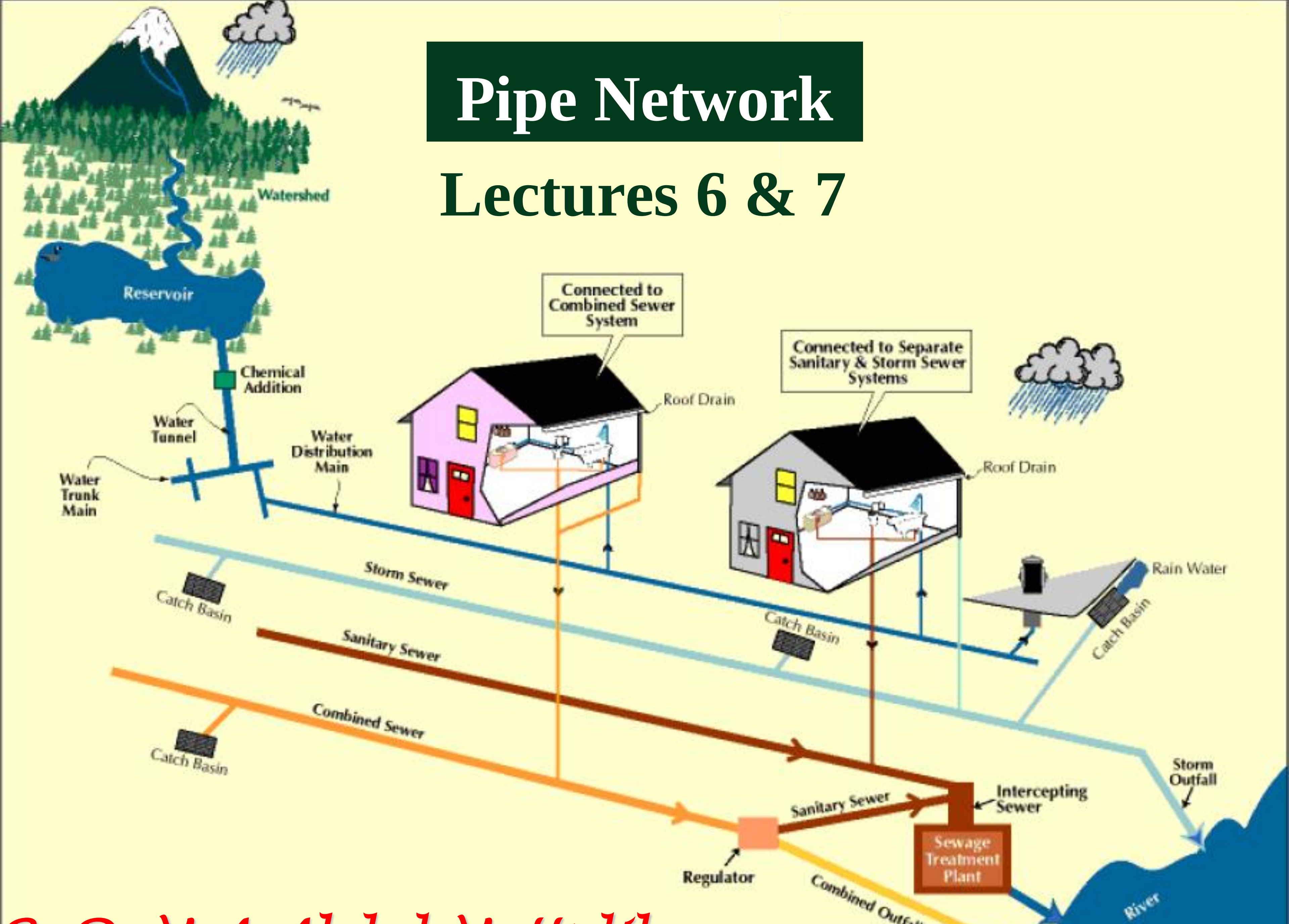
Pipe Network

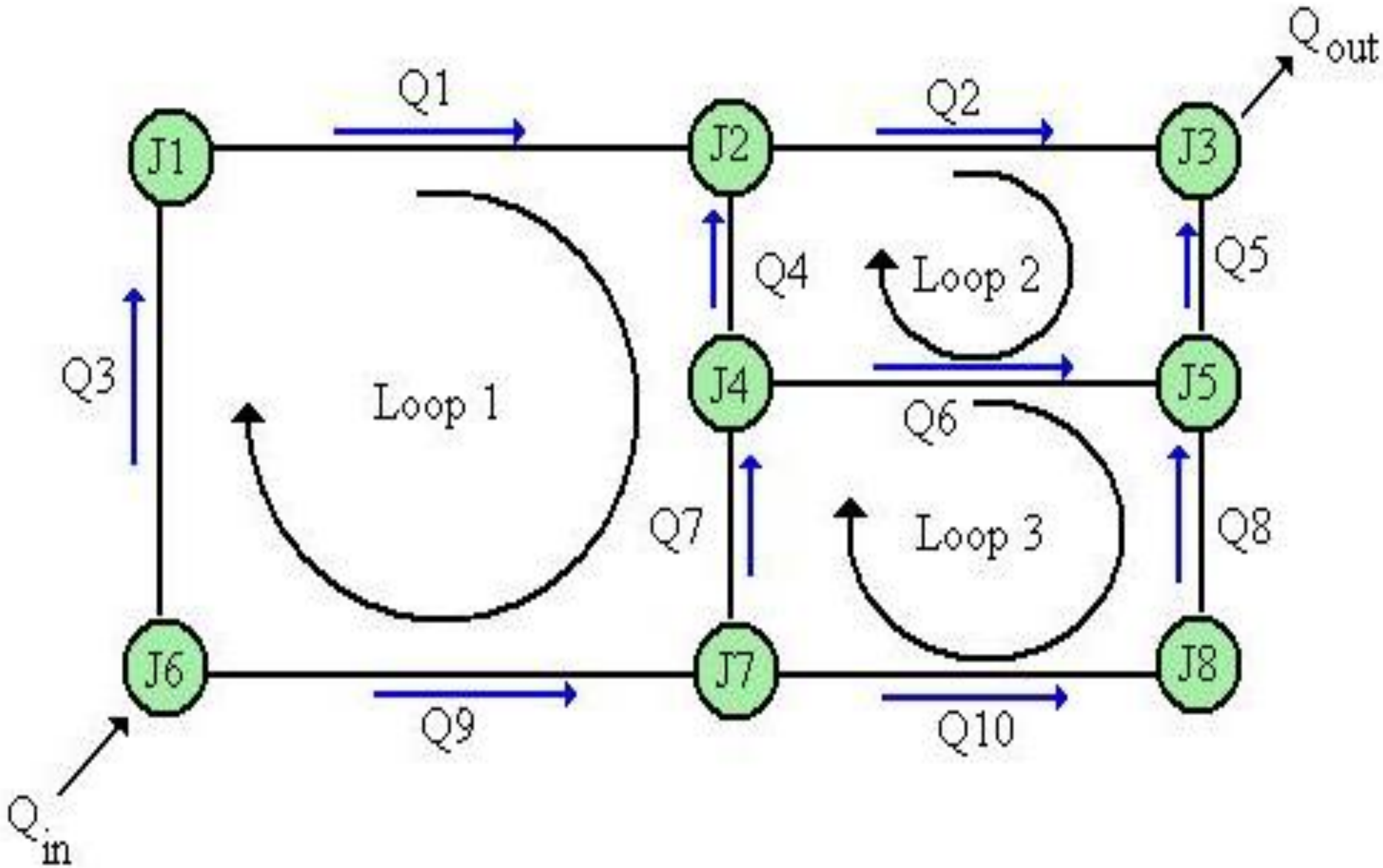
Lectures 6 & 7



By Dr.M. A. Abdul Muttalib

Lecture 3

Pipe Network



Pipe Network

- **A water distribution system consists of complex interconnected pipes, serviced from reservoirs and/or pumps, which deliver water from the treatment plant to the consumer.**

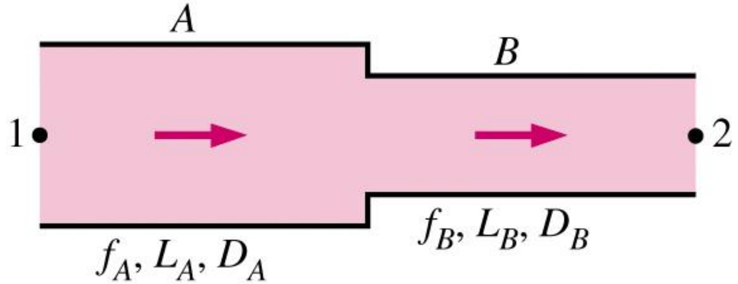
- **Pipe network analysis involves the determination of the pipe flow rates and pressure heads at the outflows points of the network. The flow rate and pressure heads must satisfy the continuity and energy equations.**

