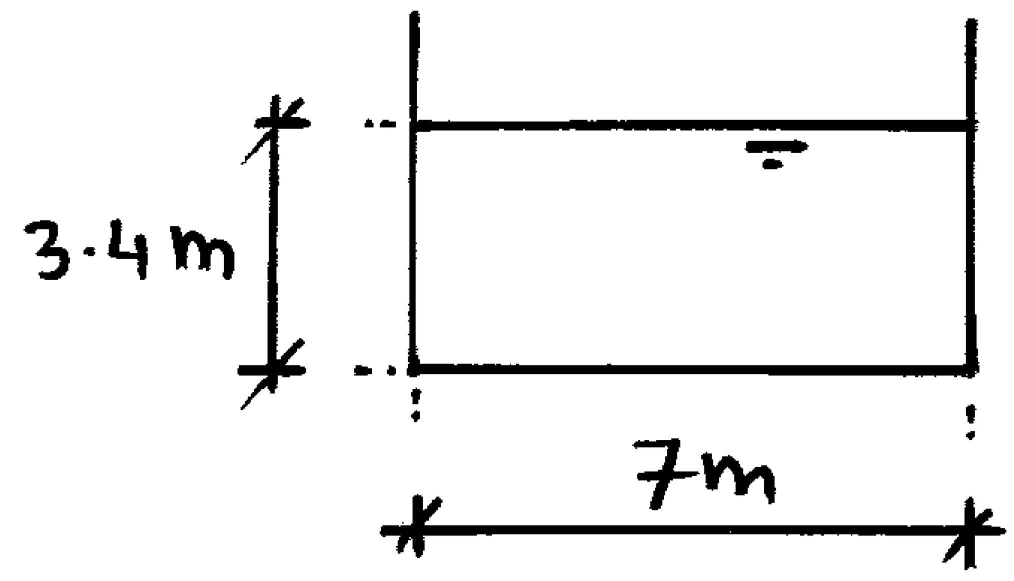
$$y_2 = 0.30\ ft$$

| $y$ | $4 - y - \frac{1}{3.02y^2}$ |
|---|---|
| 1 | 2.6 |
| 2 | 1.9 |
| 3 | 0.96 |
| 4 | −0.02 |
| 3.9 | 0.078 |
| 3.95 | 0.028 |
| 3.97 | 0.009 |

| $y$ | $4 - y - \frac{1}{3.02y^2}$ |
|---|---|
| ✓✓ 3.98 | −0.0009 |
| 0.1 | −29.2 |
| 0.5 | 2.17 |
| 0.2 | −4.47 |
| ✓✓ 0.3 | 0.02 |
| 0.29 | −0.22 |
| 0.31 | 0.24 |

