<http://www.mathalino.com/reviewer/engineering-mechanics/resultant-of-concurrent-force-system>

**Resultant** of a force system is a force or a couple that will have the same effect to the body, both in translation and rotation, if all the forces are removed and replaced by the resultant.

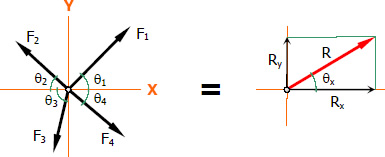
The equation involving the resultant of force system are the following

1. $ R_x = \Sigma F_x = F_{x1} + F_{x2} + F_{x3} + ... $  
   The x-component of the resultant is equal to the summation of forces in the x-direction.
2. $ R_y = \Sigma F_y = F_{x1} + F_{x2} + F_{x3} + ... $  
   The y-component of the resultant is equal to the summation of forces in the y-direction.
3. $ R_z = \Sigma F_z = F_{x1} + F_{x2} + F_{x3} + ... $  
   The z-component of the resultant is equal to the summation of forces in the z-direction.

Note that according to the type of force system, one or two or three of the equations above will be used in finding the resultant.

### Resultant of Coplanar Concurrent Force System

The line of action of each force in coplanar concurrent force system are on the same plane. All of these forces meet at a common point, thus concurrent. In x-y plane, the resultant can be found by the following formulas:



$ R_x = \Sigma F_x $

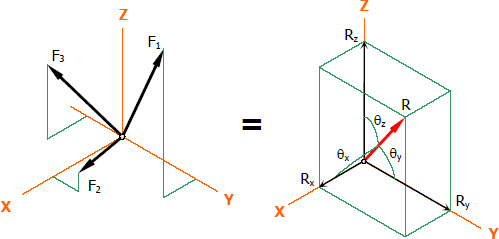
$ R_y = \Sigma F_y $

$ R = \sqrt{{R_x}^2 + {R_y}^2} $

$ \tan \theta_x = \dfrac{R_y}{R_x} $

### Resultant of Spatial Concurrent Force System

Spatial concurrent forces (forces in 3-dimensional space) meet at a common point but do not lie in a single plane. The resultant can be found as follows:



$ R_x = \Sigma F_x $

$ R_y = \Sigma F_y $

$ R_z = \Sigma F_z $

$ R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2} $

**Direction Cosines**

$ \cos \theta_x = \dfrac{R_x}{R} $

$ \cos \theta_y = \dfrac{R_y}{R} $

$ \cos \theta_z = \dfrac{R_z}{R} $

### Vector Notation of the Resultant

$ {\bf R} = \Sigma {\bf F} $  
$ {\bf R} = (\Sigma F_x) {\bf i} + (\Sigma F_y) {\bf j} + (\Sigma F_z) {\bf k} $

$ {\bf R} = R_x{\bf i} + R_y{\bf j} + R_z{\bf k} $

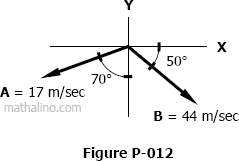
Where  
$ R_x = \Sigma F_x $ $ R_y = \Sigma F_y $ $ R_z = \Sigma F_z $

$ R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2} $

<http://www.mathalino.com/reviewer/engineering-mechanics/012-resultant-two-velocity-vectors>

### Problem 012

Find the resultant vector of vectors **A** and **B** shown in [Fig. P-012](http://www.mathalino.com/image/mech-012-two-downward-velocity-vectors).



### Solution 012: Component Method

$ R_x = \Sigma F_x $

$ R_x = 44 \cos 50^\circ - 17 \sin 70^\circ $

$ R_x = 12.31 \, \text{ m/sec to the right} $

$ R_y = \Sigma F_y $

$ R_y = -44 \sin 50^\circ - 17 \cos 70^\circ $

$ R_y = -27.89 \, \text{ m/sec} $

$ R_y = 39.52 \, \text{ m/sec downward} $

$ R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{12.31^2 + 39.52^2} $

$ R = 41.39 \, \text{ m/sec} $

$ \tan \theta_x = \dfrac{R_y}{R_x} = \dfrac{39.52}{12.31} $

$ \theta_x = 72.70^\circ $

The resultant vector R = 41.39 m/sec downward to the right at θx = 72.70°.