Pipe Network

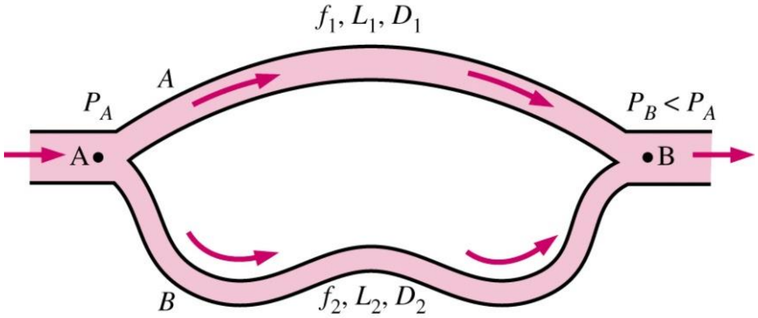
- **The earliest systematic method of network analysis (Hardy-Cross Method) is known as the head balance or closed loop method. This method is applicable to system in which pipes form closed loops. The outflows from the system are generally assumed to occur at the nodes junction.**
- **At each junction these flows must satisfy the continuity criterion, i.e., the algebraic sum of the entered flow rates equal to the algebraic sum of the exit flow rates**

Pipe Network

- Two general types of networks
  - Pipes in series
    - Volume flow rate is constant
    - Head loss is the summation of parts
  - Pipes in parallel
    - Volume flow rate is the sum of the components
    - Pressure loss across all branches is the same



$$\dot{V}_A = \dot{V}_B$$
$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$

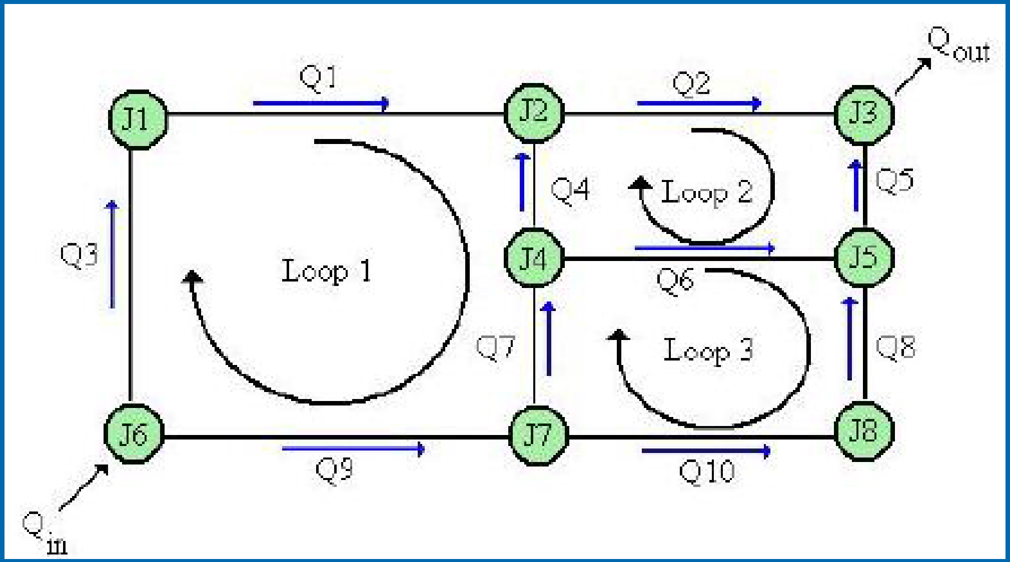


$$h_{L,\,1} = h_{L,\,2}$$
$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

Hardy Cross Method (HCM)

Example of ΔQ method

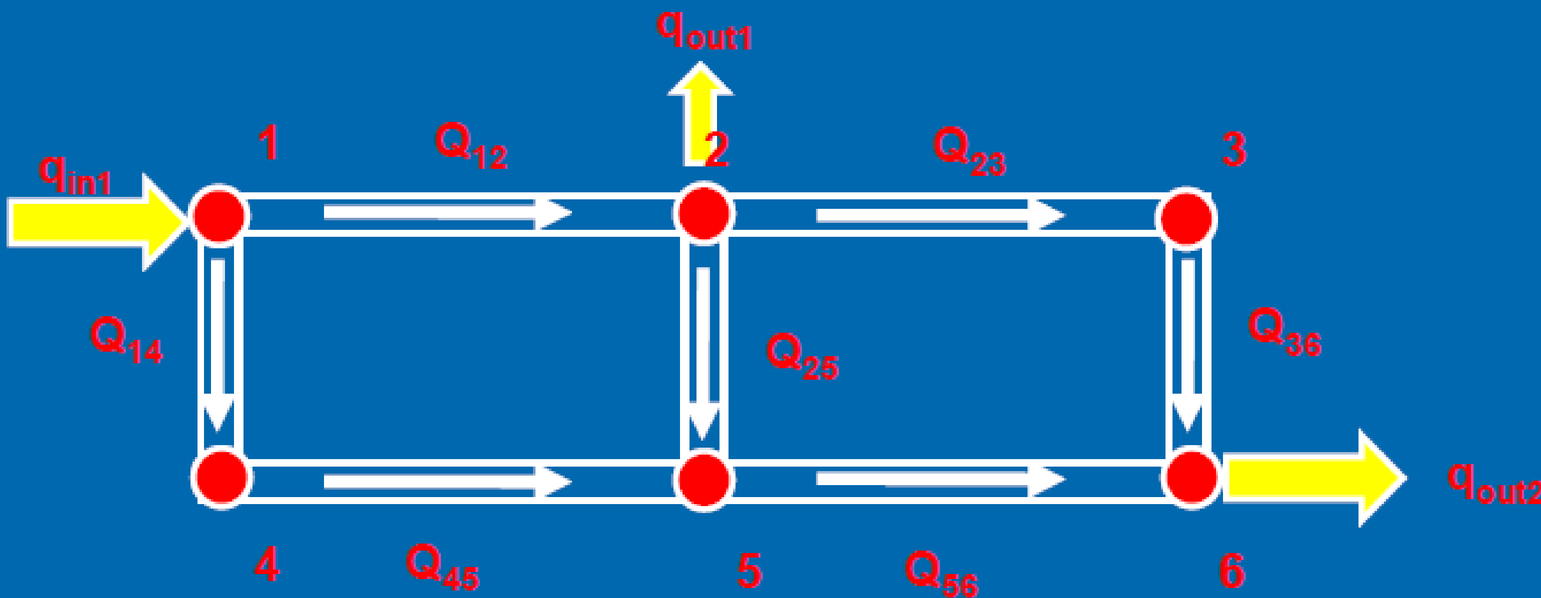
Hardy Cross 1936



The method was first published in November 1936 by Hardy Cross, a structural engineering professor at the University of Illinois at Urbana–Champaign. The Hardy Cross method is an adaptation of the Moment distribution method, which was also developed by Hardy Cross as a way to determine the moments in indeterminate structures.

Governing Equations

a. Conservation of Mass (Nodal Equations)



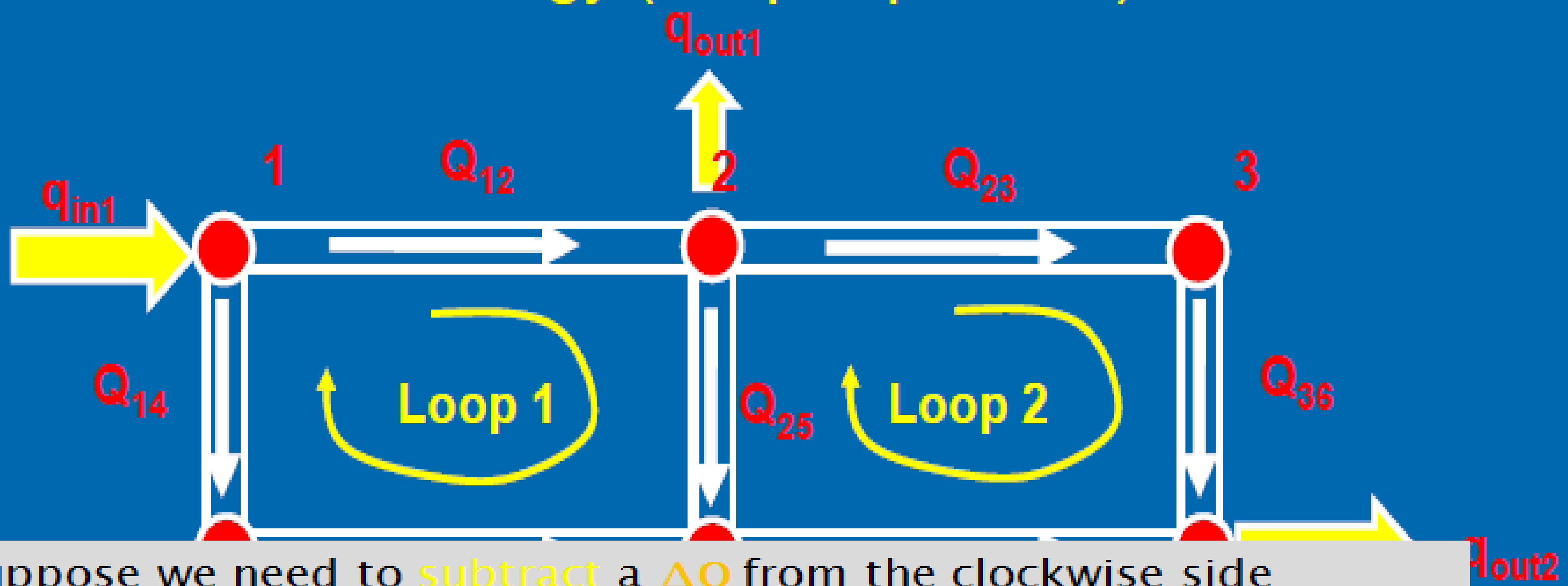
Hardy Cross Method requires an initial guesses of all pipe flows under the condition that such guesses satisfy the conservation of mass at each node.