**(5)** $Q = 160\ m^3/s$ , $n = 0.022$

$S = ?$ $\quad y_c = ?$ $\quad F_n = ?$




$$Q = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S^{1/2} A$$

$$A = 3.4\,(7) = 23.8$$

$$P = 7 + 2(3.4) = 13.8$$

$$R = \frac{A}{P} = \frac{23.8}{13.8} = 1.72$$

$$160 = \frac{1}{0.022}\,(1.72)^{2/3}\, S^{1/2}\, 23.8$$

$$\Rightarrow \quad S = \underline{\underline{0.0105}}$$

$$q = \frac{Q}{b} = \frac{160}{7} = 22.85\ m^3/s/m'$$

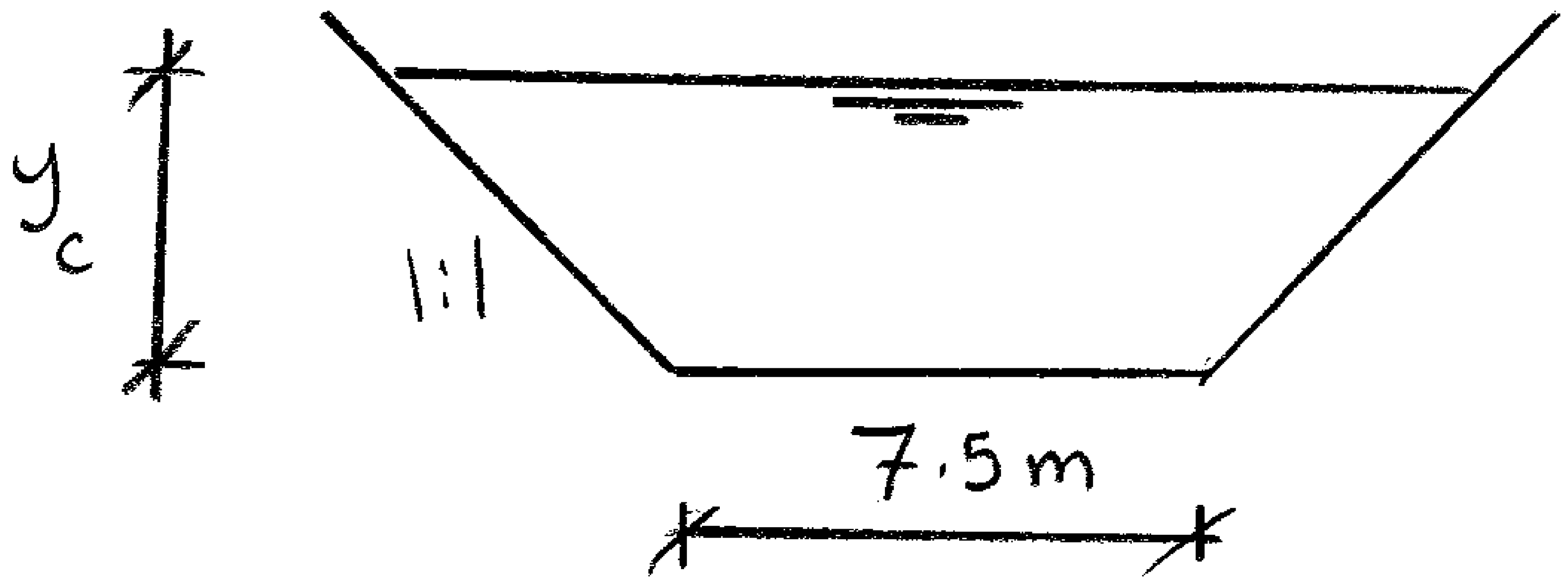
$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(22.85)^2}{9.81}} = \underline{\underline{3.76\,m}}$$

$$V = \frac{Q}{A} = \frac{160}{23.8} = 6.72$$

$$F = \frac{V}{\sqrt{gy}} = \frac{6.72}{\sqrt{9.81(3.4)}} = \underline{\underline{1.164}} > 1 \quad \text{supercritical}$$

**(6)** $Q = 40\ m^3/s \quad y_c = ? \quad S_c = ?$

$$\frac{Q^2}{g} = \frac{A_c^3}{T}$$




$$A = 7.5\,y_c + y_c^2$$

$$T = 7.5 + 2y_c$$

$$\frac{(40)^2}{9.81} = \frac{(7.5\,y_c + y_c^2)^3}{(7.5 + 2y_c)} \Rightarrow y_c = \underline{\underline{1.34\,m}}$$

$$Q = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S^{1/2}\,A$$

$$A = 7.5(1.34) + (1.34)^2 = 11.84\ m^2$$

$$P = 7.5 + 2(1.34)\sqrt{1+1^2} = 11.29\ m$$

$$40 = 40\left(\frac{11.84}{11.29}\right)^{2/3}(11.84)\ S_c^{1/2} \Rightarrow S_c = \underline{\underline{0.0067}}$$

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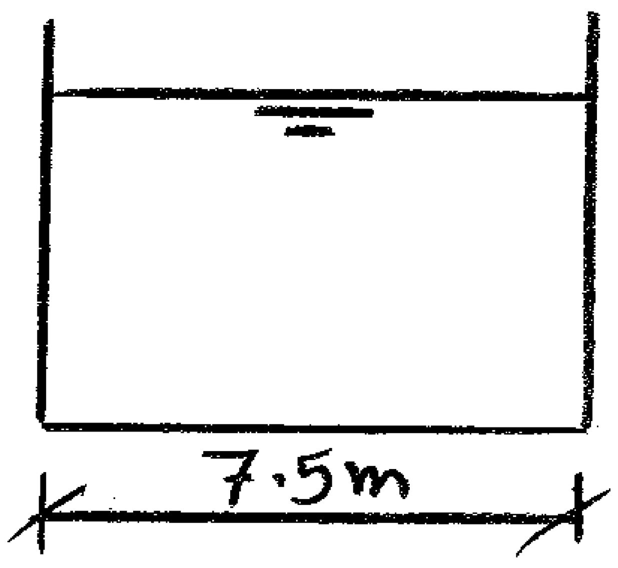



$$q = \frac{Q}{b} = \frac{40}{7.5} = 5.33$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(5.33)^2}{9.81}} = \underline{\underline{1.425\ m}}$$