### Another Solution: Vector Method

$ {\bf A} = -17 \sin 70^\circ{\bf i} - 17 \cos 70^\circ{\bf j} $

$ {\bf A} = -15.97{\bf i} - 5.81{\bf j} \, \text{ m/sec} $

$ {\bf B} = 44 \cos 50^\circ{\bf i} - 44 \sin 50^\circ{\bf j} $

$ {\bf B} = 28.28{\bf i} - 33.70{\bf j} \, \text{ m/sec} $

$ {\bf R} = {\bf A} + {\bf B} $

$ {\bf R} = (-15.97 + 28.28){\bf i} + (-5.81 - 33.70){\bf j} $

$ {\bf R} = 12.31{\bf i} - 39.51{\bf j} \, \text{ m/sec} $

$ R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{12.31^2 + (-39.51)^2} $

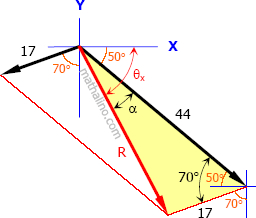
$ R = 41.39 \, \text{ m/sec} $(ok!)

$ \tan \theta_x = \dfrac{R_y}{R_x} = \dfrac{-39.52}{12.31} $

$ \theta_x = -72.70^\circ $

$ \theta_x = 72.70^\circ $downward to the right (ok!)

### Another Solution: Geometry Method

Cosine Law for the shaded triangle  
$ R^2 = 17^2 + 44^2 - 2(17)(44) \cos 70^\circ $

$ R = 41.39 \, \text{ m/sec} $(ok!)

By Sine Law  
$ \dfrac{R}{\sin 70^\circ} = \dfrac{17}{\sin \alpha} $

$ \sin \alpha = \dfrac{17 \sin 70^\circ}{R} = \dfrac{17 \sin 70^\circ}{41.39} $

$ \alpha = 22.70^\circ $

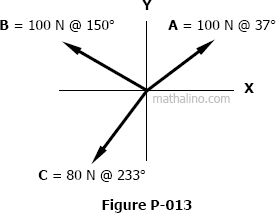
$ \theta_x = 50 + \alpha = 50 + 22.70^\circ $

$ \theta_x = 72.70^\circ $(ok!)

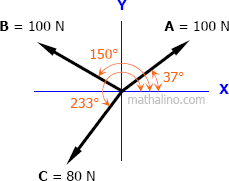
<http://www.mathalino.com/reviewer/engineering-mechanics/013-resultant-three-forces-angles-greater-90-degree>

### Problem 013

Three vectors **A**, **B**, and **C** are shown in the figure below. Find one vector (magnitude and direction) that will have the same effect as the three vectors shown in [Fig. P-013](http://www.mathalino.com/image/mech-013-three-concurrent-forces-absolute-directions) below.



### Solution 013

$ R_x = 100 \cos 37^\circ + 100 \cos 150^\circ + 80 \cos 233^\circ $

$ R_x = -54.88 \, \text{ N} $

$ R_x = 54.88 \, \text{ N to the left} $

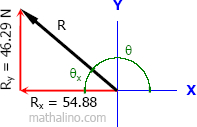
$ R_y = 100 \sin 37^\circ + 100 \sin 150^\circ + 80 \sin 233^\circ $

$ R_y = 46.29 \, \text{ N} $

$ R = \sqrt{{R_x}^2 + {R_y}^2} $

$ R = \sqrt{54.88^2 + 46.29^2} $

$ R = 71.79 \, \text{ N} $

$ \tan \theta_x = \dfrac{R_y}{R_x} $

$ \tan \theta_x = \dfrac{46.29}{54.88} $

$ \theta_x = 40.15^\circ $

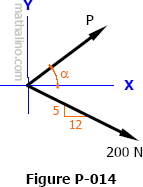
$ \theta = 180^\circ - \theta_x = 180^\circ - 40.15^\circ $