For example: Each **Node** We Could Write a Mass Conservation Equation:

Example @ Node 2: $Q_{12}=q_{out1}+Q_{23}+Q_{25}$

Governing Equations

b. Conservation of Energy (Loop Equations)



- Suppose we need to subtract a ΔQ from the clockwise side and add it to the other side for balancing head losses. Then

$$\sum k_c(Q_c - \Delta Q)^2 = \sum k_{cc}(Q_{cc} + \Delta Q)^2$$

- After applying Taylor`s series expansion & math manipulations for the above relation:

$$\Delta Q = \frac{\sum h_{fc} - \sum h_{fcc}}{2\left(\sum \frac{h_{fc}}{Q_c} + \sum \frac{h_{fcc}}{Q_{cc}}\right)}$$

$$\Delta Q = -\frac{\sum K_i Q_i^2}{\sum |2 K_i Q_i|}$$

Solution Steps Using HCM

Solution of Pipe Network via HCM is iterative as follow:

1. Consider a positive flow direction for all loops (say clock wise direction is positive;
2. Assume flow discharges for all pipes satisfying the mass conservation at each node;
3. Calculate a first approximation of the flow correction for each loop using the following equation given by Hardy Cross:

$$\Delta Q = -\frac{\sum K_i Q_i^2}{\sum |2K_i Q_i|}$$

4. Calculate the corrected Q and iterate till corrections vanish.

خطوات الحل

١- نقسم الشبكة إلى loops

٢- التأكد من أن الداخل إلى الشبكة يساوي الخارج منها

٣- فرض Q في كل فرع من أفرع الشبكة (المواسير) مع مراعاة
أن $\Sigma Q = 0$ عند أي junction

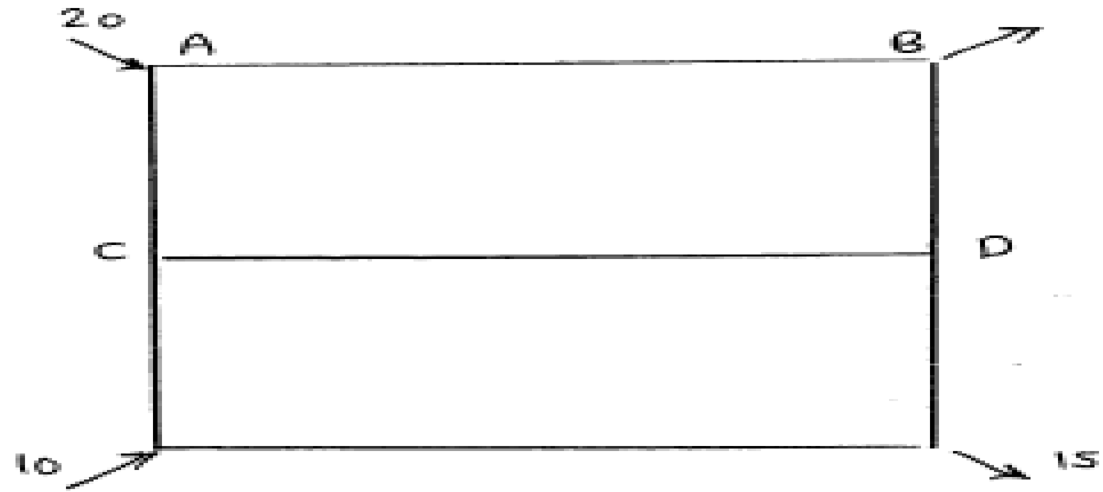
٤- حساب ΔQ من القانون السابق

٥- إذا كانت ΔQ تساوي صفر ⇐ الفرض صحيح
إذا كانت ΔQ لا تساوي صفر ⇐ نضيف ΔQ بإشارتها على Q
الموجودة في المواسير

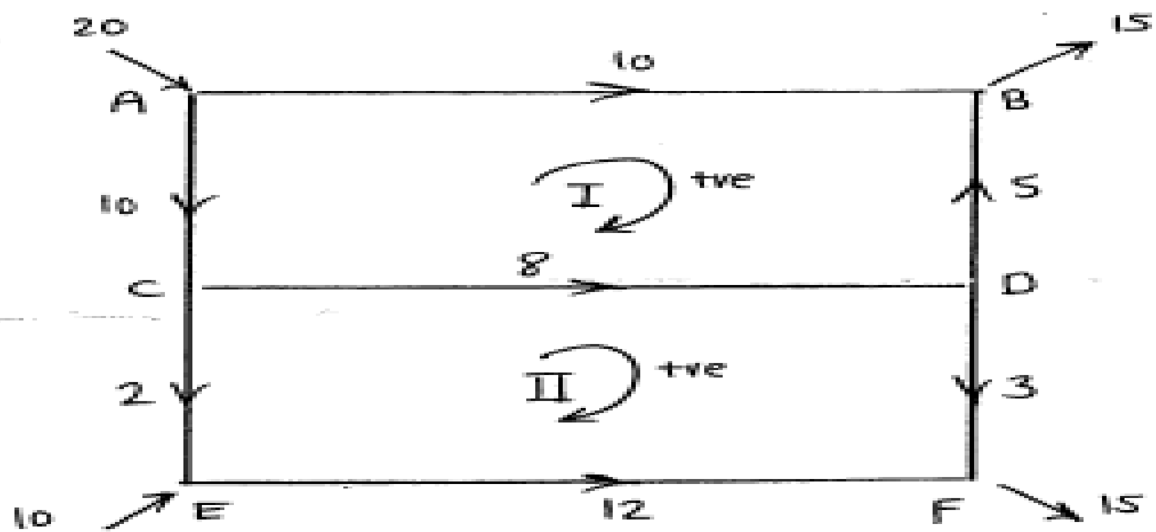
٦- نرسم الشبكة مرة أخرى بعد التصحيح (بعد إضافة ΔQ) مع مراعاة
أكثر من تصحيح لبعض المواسير [مع عقارب الساعة +ve ↻]

٧- حساب ΔQ جديدة في المواسير وهكذا حتى تقترب من الصفر
أو الاكتفاء بـ 2 trials

Example



Solution



$\Sigma Q_{in} = \Sigma Q_{out}$ ✓

ΣQ at any junction $= 0$ ✓

loop 1

Pipe	F	L	D	K	Q	KQ^n	$\lvert KQ^{n-1} \rvert$
AB					+10	+	+
BD					−5	−	+
DC					−8	−	+
CA					−10	−	+
						ΣKQ^n	ΣKQ^{n-1}

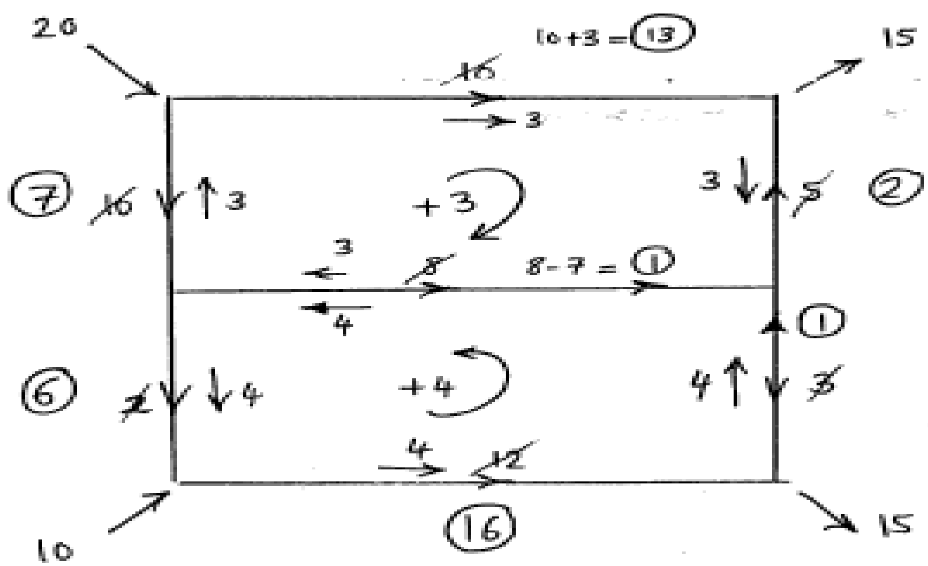
$$\Delta Q = \frac{-\Sigma KQ^n}{n \, \Sigma \lvert KQ^{n-1} \rvert}$$