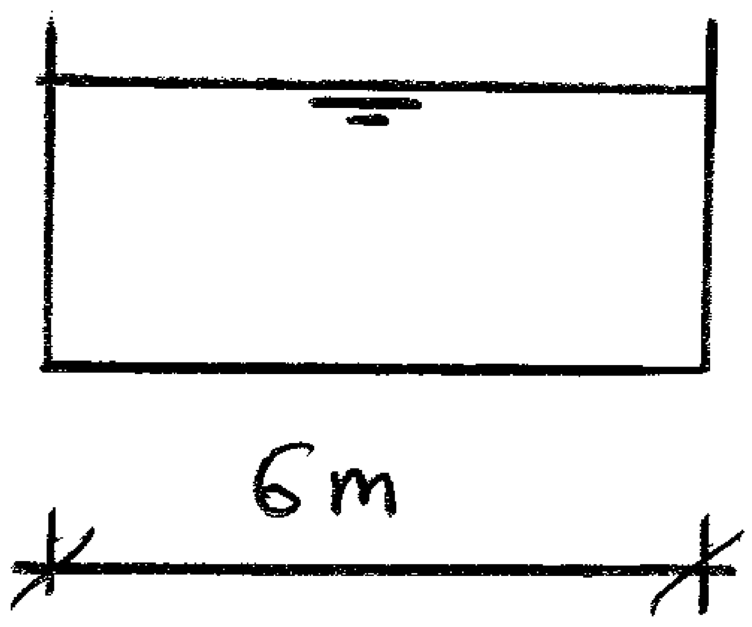
$$A = 1.425(7.5) = 10.7\ m^2$$

$$P = 7.5 + 2(1.425) = 10.35\ m$$

$$40 = 40\left(\frac{10.7}{10.35}\right)^{2/3} S^{1/2}\,(10.7) \Rightarrow S = \underline{\underline{0.0083}}$$