$ \theta = 139.85^\circ $

$ R = 71.79 \, \text{ N at } \, 139.85^\circ $ ***answer***

<http://www.mathalino.com/reviewer/engineering-mechanics/014-solving-force-given-resultant>

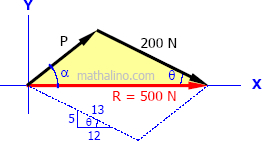
**Problem 014**

From Fig. P-014, P is directed at an angle α from x-axis and the 200 N force is acting at a slope of 5 vertical to 12 horizontal.

1. Find P and α if the resultant is 500 N to the right along the x-axis.
2. Find P and α if the resultant is 500 N upward to the right with a slope of 3 horizontal to 4 vertical.
3. Find P and α if the resultant is zero.

**Solution 014**

**Part a: The resultant is 500N to the right along the x-axis**

By Cosine law of the shaded triangle  
$ P^2 = 200^2 + 500^2 - 2(200)(500) \cos \theta $

$ P^2 = 200^2 + 500^2 - 2(200)(500)(\frac{12}{13}) $

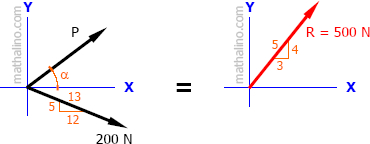
$ P = 324.63 \, \text{ N} $***answer***

By Sine law  
$ \dfrac{P}{\sin \theta} = \dfrac{200}{\sin \alpha} $

$ \sin \alpha = \dfrac{200 \sin \theta}{P} = \dfrac{200 (\frac{5}{13})}{324.63} $

$ \alpha = 13.71^\circ $***answer***

**Part b: The resultant is 500 N upward to the right with a slope of 3 horizontal to 4 vertical**



$ R_x = 500(\frac{3}{5}) = 300 \, \text{ N} $

$ R_y = 500(\frac{4}{5}) = 400 \, \text{ N} $

$ R_x = P \cos \alpha + 200(\frac{12}{13}) $

$ 300 = P \cos \alpha + 184.61 $

$ P \cos \alpha = 115.39 $

$ P = \dfrac{115.39}{\cos \alpha} $

$ R_y = P \sin \alpha - 200(\frac{5}{13}) $

$ 400 = P \sin \alpha - 76.92 $

$ P \sin \alpha = 476.92 $

$ \left( \dfrac{115.39}{\cos \alpha} \right)\sin \alpha = 476.92 $

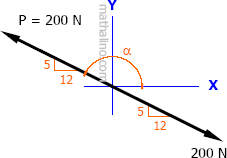
$ 115.38 \tan \alpha = 476.92 $

$ \tan \alpha = 4.1335 $

$ \alpha = 76.4^\circ $***answer***

$ P = \dfrac{115.39}{\cos 76.4^\circ} $

$ P = 490.68 \, \text{ N} $***answer***

**Part c: The resultant is zero**

The resultant is zero if P and the 200 N force are equal in magnitude, oppositely directed, and collinear.

$ \alpha = 180^\circ - \arctan \frac{5}{12} $

$ \alpha = 157.38^\circ $

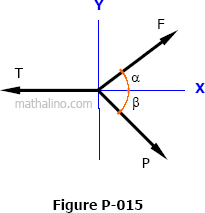
Thus, P = 200 N at α = 157.38° ***answer***

<http://www.mathalino.com/reviewer/engineering-mechanics/015-solving-force-and-its-angle-and-angle-two-forces-given-resultant>

### Problem 015

Forces F, P, and T are concurrent and acting in the direction as shown in [Fig. P-015](http://www.mathalino.com/image/mech-015-three-concurrent-forces).