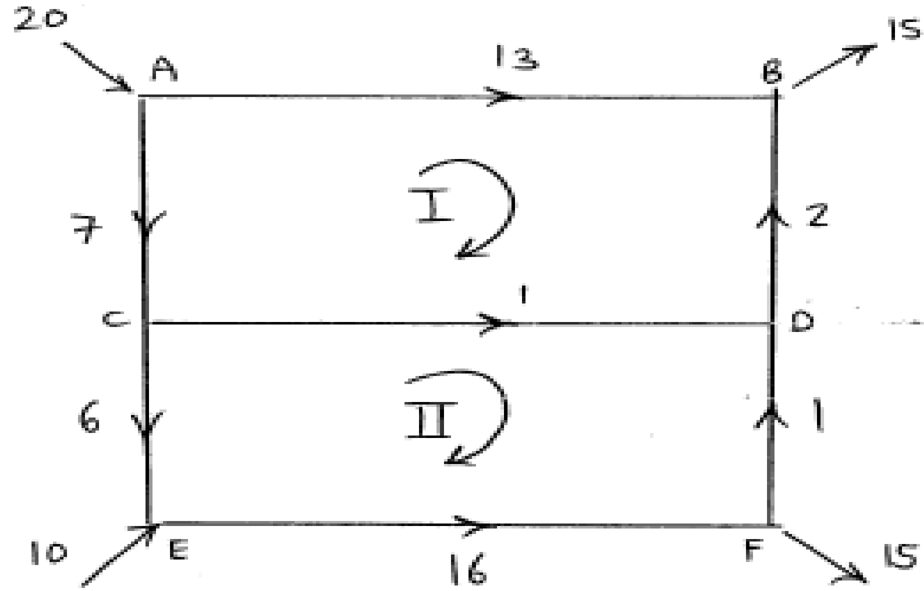
loop 2

Pipe	F	L	D	K	Q	KQ^n	$\lvert KQ^{n-1}\rvert$
CD					+8	+	+
DF					+3	+	+
FE					−12	−	+
CE					−2	−	+
						ΣKQ^n	$\Sigma\lvert KQ^{n-1}\rvert$

$$\Delta Q_2 = \frac{-\Sigma KQ^n}{n\,\Sigma\lvert KQ^{n-1}\rvert}$$

Say $\Delta Q_1 = +3$, $\Delta Q_2 = -4$ تصحيح الشبكة





2nd trial

loop	pipe	K	Q	KQ^n	$\|KQ^{n-1}\|$
I					

$\Delta Q_1 = ✓$

loop	pipe	K	Q	KQ^n	$\|KQ^{n-1}\|$
II					

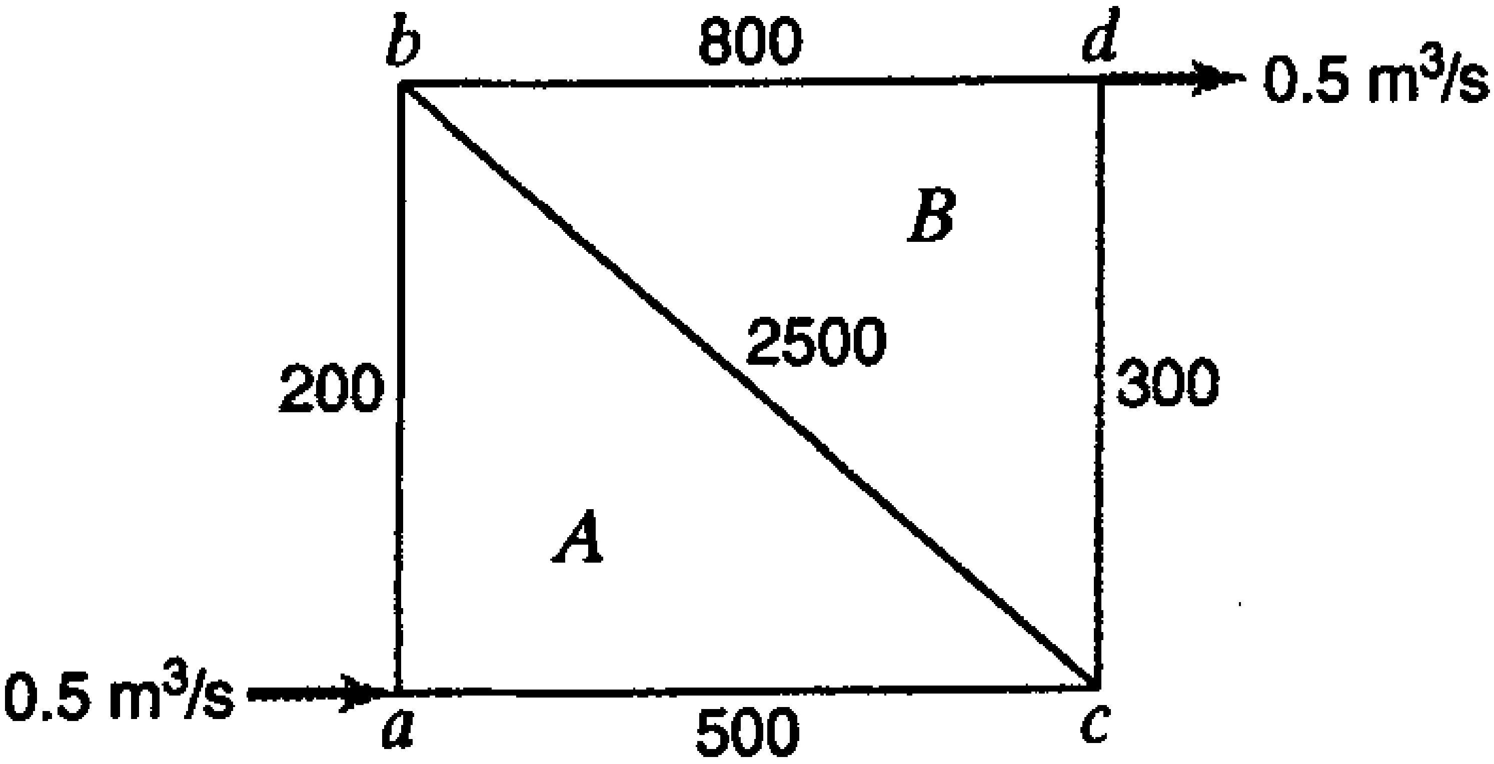
$\Delta Q_2 = ✓$

recorrect

Example

Find the magnitude and direction of the flow in network lines ab and bc (Fig. P8.118) after making two sets of corrections. The numbers on the figure are the K values of each line; take $n = 2.0$. Start by assuming initial flows as follows: 0.3 m^3/s in lines ab and cd, 0.2 m^3/s in lines ac and bd, and 0.1 m^3/s in line bc.

Example (cont.)



Example (cont.)

Given initial assumptions:

Loop A

First approx:

Pipe	K	Q	KQ^2	$\lvert 2KQ \rvert$
ab	200	+0.3	+18.0	120
bc	2500	+0.1	+25.0	500
ac	500	−0.2	−20.0	200
			+23.0	820

$$\Delta Q = -(+23/820) = -0.028$$

Example (cont.)

<u>Loop B</u>

Pipe	K	Q	KQ^2	$\lvert 2KQ \rvert$
bd	800	+0.2	+32.0	320
bc	2500	−0.1	−25.0	500
cd	300	−0.3	−27.0	180
			−20.0	1000

$$\Delta Q = -(-20.0/1000) = +0.020$$

Example (cont.)

Second approx. (after first corrections):

ab	200	+0.272	+14.80	109
bc	2500	+0.052	+6.76	260
ac	500	−0.228	−25.99	228
			−4.43	597

$$\Delta Q = -(-4.43/597) = +0.007$$

Example (cont.)

bd	800	+0.22	+38.72	352
bc	2500	−0.052	−6.76	260
cd	300	−0.28	−23.52	168
			+8.44	780