**(7)** $Q = 22\ m^3/s \qquad n = 0.017$

$$E = y + \frac{Q^2}{2gA^2}$$



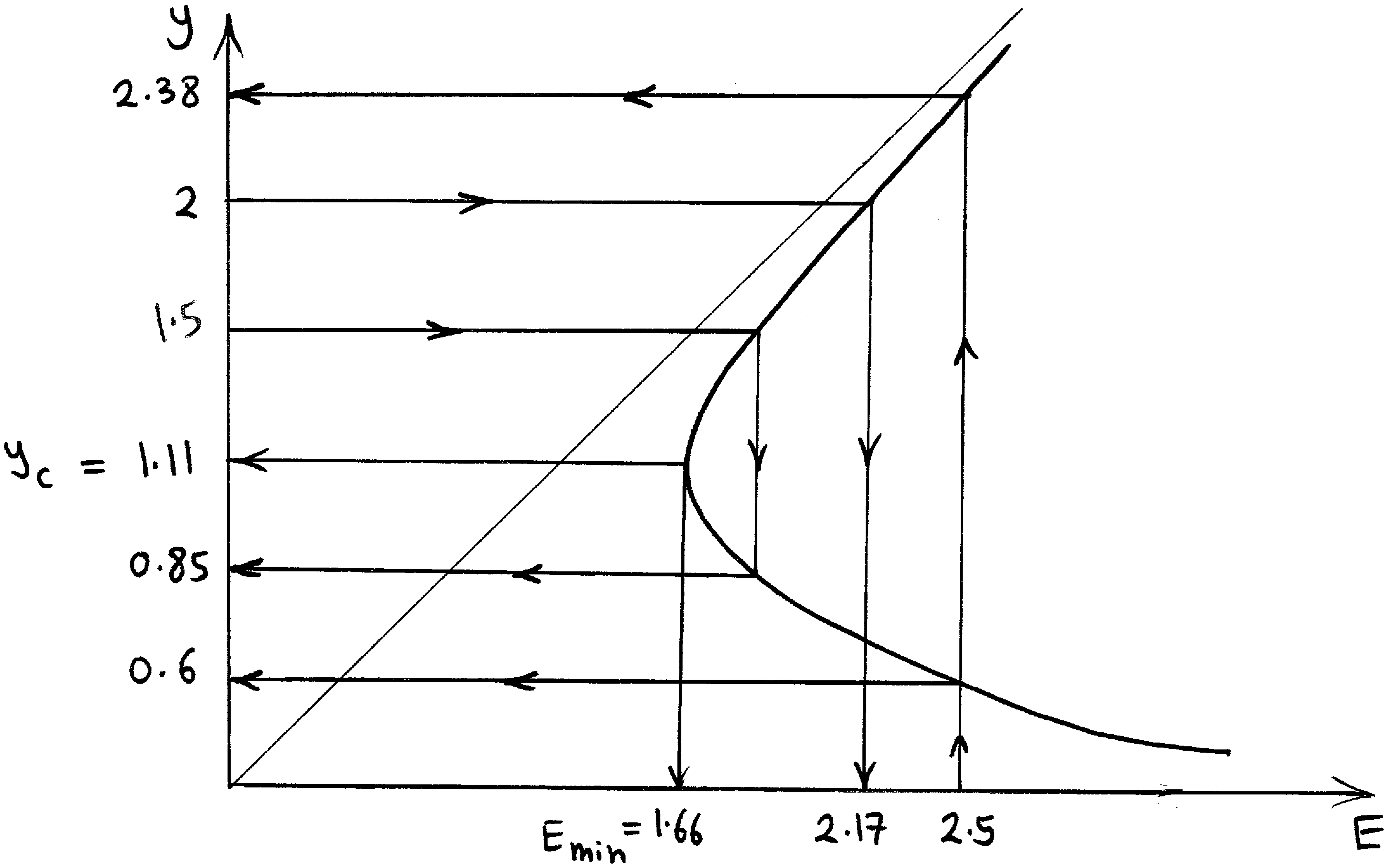

$$E = y + \frac{(22)^2}{2 \times 9.81\,(6y)^2}$$

$$E = y + \frac{1}{1.46\,y^2}$$

$$q = \frac{Q}{b} = \frac{22}{6} = 3.67$$

$$y_c = \sqrt[3]{\frac{(3.67)^2}{9.81}} = 1.11$$

| y | 0.4 | 0.6 | 0.8 | 1 | 1.11 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| E | 4.68 | 2.5 | 1.87 | 1.68 | 1.66 | 1.8 | 2.17 | 2.61 | 3.07 |



a) $y_c = 1.11$

b) $E_{min} = 1.66$

c) $y = 2m \longrightarrow E = 2.17$

d) $E = 2.5 \longrightarrow y_1 = 2.38\ ,\ y_2 = 0.6$

e) $y_1 = 1.5 \longrightarrow y_2 = 0.85$

f) $y = 0.6$ (supercritical) $y = 1.8$ (subcritical)

K) $S_c = \dfrac{g n^2}{y_c^{1/3}} = \dfrac{9.81\,(0.017)^2}{(1.11)^{1/3}} = 0.0027$

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g) at $y = 0.6$ $Q = 22\ m^3/s$ $n = 0.017$

$A = 6\,(0.6) = 3.6$

$P = 6 + 2(0.6) = 7.2$

$$22 = \frac{1}{0.017}\left(\frac{3.6}{7.2}\right)^{2/3}(3.6)\ S^{1/2}$$

$\Rightarrow S = \underline{\underline{0.027}} > 0.0027$ (( $\underline{\underline{\text{steep slope}}}$ ))