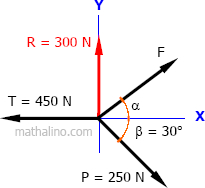
1. Find the value of F and α if T = 450 N, P = 250 N, β = 30Â°, and the resultant is 300 N acting up along the y-axis.
2. Find the value of F and α if T = 450 N, P = 250 N, β = 30Â° and the resultant is zero.
3. Find the value of α and β if T = 450 N, P = 250 N, F = 350 N, and the resultant is zero.



### Solution 015

**Part a: Unknown force and direction with non-zero resultant**

$ R_x = 0 $and $ R_y = 300 \, \text{ N} $

$ R_x = \Sigma F_x $

$ 0 = F \cos \alpha + 250 \cos 30^\circ - 450 $

$ F \cos \alpha = 233.49 $

$ F = \dfrac{233.49}{\cos \alpha} $

$ R_y = \Sigma F_y $

$ 300 = F \sin \alpha - 250 \sin 30^\circ $

$ F \sin \alpha = 425 $

$ \left( \dfrac{233.49}{\cos \alpha} \right) \sin \alpha = 425 $

$ \tan \alpha = 1.8202 $

$ \alpha = 61.22^\circ $***answer***

$ F = \dfrac{233.49}{\cos 61.22^\circ} $

$ F = 484.92 \, \text{ N} $***answer***

**Part b: Unknown force and direction with zero resultant**

$ R_x = 0 $and $ R_y = 0 $

$ R_x = \Sigma F_x $

$ 0 = F \cos \alpha + 250 \cos 30^\circ - 450 $

$ F \cos \alpha = 233.49 $

$ F = \dfrac{233.49}{\cos \alpha} $

$ R_y = \Sigma F_y $

$ 0 = F \sin \alpha - 250 \sin 30^\circ $

$ F \sin \alpha = 125 $

$ \left( \dfrac{233.49}{\cos \alpha} \right)\sin \alpha = 125 $

$ \tan \alpha = 0.5354 $

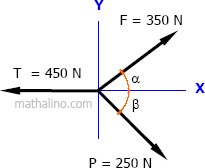
$ \alpha = 28.16^\circ $***answer***

$ F = \dfrac{233.49}{\cos 28.16^\circ} $

$ F = 264.85 \, \text{ N} $***answer***

**Part c: Unknown direction of two forces with zero resultant**

$ R_x = 0 $and $ R_y = 0 $

$ R_y = \Sigma F_y $

$ 0 = 350 \sin \alpha - 250 \sin \beta $

$ 7 \sin \alpha - 5 \sin \alpha = 0 $

$ 7 \sin \alpha = 5 \sin \beta $

$ 49 \sin^2 \alpha = 25 \sin^2 \beta $→ Equation (1)

$ R_x = \Sigma F_x $

$ 0 = 350 \cos \alpha + 250 \cos \beta - 450 $

$ 7 \cos \alpha + 5 \cos \beta - 9 = 0 $

$ 7 \cos \alpha = 9 - 5 \cos \alpha $

$ 49 \cos^2 \alpha = (9 - 5 \cos \alpha)^2 $

$ 49 \cos^2 \alpha = 81 - 90 \cos \beta + 25 \cos^2 \beta $→ Equation (2)

Equation (1) + Equation (2)  
$ 49 \sin^2 \alpha + 49 \cos^2 \alpha = 25 \sin^2 \beta + (81 - 90 \cos \beta + 25 \cos^2 \beta) $

$ 49(\sin^2 \alpha + \cos^2 \alpha) = 25(\sin^2 \beta + \cos^2 \beta) + 81 - 90 \cos \beta $

$ 49(1) = 25(1) + 81 - 90 \cos \beta $

$ 90 \cos \beta = 25 + 81 - 49 $

$ \cos \beta = \frac{57}{90} $

$ \beta = 50.70^\circ $***answer***

From Equation (1)  
$ 49 \sin^2 \alpha = 25 \sin^2 50.70^\circ $

$ 7 \sin \alpha = 5 \sin 50.70^\circ $

$ \sin \alpha = \frac{5}{7} \sin 50.70^\circ $

$ \alpha = 33.56^\circ $***answer***