$$\Delta Q = -(+8.44/780) = -0.011$$

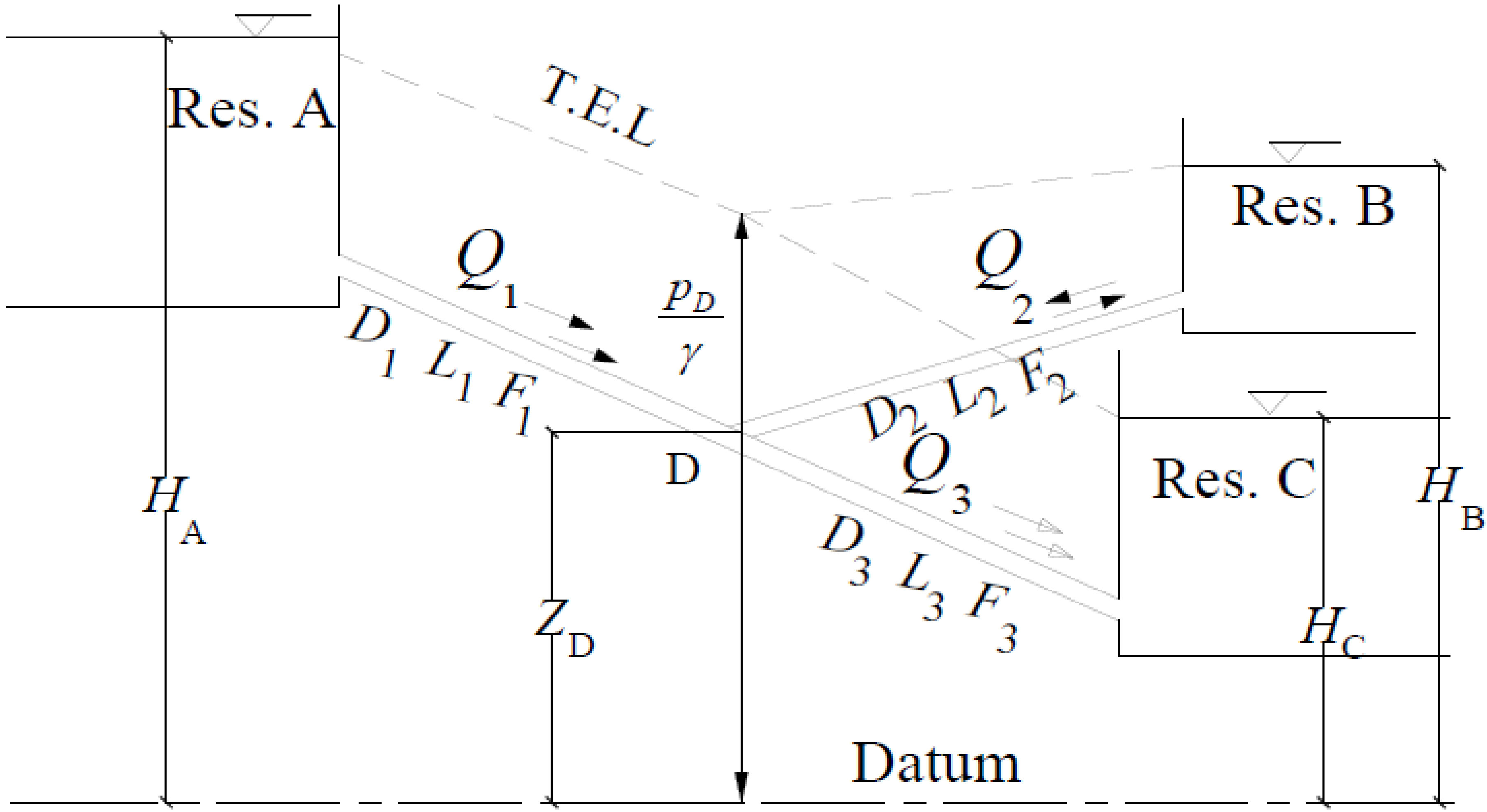
After second corrections: Flow in line *ab* = +0.272 + 0.007 = 0.279 m^3/s from *a* to *b*

Flow in line *bc* = +0.052 + 0.007 − (−0.011) = 0.070 m^3/s from *b* to *c* ◀

Three-Reservoir Problem

- Type I: Branching pipelines from a three-reservoir system asking for head loss calculations and water levels downstream
- Type II : Three reservoirs connected by pipelines asking for discharge calculations, given the pipe configuration and water level
- Type III: Three reservoirs connected by pipelines asking for sizing of pipes to meet the discharge goal

Three-Reservoir Problem cont.



Three-Reservoir Problem cont.

We have three cases:

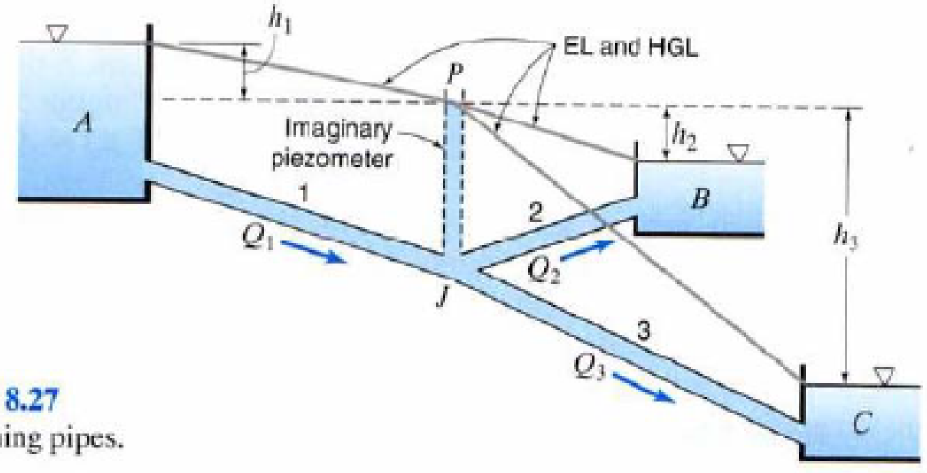


Figure 8.27
Branching pipes.

1- Flow from A,B to C

$$Q_1 + Q_2 = Q_3$$

2- Flow from A to B,C

$$Q_1 = Q_2 + Q_3$$

3- Flow from A to C

$Q_1 = Q_3$ and $Q_2 = 0$ (H.G.L at BJ horizontal)

The slope of the hydraulic gradient line gives the direction of flow

Three-Reservoir Problem cont.

Given elevations of three tanks and pipes that lead from three or more tanks to a common junction *J*
It is required to determine the discharge Q_1, Q_2, Q_3 (or Q_A , Q_B , Q_C) both magnitude and direction

Assumptions 1-minor losses are neglected
2-velocity head is neglected, i.e. T.E.L =H.G.L

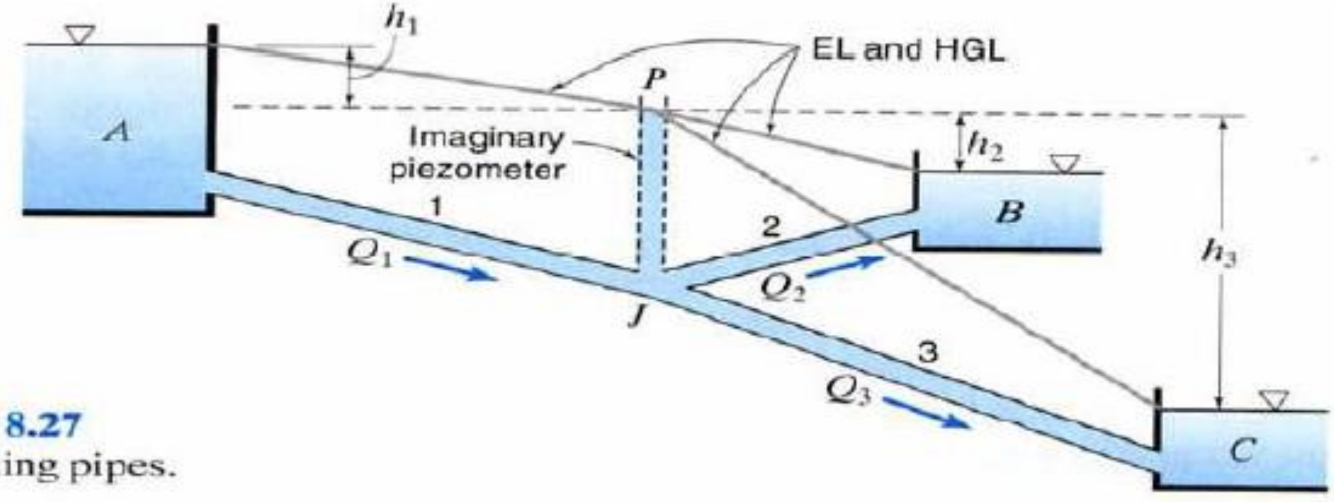


Figure 8.27
Branching pipes.

Three-Reservoir

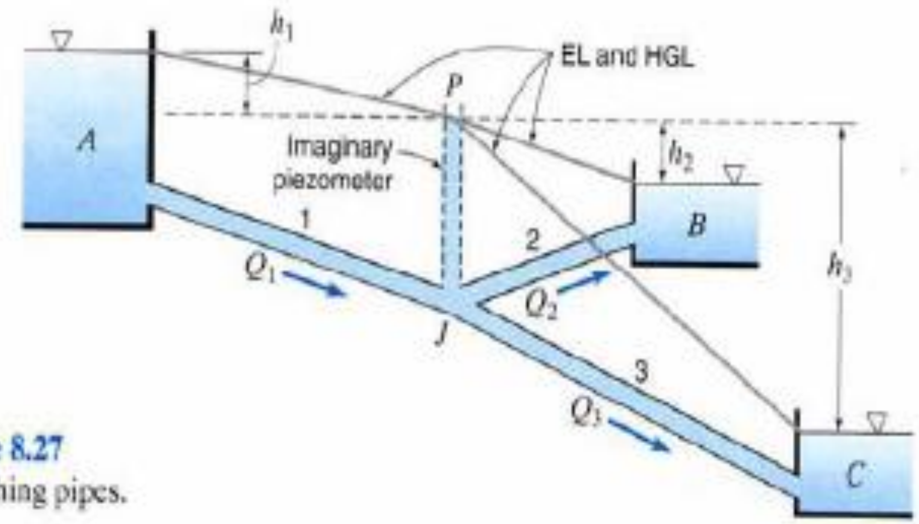


Figure 8.27
Branching pipes.

Case I

Flow from A to both B and C

$$h_F = K\,Q^2 \quad \text{where} \quad K = \frac{f\,L}{2gDA^2}$$

$$Z_A - K_A\,Q_A^2 = Z_C + K_C\,Q_C^2 \qquad (1)$$

$$Z_A - K_A\,Q_A^2 = Z_B + K_B\,Q_B^2 \qquad (2)$$

$$Q_A = Q_B + Q_C \qquad (3)$$

Three-Reservoir cont.