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at $y = 1.8$

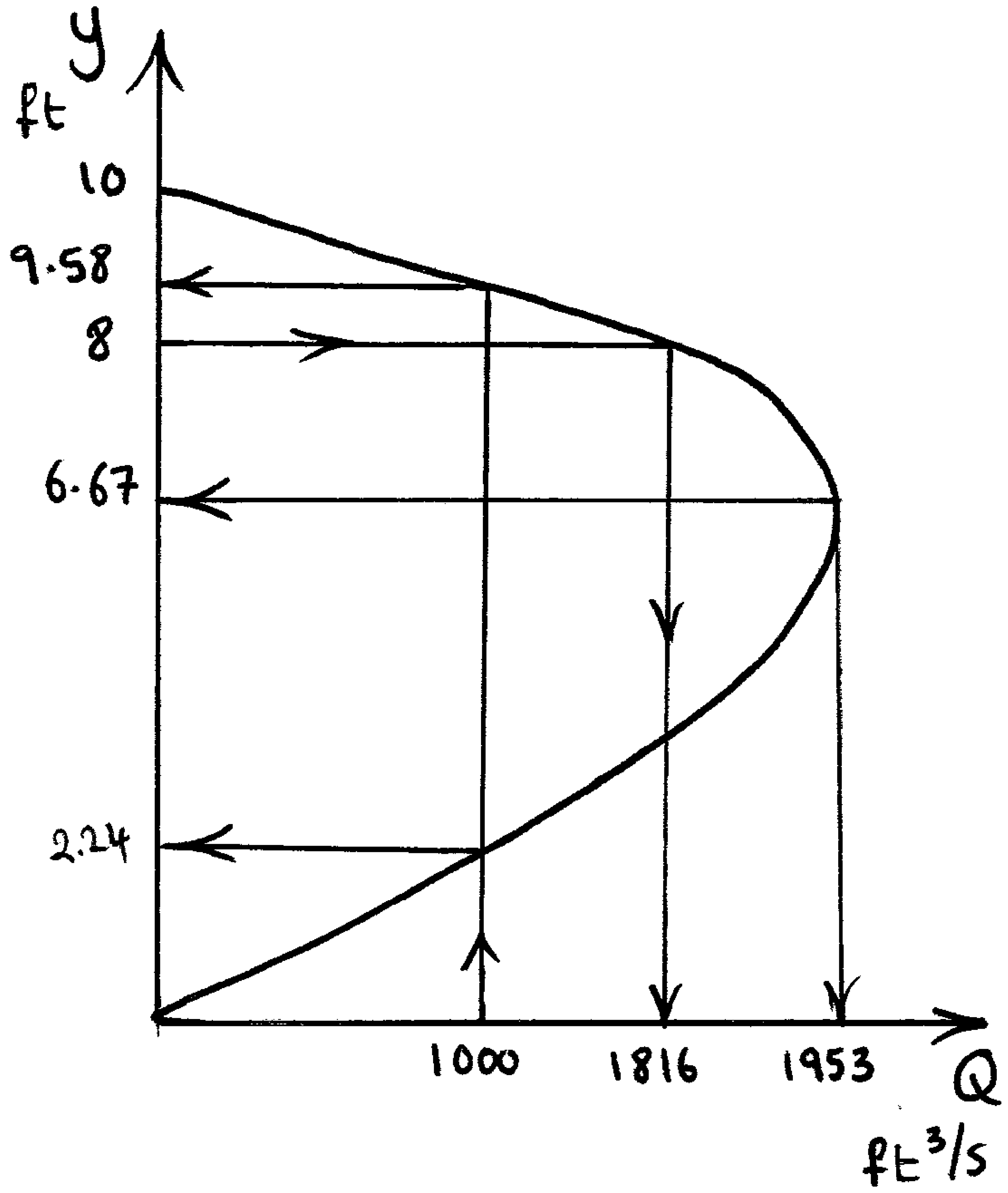
$A = 6\,(1.8) = 10.8$

$P = 6 + 2\,(1.8) = 9.6$

$$22 = \frac{1}{0.017}\left(\frac{10.8}{9.6}\right)^{2/3} S^{1/2}\ (10.8)$$

$S = \underline{\underline{0.001}} < S_c = 0.0027$ (( $\underline{\underline{\text{Mild slope}}}$ ))

**(8)**




$B = 20 \text{ ft}$

$E = 10 \text{ ft}$

$$E = y + \frac{Q^2}{2gA^2}$$

$$Q = A\sqrt{2g(E-y)}$$

$$Q = 20y\sqrt{2(32.2)(E-y)}$$

$$Q = 160.5\, y\sqrt{10-y}$$

a) $y_c = \underline{\underline{6.67 \text{ ft}}}$

b) $Q_{max} = \underline{\underline{1953.5 \text{ ft}^3/\text{s}}}$

c) $y = 8 \text{ ft} \longrightarrow Q = \underline{\underline{1816 \text{ ft}^3/\text{s}}}$

d) $Q = 1000 \text{ ft}^3/\text{s} \rightarrow y_1 = \underline{\underline{9.58 \text{ ft}}}$ , $y_2 = \underline{\underline{2.24 \text{ ft}}}$

e) $y_1$ = <u>Subcritical</u> , $y_2$ = <u>Supercritical</u>