Case II

Flow from A and B to C

$$Z_A - K_A Q_A^2 = Z_C + K_C Q_C^2 \qquad (1)$$

$$Z_A - K_A Q_A^2 = Z_B - K_B Q_B^2 \qquad (2)$$

$$Q_A = Q_C - Q_B \qquad (3)$$

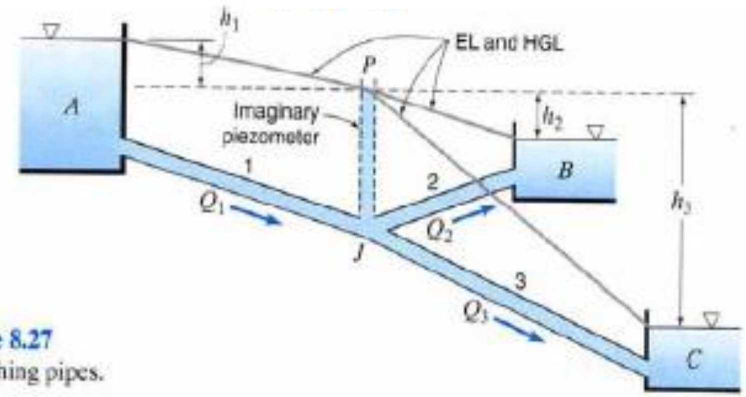
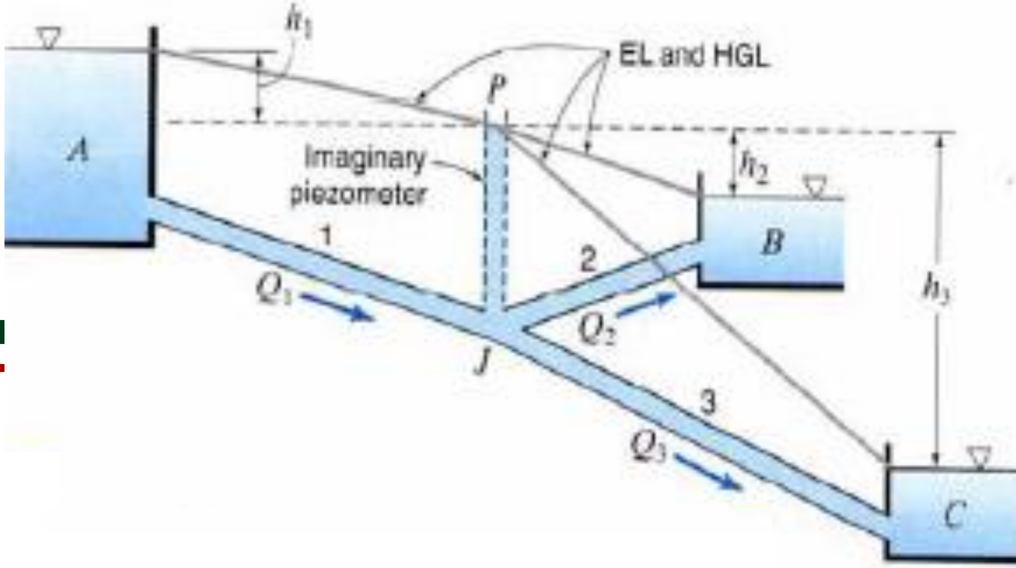


Figure 8.27
Branching pipes.

Three-Reservoir



Case III

Flow from A into C with no inflow or outflow from B, i.e. $Q_B = 0.0$

$$Z_A - K_A Q_A^2 = Z_C + K_C Q_C^2 \qquad (1)$$

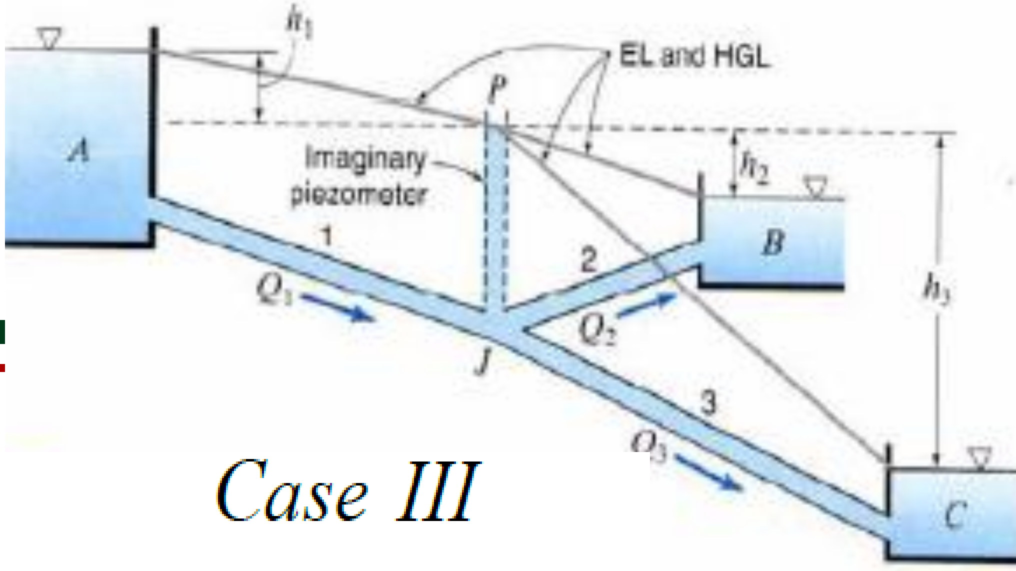
$$Z_A - K_A Q_A^2 = Z_B \qquad (2)$$

$$Q_A = Q_C \qquad (3)$$

Only one of the above cases (based on the physical flow picture) can be satisfied.

- Start with case III get Q_A and Q_C from (1) & (2).
- Check the value of Q_A and Q_C.

Three-Reservoir



IF	$Q_A = Q_C$	*Case III*
	$Q_A > Q_C$	*Case I*
	$Q_A < Q_C$	*Case II*

Having decided about the case, the direction is known and the identified set of equations can be used to solve (by trial and error) to determine the discharges Q_A ,Q_B, Q_C .

Assume junction pressure elevation i.e. elevation of point P.

If case I (P in between B and A)

If case II (P in between B and C)

Three-Reservoir Problem cont.

- These types of problems are most conveniently solved by trial and error method.

Step 1: Not knowing the discharge in each pipe, we may first assume a piezometric surface elevation, P, at the junction.

Step 2: This assumed elevation gives the head losses h_{f1}, h_{f2}, h_{f3} for each of the three pipes.

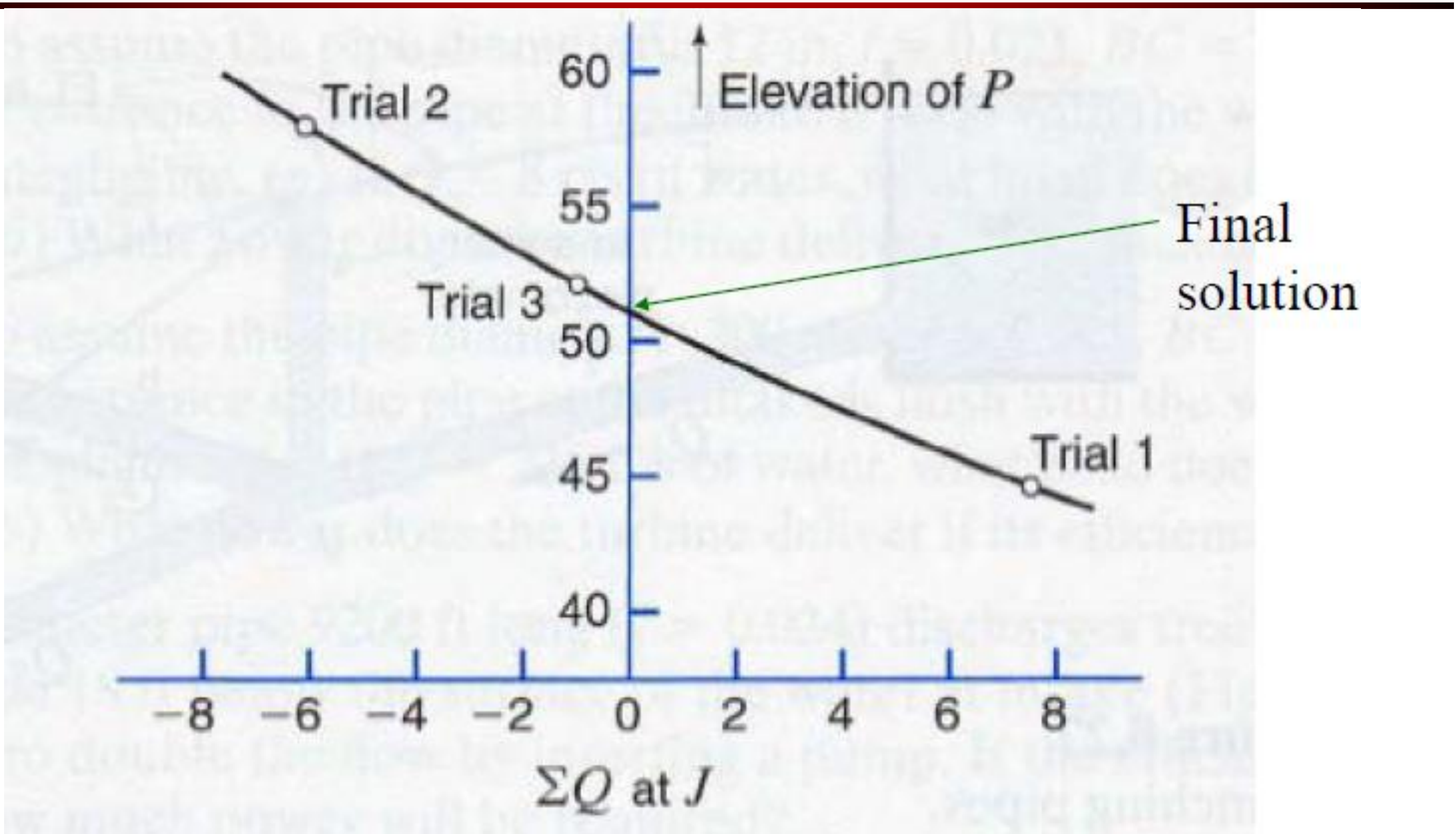
Then, the trial computation gives a set of values for discharges Q_1, Q_2, and Q_3. Check on the mass balance to see if $Q_1 = Q_2 + Q_3$ (Case1) or $Q_1 + Q_2 = Q_3$ (Case2) would be satisfied.

Three-Reservoir Problem cont.

Step 3: If not, adjust the initial guess of the assumed elevation (P) and retry it in an iterative procedure.

Step 4: If the assumed elevation is correct, the conservation of mass flow rate would hold at the junction eventually.

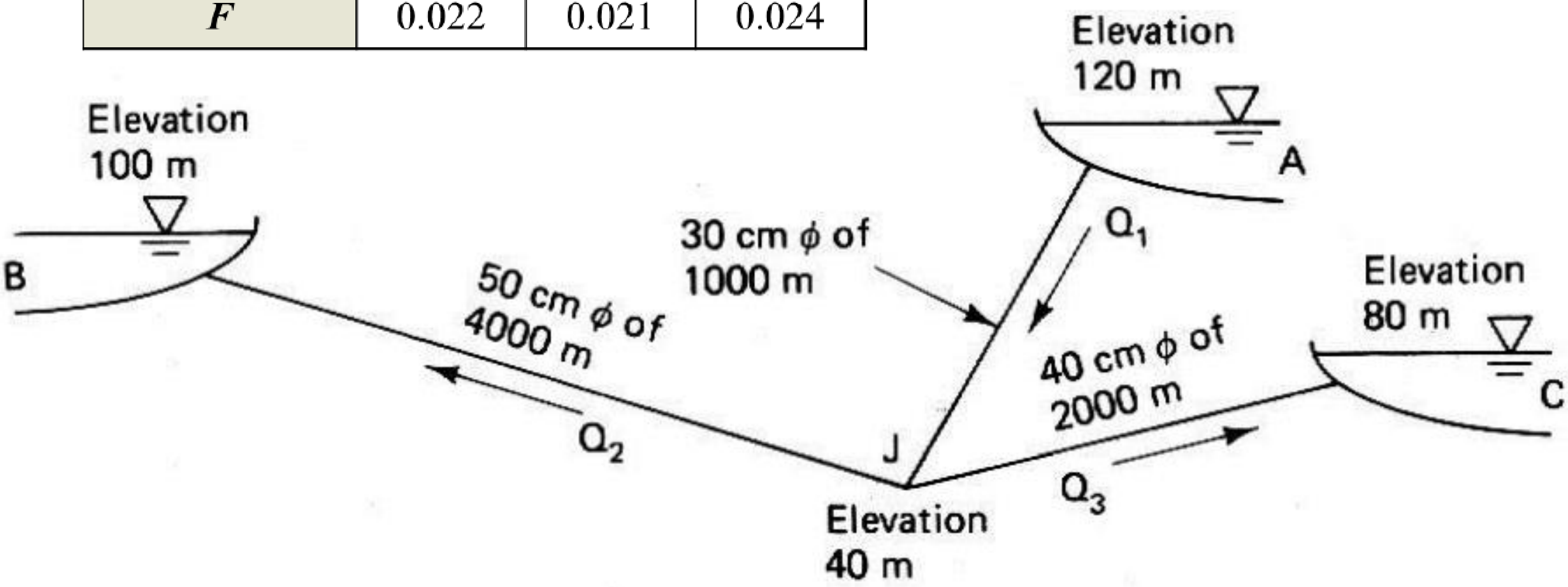
Three-Reservoir Problem cont.



Example 1

In the following figure determine the flow in each pipe.

Pipe	CJ	BJ	AJ
Length m	2000	4000	1000
Diameter cm	40	50	30
F	0.022	0.021	0.024



Example 1 cont.

Solution

Trial 1

$Z_P = 110m$

Applying Bernoulli Equation between A , J :

$$Z_A - Z_P = F_1 \frac{L_1}{D_1} \cdot \frac{V_1^2}{2g} \longrightarrow 120 - 110 = 0.024 \times \frac{1000}{0.3} \times \frac{V_1^2}{2g}$$

V1 = 1.57 m/s , Q1 = 0.111 m3/s

Applying Bernoulli Equation between B , J :

$$Z_P - Z_B = F_2 \frac{L_2}{D_2} \cdot \frac{V_2^2}{2g} \longrightarrow 110 - 100 = 0.021 \times \frac{4000}{0.5} \times \frac{V_2^2}{2g}$$

V2 = 1.08 m/s , Q2 = - 0.212 m3/s

Example 1 cont.

Applying Bernoulli Equation between C , J :

$$Z_P - Z_C = F_3 \frac{L_3}{D_3} \cdot \frac{V_3^{\,2}}{2g} \longrightarrow 110 - 80 = 0.022 \times \frac{2000}{0.4} \times \frac{V_3^2}{2g}$$

V3 = 2.313 m/s , Q2 = - 0.291 m3/s

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.111 - 0.212 - 0.291 = -0.392 \neq 0$$

Example 1 cont.

Trial 2

$Z_P = 100m$

Q1 = 0.157
Q2 = 0
Q3 = -0.237

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.157 + 0 - 0.237 = -0.08\, m^3 / s \neq 0$$

Trial 3

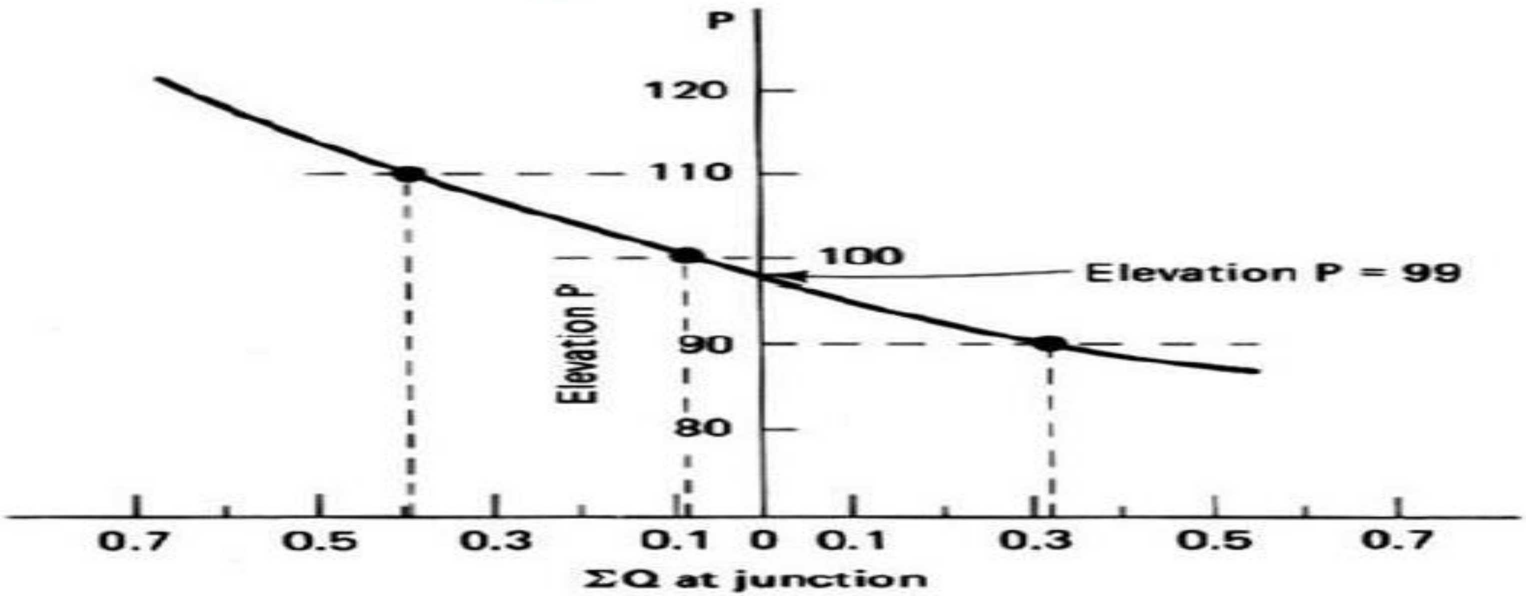
$Z_P = 90m$

Q1 = 0.192
Q2 = 0.3
Q3 = -0.168

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.192 + 0.3 - 0.168 = 0.324\, m^3 / s \neq 0$$

Example 1 cont.

Draw the relationship between $\sum Q$ and P



$$\therefore \sum Q = 0 \Rightarrow \text{at P} = 99\text{m}$$

Q1 = 0.160
Q2 = 0.067
Q3 = -0.231

$$\sum Q = Q_1 + Q_2 + Q_3 = 0.16 + 0.067 - 0.0231 = 0.004\ m^3/s \approx 0$$

Example 2

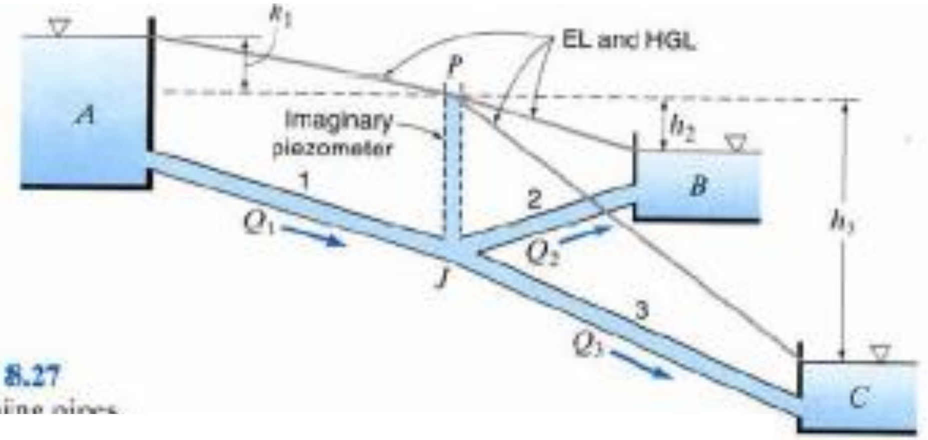


Figure 8.27
Branching pipes

Given the system

- Pipe 1 is 6,000 ft of 15 in diameter,
- Pipe 2 is 1,500 ft of 10 in diameter,
- Pipe 3 is 4,500 ft of 8 in diameter, all **asphalt-dipped cast iron.**
- The elevations of the water surfaces in reservoirs A and C are 250 ft and 160 ft, respectively,
- The discharge Q_2 of 60°F water into reservoir B is 3.3 cfs.

- Find the surface elevation of reservoir B.

Example 2 cont.

- For water at 60 °F: $\nu = 12.17 \times 10^{-6}$ ft^2/sec

Pipe	1	2	3
L (ft)	6000	1500	4500
D (ft)	1.25	10/12	8/12
e (ft)	0.0004	0.0004	0.0004
L/D	4800	1800	6750
A ft^2	1.227	0.545	0.349
e/D	0.00032	0.00048	0.0006

Example 2 cont.

Pipe Material	e(mm)	e(ft)
Glass, drawn brass, copper (new)	0.0015	0.000005
Seamless commercial steel (new)	0.004	0.000013
Commercial steel (enamel coated)	0.0048	0.000016
Commercial steel (new)	0.045	0.00015
Wrought iron (new)	0.045	0.00015
Asphalted cast iron (new)	0.12	0.0004
Galvanized iron	0.15	0.0005
Cast iron (new)	0.26	0.00085
Wood Stave (new)	0.18 ~ 0.9	0.0006 ~ 0.003
Concrete (steel forms, smooth)	0.18	0.0006
Concrete (good joints, average)	0.36	0.0012
Concrete (rough, visible, form marks)	0.60	0.002
Riveted steel (new)	0.9 ~ 9.0	0.003-0.03
Corrugated metal	45	0.15

Example 2 cont.

- Find the elevation of P by trial and error.
- Elevation of P lies between 160 and 250 ft. Calculate V or Q out of reservoir A and V or Q into reservoir C, and the surface elevation at reservoir B

Example 2 cont.

- Find the elevation of P by trial and error.
- Elevation of P lies between 160 and 250 ft. Calculate V

Elev. P	h_1	h_3	V_1	V_3	Q_1	Q_3	ΣQ	Move P
200	50	40	6.444	4.481	7.907	1.564	+3.04	Up
230	20	70	4.013	5.984	4.925	2.088	-0.463	Down

- Interpolation: (230-P)/(230-200) =0.463/(0.463+3.04)

  P= 226.03

  Q_2=3.3 cfs

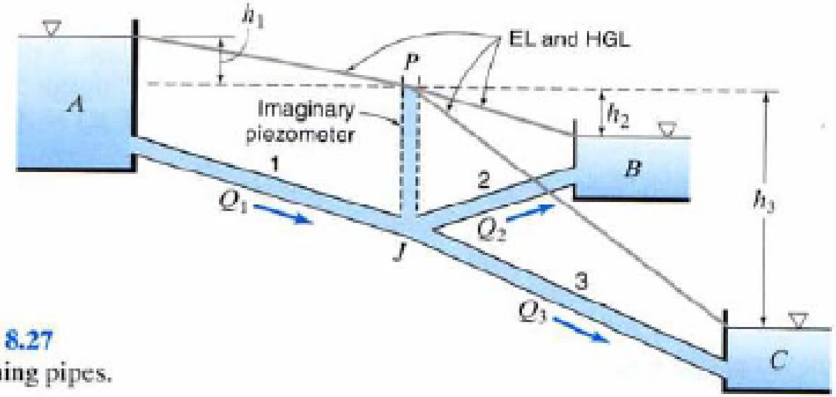


Figure 8.27
Branching pipes.

Example 2 cont.

Elev. P	h_1	h_3	V_1	V_3	Q_1	Q_3	ΣQ	Move P
226	24	66	4.412	5.805	5.414	2.026	+0.088	Up

- $V_2 = Q_2/A_2 = 3.3/0.545 = 6.055$ fps
- $R_2 = D_2V_2/\nu = 416{,}500$
- $f_2 = 0.01761$ in Moody Diagram
- So, $h_2 = 18.05$ft
- Elevation of B = Elevation P-h_2 = 226.64- 18.05 = 208.59 ft

Example 3

With the sizes, lengths, and material of pipes given in last example, suppose that the surface elevations of reservoirs A, B, and C are 525 ft, 500 ft, and 430 ft, respectively.

- (a) Does the water enter or leave reservoir B?
- (b) Find the flow rates of 60°F water in each pipe.

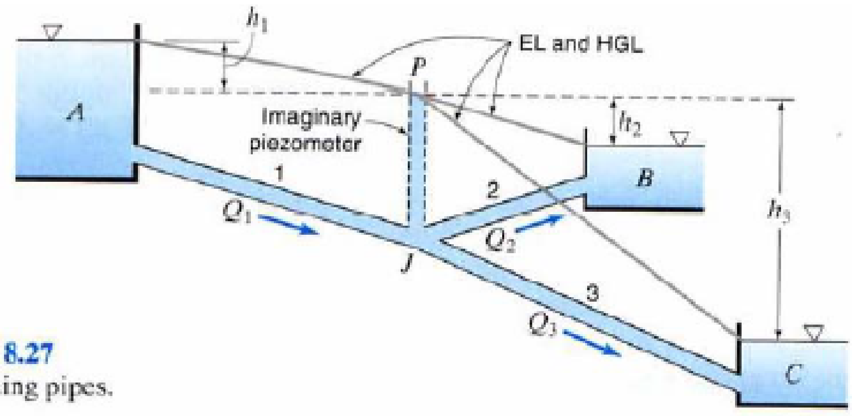


Figure 8.27
Branching pipes.

Example 3 cont.

(a) Assume P = 500 ft

Pipe	1	2	3
h (ft)	25	0	70
$(2gDh/L)^{1/2}$	0.579	0	0.817
V (fps)	4.51	0	5.98
Q = AV, cfs	5.53	0	2.09

$$V = -2\sqrt{\frac{2gDh_f}{L}}\log\left(\frac{e/D}{3.7}+\frac{2.51\nu}{D}\sqrt{\frac{L}{2gDh_f}}\right)$$

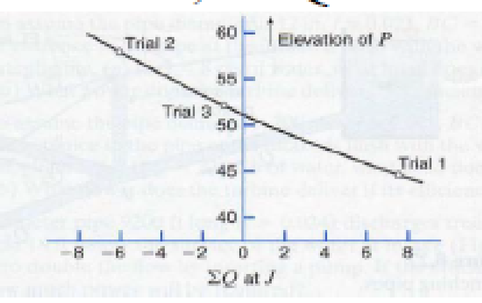
At J, Σ Q = 5.53- 2.09 = 3.44 cfs. P must be moved up.

Example 3 cont.

(a) Assume P = 510 ft

Pipe	1	2	3
h (ft)	15	10	80
$(2gDh/L)^{1/2}$	0.449	0.598	0.874
V (fps)	3.46	4.49	6.41
Check R	355,000	307,000	351,000
Q = AV, cfs	4.24	2.42	2.24

At J, Σ Q = 4.24 - 2.42 - 2.24 = -0.42 cfs.   P must be moved down



$$\frac{510 - Elev.P}{510 - 500} = \frac{0.42}{0.42 + 3.44'}$$

$$Elev.P = 508.91\,ft$$

Example 3 cont.

(a) Assume P = 508.9 ft

Pipe	1	2	3
h (ft)	16.1	8.9	78.9
$(2gDh/L)^{1/2}$	0.465	0.564	0.868
V (fps)	3.59	4.19	6.36
Check R	339,000	287,000	348,000
Q = AV, cfs	4.40	2.28	2.22